Head-Tail Stability and Linear Coupling in the Tevatron

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I. INTRODUCTION

During collider studies (1989-1990) it was observed that with large bunch intensities (>60×10^9 particles per bunch) in the Tevatron, the beam would go unstable if the machine ran close to the coupling resonance ($\nu_+ - \nu_- < 0.005$). Simple head-tail stability only requires the horizontal and vertical chromaticities (χ_x and χ_y) to be positive. Since the beam went unstable even when this condition was met, a set of experiments were performed which showed that if there is significant linear coupling the head-tail stability criterion is modified. In fact, it was observed that the Tevatron could operate stably with either the horizontal or vertical chromaticity negative if the machine was strongly coupled. Also, when the machine was strongly coupled, it was observed that with both the horizontal and vertical chromaticities positive, the beam in the Tevatron can become unstable. The purpose of this paper is to present a formalism for calculating head-tail stability. Then the predictions of the formalism are compared to data taken with the Tevatron.

II. THEORY

A. Head-Tail stability

A naive model of head-tail stability says that a bunch of sufficient intensity goes unstable when one of the chromaticities becomes negative - when

$$\chi_{(x,y)} \equiv \frac{d\nu_{(x,y)}}{d\delta} < 0.0 \tag{1}$$

Here $\delta = \Delta p/p$ is the off-momentum parameter, while ν_x and ν_y are the horizontal and vertical betatron tunes. This is an approximation referring only to the lowest order head-tail mode. Nonetheless, it is a useful rule of thumb that is adequate for most practical situations.

Without proof, we conjecture that a similar expression holds when it is not possible to implicitly ignore the effects of residual linear coupling - due to solenoidal fields, rotated quadrupoles, and displaced sextupoles, et cetera. The uncoupled tunes are simply replaced by the coupled eigentunes,

$$\chi_\pm \equiv \frac{d\nu_\pm}{d\delta} < 0.0 \tag{2}$$

so that instability is now expected when one of the "eigenchromaticities", χ_+ or χ_-, goes negative.

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B. Off Momentum Coupling Parameterization

The eigentunes of on-energy ($\delta = 0$) particles are conveniently parameterized by

$$\nu_\pm = \frac{1}{2} \{ Q_x + Q_y \pm [(Q_x - Q_y)^2 + q^2]^{1/2} \} \tag{3}$$

where $\mathbf{q}$ is the two dimensional "coupling vector"[1,2]. In the limit of negligible coupling,

$$| Q_x - Q_y | >> |\mathbf{q}| \tag{4}$$

the eigentunes become the uncoupled tunes, Q_x and Q_y. In the limit of full coupling, when the uncoupled tunes are equal, the eigentunes have their closest approach,

$$\nu_\pm = Q_0 \pm \frac{1}{2}|\mathbf{q}| \tag{5}$$

with a total separation of the length of the coupling vector. Three substitutions are made to generalize equation (3) to describe off-momentum behavior. They are

$$\begin{aligned} Q_x &\rightarrow Q_x + \chi_x \delta \\ Q_y &\rightarrow Q_y + \chi_y \delta \\ \mathbf{q} &\rightarrow \mathbf{q} + \mathbf{k}\delta \end{aligned} \tag{6}$$

where $\mathbf{k}$ may be called the "chromatic coupling vector". The general parameterization now becomes [3]

$$\nu_\pm(\delta) = \frac{1}{2} \Big\{ (Q_x + Q_y) + (\chi_x + \chi_y)\,\delta \pm [(\{Q_x - Q_y\} + \{\chi_x - \chi_y\}\delta)^2 + (\mathbf{q} + \mathbf{k}\,\delta)^2]^{1/2} \Big\} \tag{7}$$

This incidentally suggests that it may not be possible for the eigentunes to become arbitrarily close, even if the unperturbed chromaticities are set to zero, the unperturbed tunes are equal, and the coupling vector $\mathbf{q}$ has been corrected to zero. The minimum difference is approximately

$$(\nu_+ - \nu_-)_{min} \approx |\mathbf{k}| \frac{\sigma_p}{p} \tag{8}$$

proportional to σ_p/p, the beams finite momentum spread.

