Simulation and Stability of a Crab Cavity *

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Abstract

A feasibility study of the stability of a superconducting crab cavity for a B Factory was carried out. The study was done for the two trapped modes: the longitudinal TM010 and transverse TE111. It was found that for both modes the voltage induced by the beam bunches will be less than ±20 KV for the TM010 mode and ± 1KV for the TE111. Using the ZAP code it is shown that a stable frequency regime for the TM010 mode can be found in which the worse growth time is 8 times larger than the radiation damping time. For the TE111 mode the high energy ring is stable while the low energy ring will have to be stabilized by the feedback system.

1 INTRODUCTION

The Cornell B factory design employs a ±12.5 mrad horizontal crossing angle at the interaction point. The undesirable side effects of this crossing angle will be compensated by 'crabbing' [1, 2] the beams with a pair of single cell superconducting RF cavities operating in a deflecting mode. The purpose of the crab cavity is to achieve a 2 MV transverse kick by the TM110 mode at the same frequency as the accelerating cavity *i.e.*, 500 MHz as described in references [3, 4]. The crab cell design allows all modes higher in frequency than the crabbing mode to propagate out the beam pipe and be damped outside the cryostat with Ferrite-50[†] absorbers [5] located on the beam pipe . Four unwanted modes remain trapped in the cell region: the fundamental TM010 mode, two polarizations of the TE111 mode, and one polarization of TM110 mode. A study of the first two unwanted modes is being carried out with this paper giving some of the initial results.

2 $TM010$ MODE

The TM010 mode in the crab cavity is an undesirable longitudinal mode. (since the purpose of the cavity is to create a transverse kick). Its resonant frequency is 367 MHz with R/Q = 87 Ω/cell.

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[†]Product of Trans-Tech, Inc., Adamstown, MD.

2.1 *Induced Voltage*

To study the induced voltage developed in the cavity by the beam bunches, the cavity is simulated by an impedance $Z_c(\omega)$ of a shunt RLC resonant circuit with the appropriate resonance frequency ω_r and quality factor Q obtained by the code URMEL.

$$Z_c(\omega) = \frac{R}{1 + jQ\left(\frac{\omega}{\omega_r} - \frac{\omega_r}{\omega}\right)} \tag{1}$$

For the sake of simplicity, the bunched current is simulated by a train of point charges (having no synchrotron motion) separated by ΔT_b seconds between two charges and circulating around the ring with a period of T.

$$I(t) = I_o \sum_{m=0}^{N_b-1} \left\{ \sum_{n=-\infty}^{\infty} \delta(t - m\Delta T_b - nT) \right\} \tag{2}$$

where N_b is the number of bunches. Taking a Fourier transform gives the current in the frequency domain:

$$I(\omega) = \frac{1 - e^{-j\omega\,\Delta T_b\,N_b}}{1 - e^{-j\omega\,\Delta T_b}}\omega_o \cdot \left\{ \delta(\omega) + 2\sum_{n=1}^{\infty} \delta(\omega - n\omega_o) \right\} \tag{3}$$

The induced voltage developed on the cavity will be:

$$V(\omega) = I(\omega) \cdot Z_c(\omega) \tag{4}$$

or by taking the inverse Fourier transform:

$$V(t) = \frac{\omega_o R}{\pi} N_b \frac{1}{1 - j2Q} + \frac{2\omega_o R}{\pi} \sum_{n=1}^{\infty} \left\{ \frac{e^{jn\omega_o t}}{1 + jQ\left(\frac{n\omega_o}{\omega_r} - \frac{\omega_r}{n\omega_o}\right)} \cdot \frac{1 - e^{-j\omega_o\,\Delta T_b\,N_b}}{1 - e^{-j\omega\,\Delta T_b}} \right\} \tag{5}$$

The envelope of the induced voltage of this TM010 mode with ΔT_b=5 RF buckets=10 nsec and revolution frequency f_o=391.97 KHz is plotted in Figure 1. Note that the voltage is plotted at the time of arrival of each bunch giving only the envelope and thus the 367 MHz mode resonance

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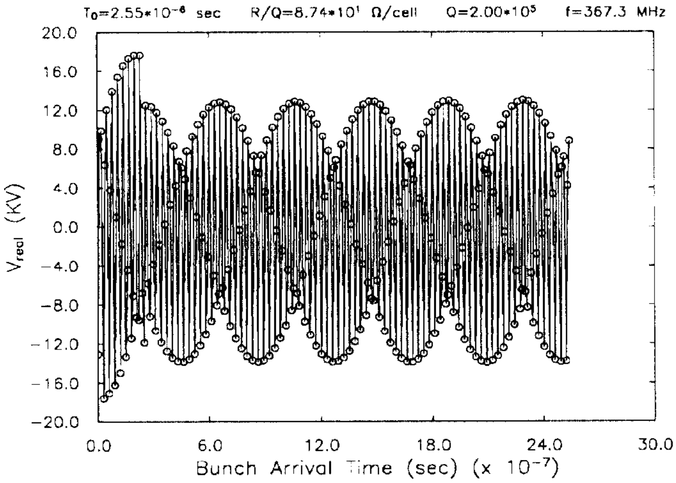

