

Realistic Modeling of Microwave Instability Effects on the Evolution of the Beam Energy-Phase Distribution in Proton Synchrotrons

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Introduction

Either bunched or coasting beam in a synchrotron may exhibit microwave instability if the momentum spread is less than^[1]

$$\left(\frac{\Delta p}{p}\right)_{\text{FWHM}}^{(\text{th})} = \sqrt{\frac{e\hat{I}(Z/n)}{F\beta^2 E|\eta|}}, \quad (1)$$

where \hat{I} is the peak current, (Z/n) is the longitudinal coupling impedance at the n -th harmonic of the beam circulation frequency divided by the harmonic number n , F is a form factor of order 1 accounting for the particle distribution (including whether bunched or not), β is v/c , E is the total energy of the beam particles, η is the time dispersion $\gamma_T^{-2} - \gamma^{-2}$; γ 's are E/m_0c^2 and T subscript labels transition energy. A useful physical picture is that beam particles are captured in buckets generated by the beam image current flowing in the longitudinal coupling impedance. Qualitatively, trapping and auto-deceleration occur when the height of the buckets exceed the FWHM energy spread of the beam. "Microwave instability" implies in addition that the coupling impedance is largest at several times the rf frequency and that the decay of the wakefield is fast enough that bunches do not affect each other. This high frequency, low Q impedance is represented in the reported work by a single resonance at 1.7 GHz (approximately microwave cutoff of vacuum chamber), $Q=1$, and $R_{\text{shunt}} = Z$. The parameters used in this paper are influenced by the Fermilab Main Ring and design of the Main Injector.^[2]

The numerical modeling uses standard features of the code ESME.^{[3],[4]} In most of the reported simulations $2 \cdot 10^4$ macroparticles and 32 values of n separated by 1113 provide the current spectrum; the justification for these choices is given in ref. [5] which gives some detail on methods. Microwave instability may be an intensity limitation during parts of the acceleration cycle where the beam is debunched or loosely bunched, perhaps at injection or high duty factor extraction. Probably of more general importance is the time near transition when the spread in circulation frequency is sharply reduced, i.e., when $\eta \approx 0$. Concrete examples are given below.

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Coasting Beam

The continuous coasting beam case involves a single time scale, the growth time. It has therefore reasonably simple evolution and is tractable analytically. However, even in this case the early decay of the highest frequency modes and the absence of a dominant mode or band of modes is difficult to calculate in detail. Because no one amplitude grows steadily, the growth time is defined here as the time required for the momentum spread to grow to the threshold value. This time is plotted against $Z/Z_{\text{th}} - 1$ for a 0.5 A beam with $\Delta E/E$ (FWHM) = ± 13.28 MeV in a parabolic energy distribution at 150 GeV with $\eta = 0.0028$ in fig. 1. The fit is second order; both linear and quadratic terms are important. The instability saturates short of the so-called "overshoot" value

$$(\Delta p/p)_{\text{in}}(\Delta p/p)_{\text{fin}} = (\Delta p/p)_{\text{th}}^2. \quad (2)$$

