Studies of Coupled-Bunch Modes in the Fermilab Booster

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Abstract

Coupled-bunch instability and the resulting longitudinal emittance growth is a major limit to beam brightness in the Booster. The data show a strong correlation between various higher-order RF cavity modes and the growth. At present, both a beam feedback system [1] and higher-order RF mode dampers are being designed to control the longitudinal oscillations. A number of experimental studies have been done to characterize the instability; however, the relevant physical parameters prove difficult to measure. We are attempting, therefore, to approach the problem from both sides: experiment and theory. This paper describes modelling the coupled-bunch motion and calculating the instability growth rates using the measured longitudinal impedance. Also described are the dipole and higher-order form factors, which give the beam response as a function of frequency and as such, scale the effective electromagnetic "force" on the beam due to a given impedance. The results show a good correspondance with the data and will potentially impact feedback system and mode damper design.

I. Introduction

The Fermilab Booster [2] is an 8 GeV (kinetic), rapid-cycling proton synchrotron using 96 combined function magnets run from a 15 Hz resonant power supply system. Injection energy is 203 MeV. The RF system consists of 17 double-gap ferrite tuned cavities which sweep from 30 to 53 MHz during the 33 msec acceleration ramp. The total RF voltage is about 1 MV. Figure 1 shows a schematic of a Booster cavity.

At present, the maximum intensity in the Booster is 3.6E10 protons per bunch. While the Linac upgrade [3] should raise the space charge limit by a factor of three, beam brightness is expected to be limited by a long observed coupled-bunch instablity and resulting longitudinal emittance growth. Prior studies [4] have shown that the coupled-bunch motion is likely due to parasitic modes in the RF cavities.

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Spectral measurements have indicated unstable growth of oscillations centered around modes 16 and 36, out of 42 possible coupled-bunch modes (h=84.) This is shown in Figure 2 and reported elsewhere in this conference [5]. The growth rate and coupling impedances, important in specifying gain and bandwidth for a beam feedback system, are very difficult to determine experimentally. These were calculated, therefore, using a beam model and measured single-gap impedances of the higher-order RF cavity modes. The model and growth rate calculations are described below.
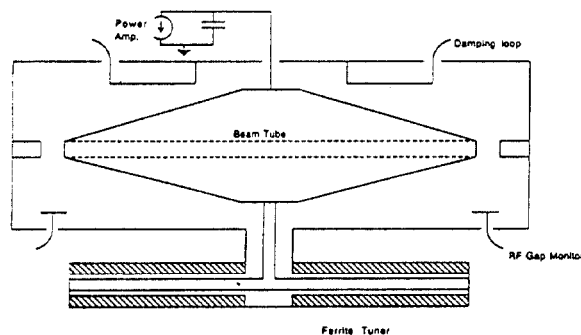

Figure 1. Booster RF cavity schematic. Not shown are two additional tuners located at the front and back.
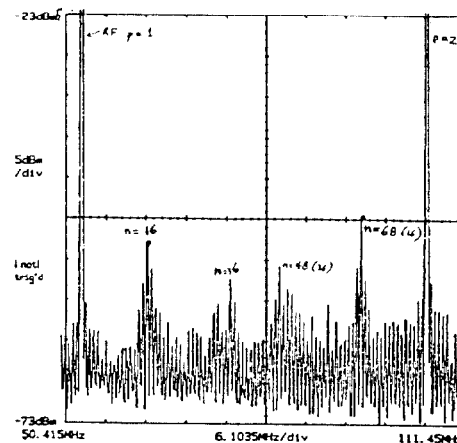

Figure 2. Booster beam spectrum showing coupled-bunch mode growth at n=16 and 36 (at 25 msec in the cycle.) The two sharp lines are the RF harmonics. The data were recorded on a 2 GHz digitizing oscilloscope using a broad band resistive wall pickup.

II. Beam model: coupled-bunch motion

Coupled-bunch instability may arise in a synchrotron if wake fields excited as the beam passes through structures in the ring feed back on the beam coherently. For low intensities and amplitudes, the characteristic frequencies of this coherent motion are harmonics of the synchrotron frequency. There are a number of references giving the complete treatment; we have used Laclare [6]. A few illustrative intermediate results are presented below. The intensity signal for a perturbed synchrotron beam undergoing coupled-bunch motion has a discrete spectrum with frequencies

$$\omega_n = (n+ph)\omega_0+m\omega_s \qquad (1)$$

where n is the coupled-bunch mode number, p the RF harmonic, h the number of bunches, m the synchrotron mode number, ω_0 the revolution frequency, and ω_s the synchrotron frequency. The amplitudes, or rather envelopes, of the perturbed beam spectra for each m are given by form factors F_m. These give the effective response of the beam to an impedance at ω_n for coherent dipole (ω_s), quadrupole $(2\omega_s)$ oscillation, etc. As such, the form factors also give the response of the beam to a feedback system.

