ION TRAPPING IN THE CESR B–FACTORY*

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Abstract

Analysis of the ion trapping expected for the Cornell B–factory shows that the presence of a 10% gap in the bunch train will be very effective in reducing the ultimate ion density. With a gap the average ion lifetime is seen from simulations to be under 10^{-5} seconds or about a few turns. While the average lifetime is short both simulation and analysis show that there exists near the center of the beam a small core of long lived ions. The resulting ion density distribution is then substantially different from the (gaussian) beam distribution.

Introduction

The phenomena of ion trapping by an electron beam has appeared in the majority of electron storage rings and there is no reason to suppose that the planned B–factory planned at Cornell, CESR–B, will be free of this problem. Indeed the large time averaged electron beam densities that will be present in CESR–B will produce unacceptably large ion densities unless the average ion lifetimes are kept to of order 10^{-5} seconds or less. The use of ion stripping electrodes in this case is not effective since the ions would not have enough time to migrate to the electrodes. In order to keep the ion density at a reasonable value it is necessary to introduce a gap in the bunch train in which a number of consecutive bunches are empty. The use of a gap has been used previously giving a significant enhancement in performance [1]. In CESR–B a gap is especially effective in clearing ions in consequence of the large velocities attained by the ions due to the large beam current.

Ion Stability and Density

Given a bunch train where a number of consecutive bunches are empty if the distance between bunches is small then the condition for linear stability in the horizontal (x) or vertical (y) plane is [4]

$$0 \le \theta_u \ (\mathrm{mod}\ \pi) \le \theta_{uc} \quad u = x \text{ or } y\,, \tag{1}$$

$$\theta_{uc} \equiv 2\tan^{-1}\left(\frac{2}{\omega_u t_{gap}}\right)\,, \tag{2}$$

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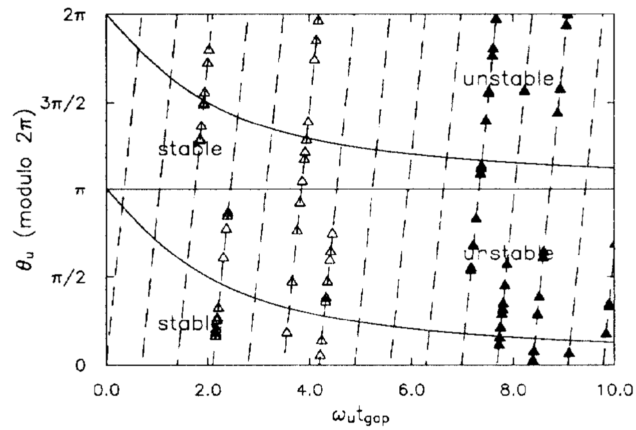

Figure 1: Stability region as a function of θ_u and $\omega_u t_{gap}$. The closed triangles (vertical) and open triangles (horizontal) correspond to the operating points at 50 'random' places in a CESR–B lattice.

where t_{gap} is the period of the gap, ω_u is the oscillation frequency given by

$$\omega_u = \left(\frac{2N\,c\,r_p}{A(t_0 - t_{gap})\sigma_u(\sigma_x + \sigma_y)}\right)^{1/2}, \tag{3}$$

$\theta_u = \omega_u(t_0 - t_{gap})$ is the total 'rotation angle' of the ion during the non-gap period, A is the atomic weight of the ion, N is the number of relativistic electrons, t_0 is the bunch revolution period, and σ_x and σ_y are the beam sigmas.

Ion stability as a function of θ_u and $\omega_u t_{gap}$ is shown in figure 1. With a 10% gap, an ion's 'operating point' will lie upon one of the dashed lines in figure 1. In the limit $\omega_u t_{gap} \to 0$ the ion is unconditionally stable while for $\omega_u t_{gap} \to \infty$ the width of a stability region decreases as $\delta\theta_u^{stable} \approx 4/\omega_u t_{gap}$. For $\omega_u t_{gap} \gtrsim 1$ the ratio of the width of the stable region $\delta\omega_u^{stable}$ to ω_u and the ratio of the width of the stable region to the distance between stable regions $\Delta\omega_u$ $(= \pi)$ is given by [4]

$$\frac{\delta\omega_u^{stable}}{\omega_u} \approx \frac{4}{\omega_u(t_0 - t_{gap})\omega_u t_{gap}}\,, \tag{4}$$

$$\frac{\delta\omega_u^{stable}}{\Delta\omega_u} \approx \frac{4}{\pi\omega_u t_{gap}}\,. \tag{5}$$

Table 1 gives values for ω_u, $\omega_u t_{gap}$, and θ_u. The values shown in the table were obtained using CESR–B parameters with a revolution time of $2.54 \cdot 10^{-6}$ sec, a beam