C. Eigenchromaticities and stability

In principle the problem is now solved, since it is formally possible to differentiate equation (7) with respect to δ, and then impose the constraints of equation (2). This is rather messy and unilluminating. Instead, consider the following practical problem: if the chromaticities are χ_x and χ_y when Q_x and Q_y are far apart, will the beams be stable when the

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tunes are brought together? When $Q_x = Q_y = Q_0$, equation (7) becomes

$$\nu_\pm(\delta) = Q_0 + \frac{1}{2}(\chi_x + \chi_y)\,\delta \pm \frac{1}{2}[\,\{\chi_x - \chi_y\}^2\delta^2 + (\mathbf{q} + \mathbf{k}\,\delta)^2\,]^{1/2} \tag{9}$$

Now consider two cases - when the on-energy lattice has been perfectly decoupled, so that $\mathbf{q} = 0$, and when δ is small.

In the case of perfect decoupling the eigenchromaticities are

$$\chi_\pm = \frac{1}{2}(\chi_x + \chi_y) \pm \frac{1}{2}[(\chi_x - \chi_y)^2 + \mathbf{k}^2]^{1/2} \tag{10}$$

and equation (2) reduces to a single stability criterion

$$\chi_x\,\chi_y > \frac{\mathbf{k}^2}{4} \tag{11}$$

This shows that the area of stability in the upper right quadrant of (χ_x,χ_y) space shrinks significantly if $\mathbf{k}$ is large. Instability near the origin of this space has been observed in the Tevatron, when the unperturbed tunes are brought together.

In the second case, expanding (9) to first order in δ gives

$$\chi_\pm(\delta) = \frac{1}{2}[\,\chi_x + \chi_y \pm \frac{\mathbf{k}\cdot\mathbf{q}}{|\mathbf{q}|}\,] + O(\delta) \tag{12}$$

and the stability condition becomes

$$\chi_x + \chi_y > \frac{|\mathbf{k}\cdot\mathbf{q}|}{|\mathbf{q}|} \tag{13}$$

This is less restrictive than equation (11), since the boundary line is no longer a hyperbola, but a straight line entering three of four quadrants in (χ_x,χ_y) space. Instability has also been observed in Tevatron collider runs when the unperturbed tunes are <u>separated</u>, after the unperturbed chromaticities have been accidentally tuned so that one of them is large and positive, while the other is small and negative.

The apparent contradiction between equations (11) and (13) that arises when $\mathbf{q}$ is allowed to become small is resolved by properly considering the higher order term in equation (12). Note that both stability conditions (11) and (13) implicitly assume that it is the $\delta = 0$ eigenchromaticity that is important, in keeping with the spirit of equations (1) and (2).

III. EXPERIMENT

A. Setup

The Tevatron was operated with horizontal and vertical tunes of 20.408 and 20.419, respectively. The machine was decoupled using two somewhat orthogonal skew quadrupole circuits. The angel between the coupling vector associated with the two circuits was measured to be approximately 31°. The minimum tune split that could be achieved using these circuits was approximately 0.002, and 0.003 for uncoalesced and coalesced beam, respectively. High intensity beam was achieved by coalescing 11 bunches with approximately 8E9 particles/bunch into one bunch with intensity of approximately 80E9. The tune split $(\nu_+ - \nu_-)$ is adjusted using the correction quadrupole circuit Q_x, which in principle changes only the horizontal tune (ν_x). The chromaticity is adjusted using the chromaticity sextupole circuits (C_x,C_y). The chromaticity is measured by varying the RF frequency and measuring the change in tune. The chromaticity measurement is also made with $\nu_y - \nu_x > 0.01$. The tunes are measured by spectrum analysis of signals from horizontal and vertical Schottky plates[4].