Figure 1: Voltage induced in the TM010 mode during one period

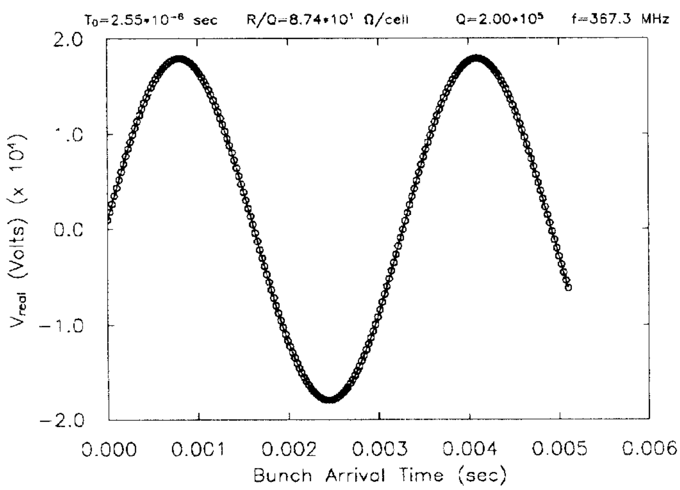

Figure 2: Induced Voltage seen at the gap during a thousand periods

frequency is not seen. Also note that since the spacing between the bunches is a multiple of the accelerating frequency *i.e.*, 500 MHz, and the longitudinal mode of the crab is oscillating at 367 MHz, the bunches will arrive at the cavity with different phases and as a result some of them will be accelerated and some decelerated causing to an irregular envelope of the built-up voltage. The amplitude of the induced voltage at small values of 't' is higher due to the gap of 151.2 nsec. at the end of the bunch train. Figure 2 shows the voltage seen at the gap during one thousand revolutions and Figure 3 shows the dependence of the induced voltage on the quality factor.

The amplitude of the induce voltage is smaller than ± 20 KV which is less than 2% of the driving voltage and can be neglected.

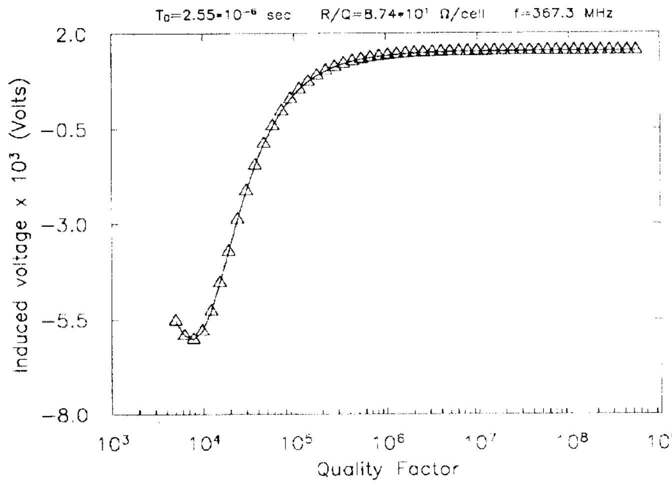

Figure 3: Induced voltage as function of quality factor

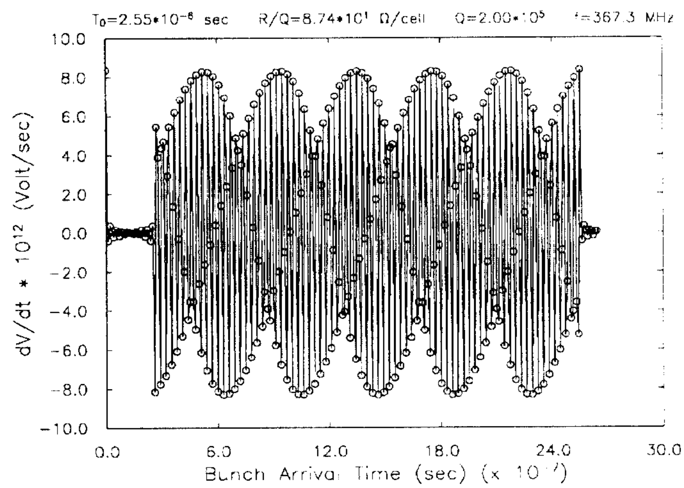

Figure 4: The derivative of the induced voltage as seen by the bunches in one period

2.2 *Bunch Lengthening*

The other concern is a possible bunch lengthening due to the slope of the voltage developed in this TM010 mode. Figure 4 shows the slope of the voltage at the time of bunch arrival during one revolution obtained numerically from the simulation described in the previous section. This time of arrival would be different for each bunch if synchrotron motion had been in the bunch train. Since we are not interested in the slope of a specific individual bunch but in the range of all possible slopes, the simulation result is still valid. The slope obtained from tracking a bunch for 1000 revolutions is less than 10^{13} V/sec compared with a slope of 10^{16} V/sec from the main accelerating cavities so the perturbation in bunch length is negligible.