Initially, the description is simplified by treating the beam as a single particle moving in $(\tau, d\tau/dt)$ phase space $(\tau=t-t_s)$. For coherent coupled-bunch motion, phase modulation occurs at the synchrotron phase such that $\tau=\tau_s\cos(\omega_s t+\psi_0)$, where τ_s is the amplitude of the motion and ψ_0 is an initial phase. For simplicity, we assume throughout that ω_s is constant, i.e. $\omega_s \neq \omega_s(\tau_s)$. The time-domain signal is Fourier analyzed, and we find that the amplitudes F_m of the single-particle spectra are Bessel functions $J_m = J_m(p\omega_0\tau_s)$.

This treatment is now extended to the case of multiple bunches. A parabolic amplitude longitudinal bunch distribution is assumed, which is consistent with measurements using a fast (2 GHz) oscilloscope. The amplitudes F_m are now integrals of the J_m and the bunch distribution over phase space. The stationary spectrum m=0 is solved simply and gives harmonics of the RF. The perturbed spectra are found by solving Vlasov's equation for the time evolution of the particle density function (with assumed longitudinal distribution and time dependence.) Laclare gives the result:

$$F_m(x) = \frac{2(|m|+1)}{x^2}\int_0^x J_m^2(u)\, u\, du \; ; \quad x = \omega_n\frac{\tau_L}{2} \qquad (2)$$

Here τ_L is the bunch length. The form factors for m=0,1,2,3 corresponding to stationary, dipole, quadrupole, and sextapole coupled-bunch motion, respectively, are plotted in Fig. 3 as a function of x. Also shown in this figure are these form factors plotted as a function of frequency for various times in

the Booster cycle. This result has immediate consequences for feedback system design. To feed back on dipole motion, for example, one might wish to operate the system at the peak of the dipole envelope for maximum beam signal. However, the RF signal here is large, yielding a poor signal-to-noise. If the RF signal is not filtered, one might choose instead to operate at the minimum of the RF envelope. As can be seen in Fig. 3, the bunch length varies through the Booster acceleration cycle as does, therefore, the optimum operating point. The final design is a compromise, considering also the observed onset in the cycle of the instability, which is around transition (19 msec.)
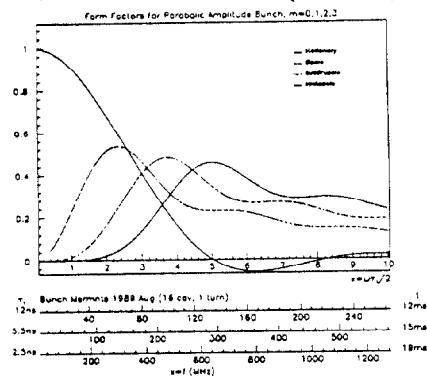

Figure 3. Form factors for stationary, dipole, quadrupole, and sextapole coupled-bunch motion in the Booster. Given at the bottom is the scaling with frequency for three different times in the cycle.

III. Higher-Order RF Mode Impedance Measurements

The higher-order RF cavity mode impedances have been measured at Fermilab using a variety of techniques. A description and critical survey of these impedance measurements is given by another paper in this conference [7]. For this paper, the impedance was measured at one gap on an RF cavity test stand for different tuned frequencies using a stretched wire. The single-gap stretched wire assembly, which fits into one end of the cavity beam pipe, is shown schematically in Fig. 4. Plotted in Fig 5 is the impedance when the cavity is tuned to a fundamental frequency of 52.2 MHz (corresponding to about 19 msec in the cycle.) The results for the single-gap impedances were multiplied by twice the total number of cavities (two gaps each) used during emittance growth measurements [4]. In this way, calculations may be compared to the data. The changing velocity of the beam through the cycle was not taken into
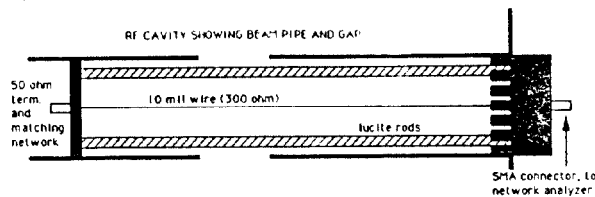

Figure 4. Schematic of single-gap stretched wire assembly used to measure gap impedance on an RF cavity test stand.