B. Part I

A high intensity proton bunch (>60E9) was injected into the Tevatron. The chromaticity was set such that $\chi_x = \chi_y = \chi_0$. Then, the Q_x circuit was slowly varied to reduce the tune split from 0.010 to 0.003. The above procedure was repeated for several values of χ_0 (4,3,2, and 1 respectively). For χ_0 less than or equal to 2 units, the beam went unstable when Q_x was set to minimize the tune split (0.003). Substituting this result into equation 11 gives.

$$k \leq 4.$$

Point A in figure 1 shows our result. Equation 13 implies that the cross hatched area for $\chi_x > 0$ and $\chi_y > 0$ will be unstable when the tune split is small (machine is coupled) and stable when the tune split is large (machine is decoupled).

Next, the Q_x circuit was used to place the tunes on top of one another (minimum tune split 0.003). χ_x and χ_y were set to 8 and -3 units, respectively, and the machine was operating stably at point B in figure 1. Then the Q_x circuit was used to separate the tunes. When the tune split was about 0.01 the vertical tune line on the spectrum analyzer went coherent (became very narrow) and total beam loss occurred in less than a second (i.e. beam went unstable).

Point B in figure 1 shows our result, and implies that the cross hatched region , excluding the first quadrant, will be stable when the beam is coupled and unstable when the beam is decoupled.

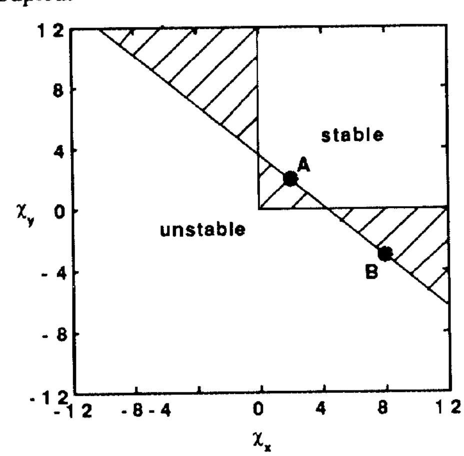

Figure 1: χ_x verses χ_y stability curve. The points A and B are either stable or unstable operating points as demonstrated in Part I. The cross hatched region is also believed to be either stable or unstable depending the amount of coupling.

C. Part II

The second part of the study used 20 uncoalesced bunches with approximately 8E9 particles/bunch in the Tevatron. Next the momentum (δ=dp/p) of the Tevatron was varied by changing the RF frequency. The eigentunes ($\nu_\pm$) were measured as a function of δ for three different settings of the Q_x circuit. The results of these scans are shown in figure 4. The sums and differences of the eigentunes (figures 2 and 3, respectively) were used to fit for the parameters in equation 7. The interesting results of the fits are that χ_x,χ_y, and q vary with Q_x settings (see table 1). Any attempt to hold these parameters constant caused the fit to become rather poor for δ>0. Also, a fit was done excluding the k term in equation 7. The results of the fit again required χ_x, and χ_y to vary with Q_x while q remained constant at approximately 6.0E-3. One justification for including the k term in the fit was that it keeps the value of q consistent with the minimum tune split measurement (q<3.0E-3).

Table 1: Fitted Parameters

ΔQ_x circuit	.0000	.0030	.0060	Average value	σ
	fitted parameters				
Q_x	.4081	.4103	.4125		
Q_Y	.4183	.4189	.4197	.4190	.0007
χ_x	8.2	7.0	5.0	6.7	1.6
χ_y	-2.6	-1.8	-0.9	-1.8	1.0
q	.0041	.0030	.0025	.0032	0.0008
cos(**k**·**q**)	1	1	1	1.11	0.07
k	3.6	3.9	4.0	3.8	0.2

D. Conclusion

There are two measurements that require the k term in equation 7. First, the unstable region in the first quadrant of figure 1 cannot be explained without including the k term. Second, the fit to the $\nu_\pm$ vs δ was not consistent with the measured minimum tune shift if the **k** term was ignored.