2.3 *Stability*

The longitudinal coupled bunch instability can be effectively controlled by tuning the resonant frequency of the

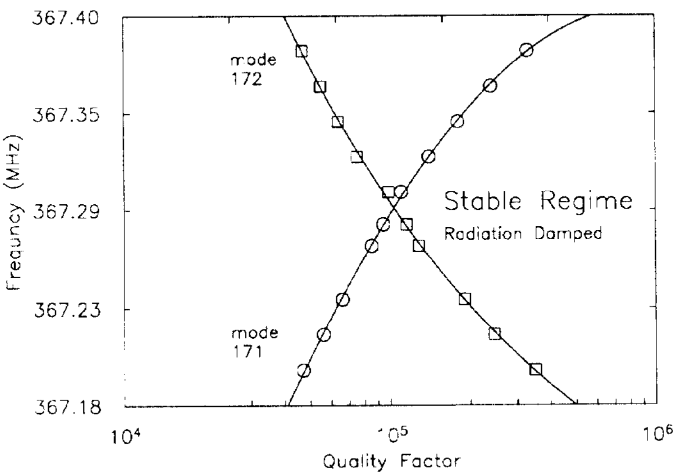

Figure 5: The stable frequency regime with growth time higher than 100 msec, in the TM010 mode as function of quality factor, for the low energy ring

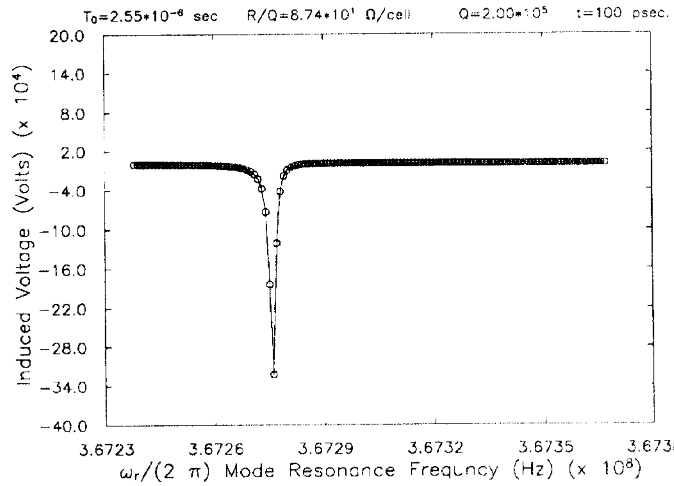

Figure 6: Change of the induced voltage due to resonance mode tuning.

TM010 mode. A range of frequencies for which the beam will be either stable or Landau damped is searched for using the program ZAP [6]. The program calculates the frequency shift and growth time of all modes using the Wang formalism for electrons in gaussian bunches. For the low energy ring (3.5 GeV and $1.37 \cdot 10^{11}$ electrons/bunch) bunch modes 171 and 172 have the fastest growth time (few msec) and their synchrotron frequency shift is too large (1000 Hz at $Q=5 \cdot 10^3$) to use Landau damping. If we allow higher values of quality factor Q the bunch mode growth time can be increased by controlling if the mode frequency to allow damping by synchrotron radiation. Figure 5 shows the stable frequency regime as function of Q for which the worse growth time is higher than 100 msec which is 8 times the radiation damping time in the CESR-B low energy ring. It can be seen, for example, that for $Q=2 \cdot 10^5$ the stable frequency range is from 367.235 MHz to 367.347 MHz, while the frequency range is zero for $Q=1 \cdot 10^5$.

The change of the induced voltage during the tuning of the resonance frequency of the resonator using equation 5 can be seen in figure 6. The strong dip in the voltage occurs when the harmonic revolution frequency falls on the resonance frequency. The width of the voltage dip is very narrow due to the small frequency band of the mode resonance (1.8 KHz). In the high energy ring (8 GeV) the number of particles per bunch is smaller ($0.6 \cdot 10^{11}$) making the stability demands less severe. Bunch modes 85 and 86 have the shortest growth time but they will also be radiation damped for $Q=2 \cdot 10^5$.

3 $TE111$ MODE

A similar study done for the parasitic transverse mode with a resonance frequency of 484.7 MHz and transverse R_T/Q of 5.5 $\Omega/cell$ (obtained from the code URMEL) gives an induced voltage smaller than 1 KV. Assuming a tune spread of $2 \cdot 10^{-5}$, the beam in the high energy ring is stable with a frequency shift of less than 10 Hz and a growth time longer than 47 msec. For the low energy ring the growth time is greater than 20 msec but the beam is unstable and will have to be stabilized by the transverse feedback system.

4 *Conclusions*

By making the quality factor in the TM010 mode higher than $2 \cdot 10^5$,and by tuning its resonant frequency it is possible to make the growth time of the unstable modes to be greater than the radiation damping time. By avoiding the resonant frequency of 367.277 MHz (where it falls on the harmonic revolution frequency) the induced voltage is less than 20 KV and the bunch length distortion is negligible. In the transverse mode TE111 the high energy ring is stable while the low energy ring must be stabilized by the transverse feedback system.

References

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