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account. This will change the relative phase between the beam and the voltage at the two gaps; however, this is expected to be a small correction.
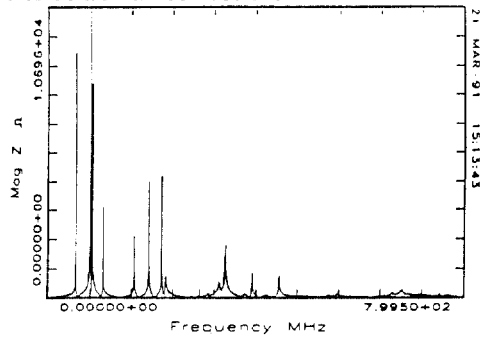

Figure 5. Single-gap impedance for a fundamental cavity tune of 52.25 MHz (bias current 1800 amps) using the probe shown in Fig. 4 above. This corresponds to about 19 msec in the cycle.

IV. Growth Rate

A program was written to calculate the form factors, read the impedance data, and calculate growth rates for coupled bunch modes. Only the impedances at $\omega = \omega_n$ contribute. The growth rate is the imaginary part of the coherent mode frequency shift $\Delta\omega_{cm} = \omega_{cm} - m\omega_s$ and is given by [6]

$$\text{Im} \left(\Delta\omega_{cm} \right) = \frac{- N e m \omega_s}{f_0 V \cos \phi_s (m+1)} \sum_{p=-\infty}^{\infty} \frac{\text{Re } Z(p)}{p} F_m \quad (4)$$

The results corresponding to 19 msec are shown in Fig.6. It is interesting to compare these results to the data in Fig.2, even though the times are different. The observed coupled-bunch mode 16(15) is predicted in the calculation. The calculation shows faster growth in modes 51(84-33) and 45(84-39.) In fact, experimentally, mode 16 saturates early in the cycle and we observe more pronounced growth around modes 32, 36. The data shows a narrow band at mode 16, but a broad band from 32-38. The latter could be driven by two cavity modes sweeping as the RF frequency sweeps.

The total effective impedance driving the instability is also calculated, as well as the dominant offending RF cavity mode. The maximum effective total coupling impedance due to the higher-order cavity modes was found to be 60 kohms early in the cycle and 15 kohms near extraction. This corresponds to an effective voltage on the beam of 18 and 4.5 kV, respectively, at maximum intensity. The offending RF modes appear to be 163 MHz for mode n=16 and 84, 79 MHz for n=32, 36.

V. Conclusions

These recent analyses confirm prior results showing a relationship between the RF cavity higher-order modes and emittance growth. They go further in actually predicting the excited coupled-bunch modes with associated growth rates comparing qualitatively well to the data. We plan to further develop the beam model to include in the form factors F_m the spread in synchrotron frequency with particle oscillation amplitude $\omega_s(\tau_s)$. Impedance measurements were performed with the cavity "shorted;" the short is a ferrite-loaded rod connecting the inner and out conductors. Also, impedance measurements were performed with prototype mode dampers installed on the RF cavity test stand. The effect of both the shorts and the dampers is to lower both the impedance and the Q of the higher-order modes. The theory assumes excitation by a high-Q resonator; it must now be further developed to bridge the gap between high-Q and broad band resonators. We plan to then analyze these data to compare calculated instability growth rates to emittance growth data taken with a few cavities shorted. The gap impedance measurements themselves are ongoing; we are currently assessing the various techniques employed for accuracy and ease of error elimination. Finally, we plan to do a dynamic study of coupled-bunch instability growth through the cycle using high-resolution impedance data.
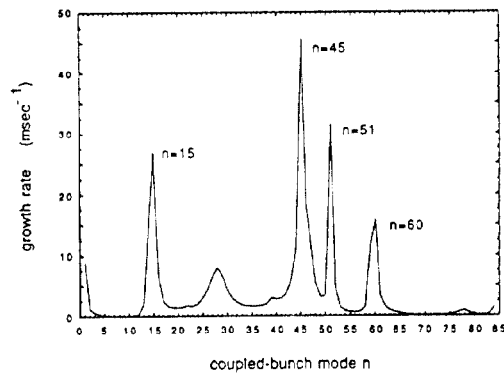

Figure 6. Calculated growth rates corresponding to 19 msec in the cycle. The spread in synchrotron frequencies has not been taken into account; this will reduce the absolute magnitudes. We expect growth times on the order of 1 msec in the Booster.

VI. References

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