IV. REFERENCES

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[4] D.Martin, "A Resonant Beam Detector for the Tevatron Tune Monitoring", FERMILAB-Conf-89/74.

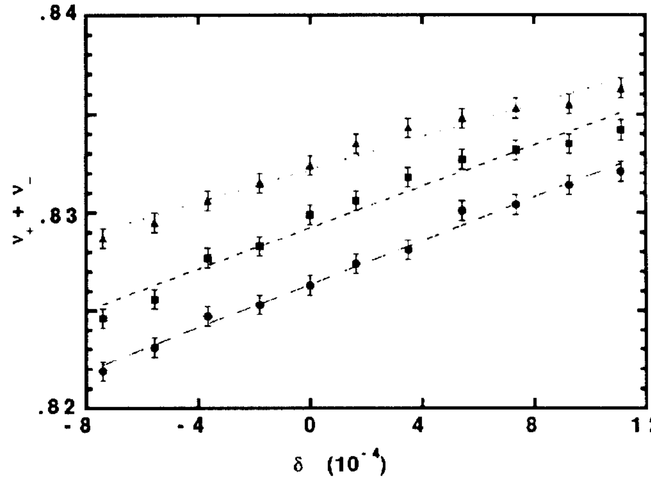

Figure 2: $\nu_+ + \nu_-$ verses δ - the curves are the best fit of the data to a straight line.

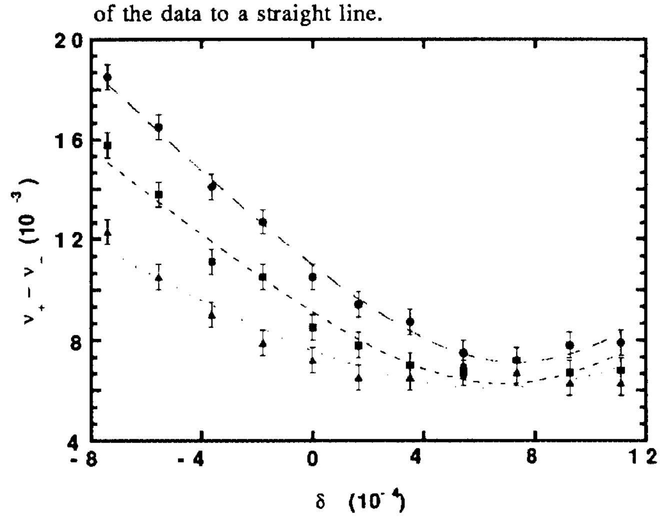

Figure 3: $\nu_+ - \nu_-$ verses δ - the curves are the best fit to the data. Table 1 gives the fit parameters.

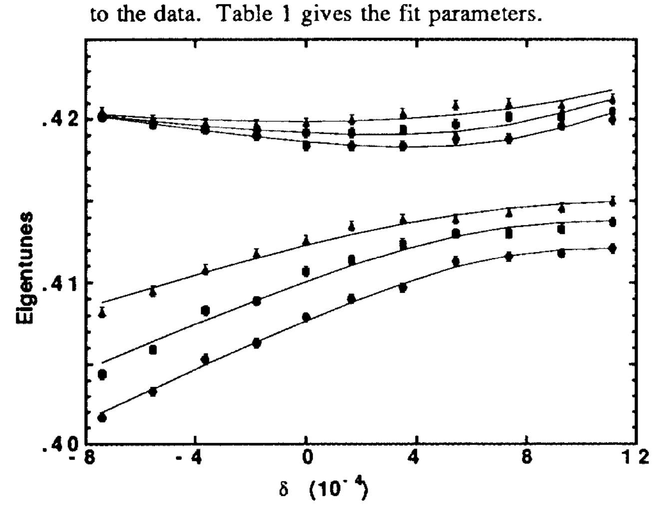

Figure 4: Eigentune verses δ - the points are the measurements and the curves are equation 7 with the parameters from table 1.