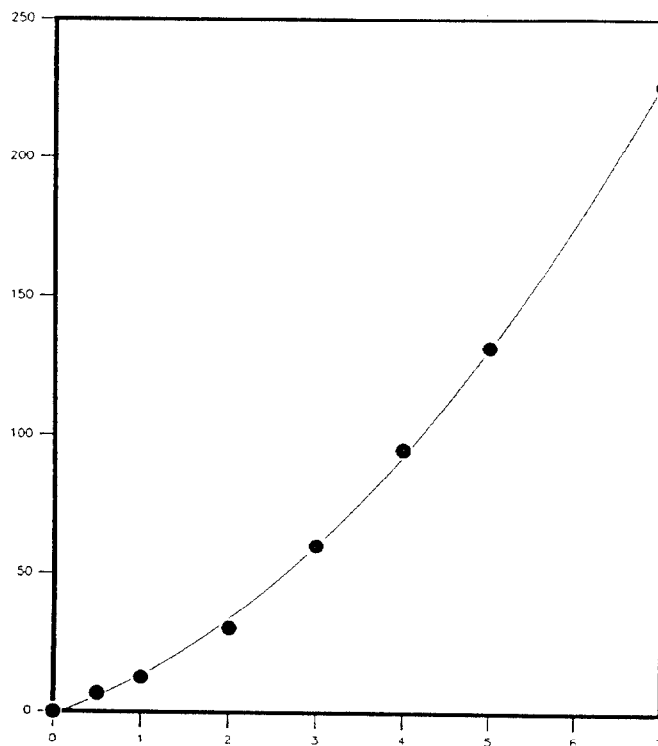


Figure 1: Rate of growth of coasting beam instability to threshold $[s^{-1}]$ vs. $Z/Z_{\text{th}} - 1$

Debunching with RF Off

If the rf voltage is removed at fixed momentum, instability appears when the bunches shear to the point that local momentum spread drops below threshold. Note from the phasespace plot in fig. 2 that the total Δp of the bunch is not what determines stability. Note also that the instability proceeds independently for each bunch even after overlap, a result predicted by Ng.^[6] There is a growth time and a debunching time dependence in this example; therefore, the dependence of the blowup on parameters is more complex.

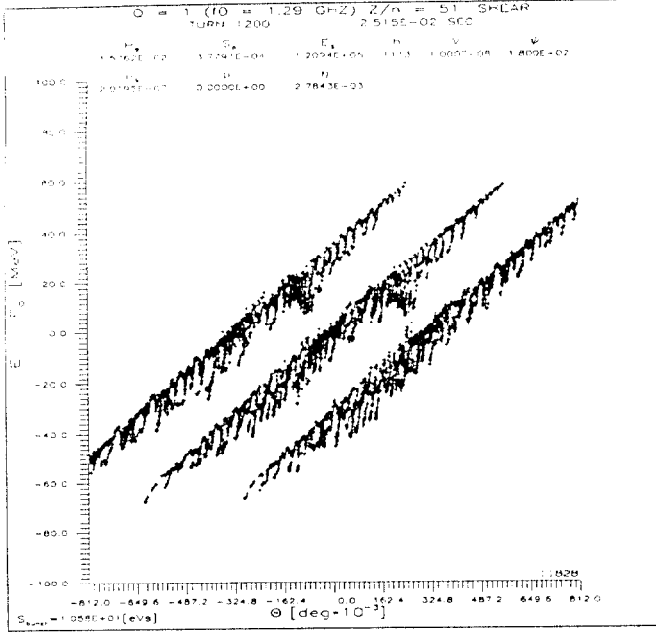


Figure 2: RF off debunching — Main Ring, $Z/n = 51\Omega$

Bunched Beam

For bunched beam the time scales are set by growth time and the synchrotron period. The long term evolution of the instability has two distinct courses depending on whether or not the instability is strong enough to decelerate beam out of the bucket. Fig. 3 illustrates a stage of the latter course. If the voltage is being reduced for adiabatic debunching, the voltage reduction is so slow that the threshold momentum spread gives a good estimate of the minimum attainable.

Transition

Although the momentum spread is generally close to its maximum for the cycle near transition energy, the threshold criterion eq. 1 predicts instability as $\eta \rightarrow 0$. The dependence of orbit length on momentum difference may be written

$$\Delta C/C = (\alpha_0 + \alpha_1 \delta + \dots) \delta, \quad (3)$$

where δ is $\Delta p/p$ and $\alpha_0 = \gamma_T^{-2}$. To first order the momentum dependence of η is

$$\eta = \eta_0 + \eta_1 \delta \approx \alpha_0 - \gamma^{-2} + (\alpha_1 + 3\alpha_0/2) \delta. \quad (4)$$

With parameters similar to the Main Ring ($\gamma_T = 18.75$, $\alpha_1 = .005689$, $Z/n = 9\Omega$, $\delta = 0.34\%$) the coasting beam threshold is ~ 0.75 A, and emittance growth about one percent is evident in 10 ms at 1.5 times threshold.