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u	ω_u (1/sec)	θ_u (rad)	$\omega_u t_{gap}$	% stable
x	$1.2 \cdot 10^7$	28	3.2	40%
y	$3.5 \cdot 10^7$	81	9.0	14%
x & y				6%

Table 1: Values for ω_u, θ_u, and $\omega_u t_{gap}$. The values were calculated for CO^+ using CESR–B parameters and a 10% gap

current of 2 Amps, a beam size of $\sigma_x \sim 1.6 \cdot 10^{-3}$ meters and $\sigma_y \sim 2.0 \cdot 10^{-4}$ meters, and a gap width of 10%. The values in table 1 are only approximate since variation in σ_x and σ_y from point to point in the ring will result in variation in ω_u and hence in variation of θ_u and $\omega_u t_{gap}$. Changes in ω_u will move the ion's operating point along a dashed line in figure 1. From equation (4) a 0.5% change in ω_y or a 4.5% change in ω_x is enough to destabilize an ion. In CESR–B the ions will be more or less uniformly distributed in a limited region of the stability diagram and thus the percentage of the ring that is stable can be calculated from equation (5). As tabulated in table 1, 40% of the ring is stable horizontally while 14% of the ring is stable vertically. Since the horizontal and vertical stable regions are uncorrelated, the percentage of the ring that is both horizontally and vertically stable is 6% (= 40% · 14%) with the length of a stable region being typically 0.4 meters. For CESR–B the above analysis is not effected by the dipole magnetic fields since the beam-ion force dominates the force due to the dipoles.

Besides the presence of the gap, a second major factor limiting the ion density is the nonlinearity of the transverse beam-ion force. This nonlinearity, in conjunction with the presence of a gap, produces destabilizing resonances. The widths of these resonances in frequency space are an increasing function of the ion's oscillation amplitude and for large enough amplitudes the ion motion will be unstable. Considering only cubic resonances the average amplitude beyond which the motion is unstable is given by [5]

$$A_x \simeq \sigma_x \sqrt{\frac{\delta\omega_x^{stable}}{\omega_x}}, \qquad (6)$$

$$A_y \simeq 2\sigma_y \sqrt{\frac{\delta\omega_y^{stable}}{\omega_y}}, \qquad (7)$$

Simulations show that for ions with amplitudes greater than the critical amplitude the resonance driven motion can quickly remove an ion from the vicinity of the beam.

The analysis of ion trapping using CESR–B conditions shows that one can naturally divide the ions into two classes depending upon their lifetimes. The ions with the shortest lifetimes, which will be referred to as 'background' ions, are the ions that are either born in a linearly unstable region of the ring or are born in a stable region with sufficient initial oscillation amplitude to have an unstable trajectory. Simulations have shown that for CO^+ the time that the background ions spend within 3σ of the beam is on average about $5 \cdot 10^{-6}$ seconds or about $2t_0$. The second class of ions are those ions that are born in a linearly stable region and have a small enough amplitude to not be destabilized by the nonlinearities in the beam-ion force. These ions will be referred to as 'core' ions since they only inhabit the core of the beam. Simulations show that with CESR–B conditions this core has a characteristic width of order $\sigma/10$ either horizontally or vertically. The core ions can be further subdivided into two classes depending upon whether or not the ion is near a minimum in the longitudinal potential produced by the longitudinal component of the beam-ion force [3]. Core ions near a longitudinal potential minimum will be called '(longitudinally) stable core' ions while the other core ions will be denoted as '(longitudinally) unstable core' ions.

The lifetime for the unstable core ions is determined by the time it takes the ions to be pulled into an unstable region by the longitudinal component of the beam-ion force. With CESR–B conditions the longitudinal potential is of the order of 5 eV/meter which, for CO^+, gives a lifetime for the unstable core ions of typically $1.2 \cdot 10^{-4}$ seconds. The total percentage of the ring inhabited by the unstable core ions is the 6% of the ring that is linearly stable. The integrated density of the unstable core ions integrated over the entire ring is then comparable to the integrated density of the background ions but the transverse distribution of the unstable core ions is, as explained above, quite different.

The stable core ions, on the other hand, have a lifetime determined by the time for double ionization since the upward frequency shift experienced by the ion upon further ionization will almost certainly make the ion unstable. With CESR–B conditions this translates into a lifetime of about $6 \cdot 10^{-4}$ seconds. The total percentage of the ring inhabited by the stable core ions is determined by the number of longitudinal potential minimums in the ring that also correspond to a linearly stable region. A calculation[4] shows that this number is about 1% so that the integrated density of the stable core ions is similar to the integrated density of the unstable core ions.

With a design pressure of $5 \cdot 10^{-9}$ Torr, an ion tracking simulation program was used to calculate the expected density distribution. The resulting density profiles along the x and y axes are shown in figure 2 along with the gaussian beam density profile for comparison. As shown in the figure, the large ion density near the beam center is indicative of the presence of the core ions. The long tail of the ion distribution is due to the ions that are 'streaming' towards the vacuum chamber walls.