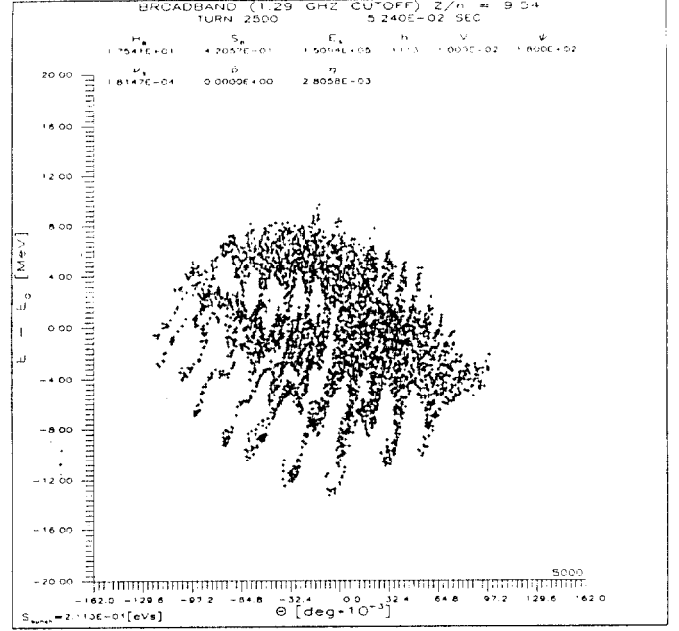


Figure 3: 53 MHz bunch in Main Ring — $2 \cdot 10^{10}$ proton, $Z/n = 9.54\Omega$, $f_{co} = 1.3$ GHz, $V_{rf} = 10$ kV

By reversing the sign of α_1 one meets the condition $\alpha_1 = -2\alpha_0$ which makes the lattice isochronous through second order at transition. Figure 4 shows the growth of $\Delta \epsilon_1/\epsilon_1$ for coasting beam in 0.0119 s at Main Ring transition with α_1 set for isochronicity. With the sign of α_1 reversed growth is not visible on this scale below 2 A.

The practical questions for transition crossing are what is the emittance blowup and what parameter choices minimize the blowup or beam loss for a given beam current. Figure 5 shows one cut through parameter space for a so-called duck-under scheme specifying the rf voltage in the transition region. The plot shows growth of an initial 0.1 eVs bunch during transition crossing *vs.* \hat{I} in the range 0. to 4.57 A for an accelerator like the Main Injector ($\gamma_T = 20.4$, $\alpha_1 = 0.$) except that the ramp is slower ($\dot{\gamma} = 107$) and Z/n is higher (10Ω). At 7.1 A there is 6% beam loss making emittance incomparable.

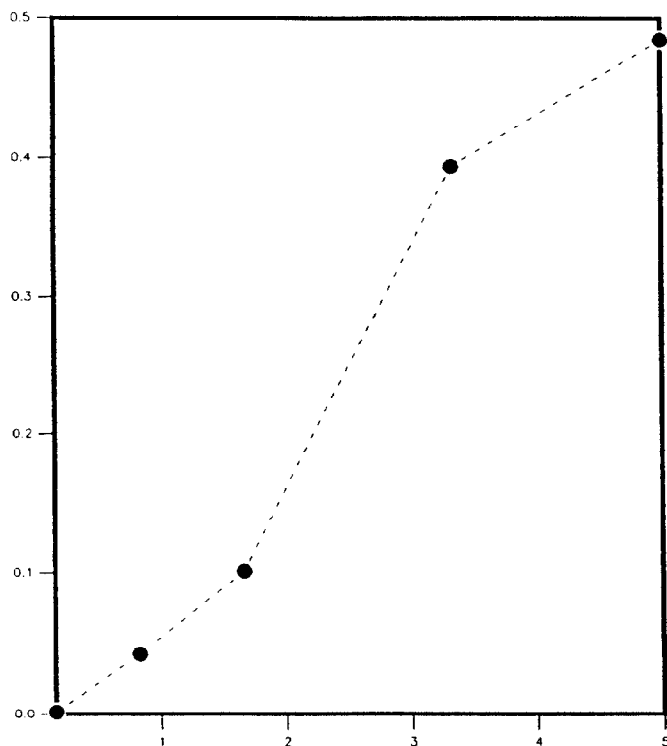


Figure 4: Growth of $\Delta\epsilon_l/\epsilon_l$ for Main Ring coasting beam at transition with $\alpha_1 = -2\alpha_0$ for isochronous orbit vs. number of protons per bunch [10^{10}]