Tune Shift

From the calculated ion density profile and an electron beam energy of 3.5 GeV it is possible to calculate the amplitude dependent tune shift felt by electrons of the beam due to the ions. In this case the vertical tune shift is greater

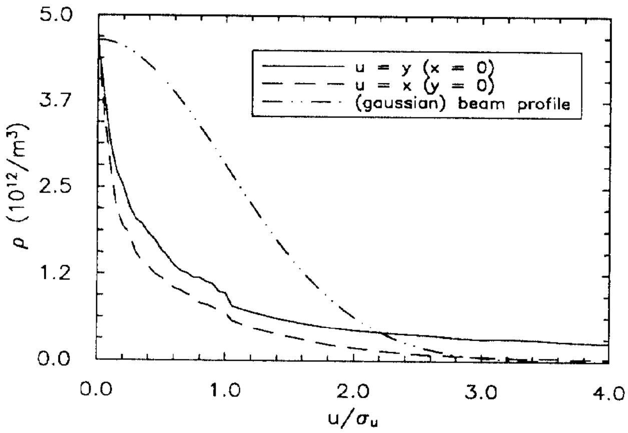

Figure 2: Ion density for a 10% gap. The gaussian beam profile shown for comparison has been normalized to be the ion density at $x = y = 0$

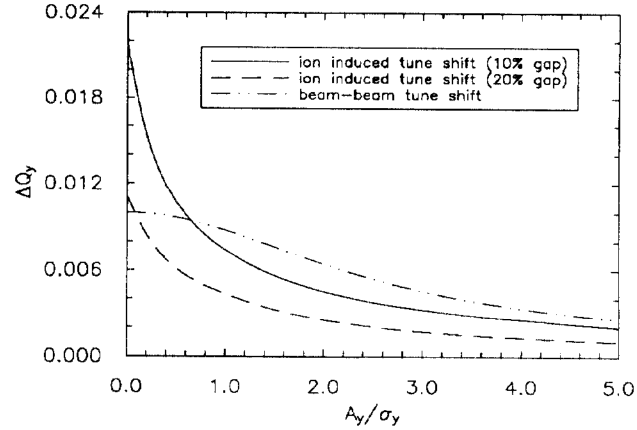

Figure 3: vertical tune shift vs. vertical amplitude

than the horizontal tune shift and so only the vertical tune shift needs to be considered. The vertical tune shift as a function of vertical amplitude is shown in figure 3. Also shown in the figure for comparison is the beam-beam tune shift scaled so that the zero amplitude tune shift is 0.01. As can be seen from the figure the ion induced tune shift falls off rapidly with increasing amplitude which is just a reflection of the large ion density near the center of the beam due to the core ions. This rapid fall off in tune shift is an advantage in terms of any incoherent instability since it is the electrons with amplitude of order the beam size or larger that will contribute to lifetime or beam blowup effects. Another consideration is that, unlike the beam-beam interaction, the ions are distributed around the ring so that the contribution of the ions to any beam resonances will be diminished due to phase averaging [2]. With the above considerations the ion induced tune shift is within tolerable levels in CESR–B.

The effect of increasing the gap to 20% is shown in figure 3. A 20% gap brings the tune shift well below the 0.01 beam-beam interaction tune shift for all amplitudes except at the lowest amplitudes. While the larger gap is effective in further suppressing the ions in terms of further increases in the gap size one is quickly reaching a region of diminishing returns since for a given current there is a minimum ion density that will accumulate regardless of the width of the gap. This can be seen by considering the fact that an ion will have stable oscillatory motion between the time of its creation and the time that the gap first passes by it. With CESR–B parameters this minimum density is about $3 \cdot 10^{11}/\mathrm{m}^3$ near the center of the beam.

Coherent Instabilities

Given a coherent normal mode of the beam-ion system, if any ions have a lifetime that is short compared to the growth time of the mode then the ions will not contribute to the growth of that mode. This is true since for mode growth there must be phase stability between ions and beam and the creation and destruction of ions will tend to randomize this phase. The background ions with their very short lifetimes will thus not contribute to coherent growth.

The analysis of the coherent interaction between the beam and the core ions is complicated by the time dependence in the beam-ion interaction introduced by the gap. There is also a further complication introduced by the differences in the density distributions between the core ions and the beam. An analysis of this situation[5] shows that for the dipole modes the effect of the gap is to increase the response of the ions to a given perturbation roughly in proportion to $\omega_u t_{gap}$. It can also be shown that for the dipole modes (the dipole modes are the most dangerous modes) the 'effective' ion density is the ion density that one would have if one took the core ions and spread them out over the whole beam. Taking the above into account the calculation of the effect of effect of Landau damping upon a mode can proceed upon standard lines [6]. With CESR–B parameters a calculation shows[5] that the dipole modes will be suppressed by Landau damping.

References

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