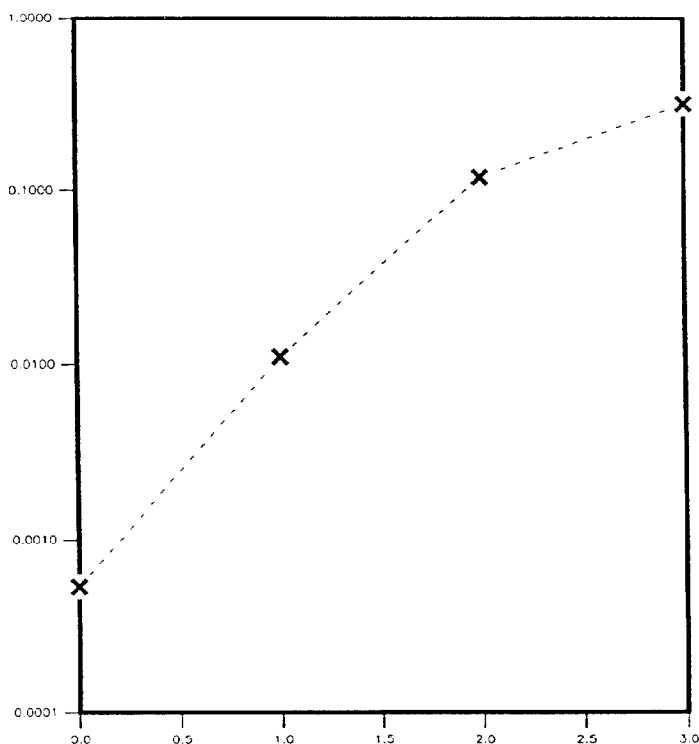


Figure 5: $\Delta\epsilon_l/\epsilon_l$ [eVs] for "duck-under" transition crossing of 0.1 eVs bunch in modified Main Injector vs. number of protons per bunch

Remarks

Some situations in which microwave instability can set the effective limit to accelerator performance have been examined by particle tracking simulation. The emphasis has been on the practicability of testing alternative parameters by numerical experiment for processes with multiple time scales which would not be tractable analytically. It is not possible to exhibit here the evolution of the instability to lend intuitive support for the interpretations offered nor to make thorough comparison with analytic results. Little space has been devoted either to fundamental questions of dynamics or details of numerical method. These matters are treated more fully in ref. [5] by MacLachlan, Kourbanis, Ng, and Wei. The modeling approach may have practical benefit if it leads designers to evaluate the overall growth of emittance in a process instead of considering only the instability threshold.

References

- [1] A. Hofmann, "Longitudinal Instabilities" in "Theoretical Aspects of the Behavior of Beams in Accelerators and Storage Rings", CERN 77-13 (19 July 77) p139ff
- [2] "Fermilab Upgrade: Main Injector Conceptual Design Report", Rev. 2.3, Fermilab (April 90)
- [3] J. A. MacLachlan, "Fundamentals of Particle Tracking for the Longitudinal Projection of Beam Phase-space in Synchrotrons", Fermilab note FN-481 (15 April 90)
- [4] S. Stahl and J. MacLachlan, "User's Guide to ESME v. 7.1", Fermilab internal note TM-1650 (26 Feb. 90)
- [5] Proceedings of the Fermilab III Instabilities Workshop", Fermilab internal note TM-1696 (Dec. 90)
- [6] K.-Y. Ng, "Microwave Instability for Overlapped Bunches", Fermilab note FN-432 (May 86)