Analysis of the Longitudinal Instability in an Induction Linac*

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Abstract

A theoretical model for the longitudinal instabilities of intense beams in a transport channel with complex wall impedances is presented. A dispersion equation is derived to relate the growth rates of the slow waves to all the relevant parameters including the space charge, the real and imaginary parts of the complex wall impedances. The growth rates for different wall structures are compared and discussed.

I. INTRODUCTION

The induction linac is a promising driver for Heavy Ion Inertial Fusion. An important issue in the design of such induction linacs is the longitudinal instability [1,2,3,4] which is predicted to develop and is detrimental to the beam. In contrast with the accelerators for high energy physics, the beam current in induction linacs as drivers for heavy ion inertial fusion is as high as kilo-amperes, while the beam particle velocity is only sub-relativistic. Thus, the space charge force plays an important role in the instability analysis. Another important factor is the interaction between charged particles and induction gaps which are usually modeled by discrete R, L, C circuits in the low frequency limit. Studies show that the impedance of induction gaps is a function of frequency and could change from inductive to capacitive as the frequencies shift. This requires different circuit models at different frequency ranges. Thus, it is desirable to have a theory which can analyse the instability in a transport channel with general wall impedances, rather than a specific circuit configuration. In a previous paper [5] we developed a theoretical model to relate the growth rates of the slow waves to the beam space charge and the complex impedances of the transport channel. This paper is an extension of the previous one. The derivation of the dispersion equations and the expressions for the general longitudinal field taking into account the real and imaginary parts of the wall impedance is given. The physical consequences of the resulting equations are discussed. The growth rates for the wall modeled by three different circuits are also compared. The results show explicitly how a capacitive wall can dramatically reduce the growth rate of the instability.

II. DERIVATION OF THE DISPERSION EQUATIONS

The longitudinal beam dynamics can be described by the linearized cold, one-dimensional fluid model which consists of the continuity equation and momentum transfer equation:

$$\begin{cases} \dfrac{\partial \lambda_1}{\partial t} + v_0 \dfrac{\partial \lambda_1}{\partial z} + \lambda_0 \dfrac{\partial v_1}{\partial z} = 0 \\ \dfrac{\partial v_1}{\partial t} + v_0 \dfrac{\partial v_1}{\partial z} \approx \eta E_z \end{cases} \tag{1}$$

where $\lambda(z,t)$ and $v(z,t)$ are the line charge density and the particle velocity, the subscripts 0 and 1 representing the unperturbed and perturbed quantities, respectively, $E_z(z,t)$ is the longitudinal electrical field induced by the a. c. component of the beam current, and $\eta = q/(m\gamma^3)$ denotes the ratio of the charge and the "longitudinal mass" of the charged particles. Under the long wavelength condition, the field $E_z(z,t)$ can be calculated as [6]

$$E_z(z,t) = -\frac{g}{4\pi\varepsilon_0\gamma^2}\frac{\partial \lambda_1}{\partial z} + E_w(z,t) \tag{2}$$

where ε_0 is the permittivity of free space, g is a geometric factor of order unity, and $E_w(z,t)$ is the field induced on the pipe wall of the transport channel and depends on the perturbed beam current and the properties of the wall. In the complex frequency domain this field is related to the perturbed beam current and the wall complex impedance $Z_w(s)$ per unit length as

$$\begin{aligned} E_w(k,s) &= \int_{-\infty}^{+\infty} dz \int_0^{\infty} dt E_w(z,t) e^{-(ikz+st)} \\ &= -Z_w(s)\left[v_0\lambda_1(k,s) + \lambda_0 v_1(k,s)\right]. \end{aligned} \tag{3}$$

Solving Eqs. (1) to (3) by using Laplace transform in time and Fourier transform in space leads to the dispersion equation

$$(s + ikv_0)^2 + \frac{k^2\eta g\lambda_0}{4\pi\varepsilon_0\gamma^2} + s\eta\lambda_0 Z_w(s) = 0 \tag{4}$$

Though this equation is exact in this theoretical model, its solution depends on the wall properties, namely $Z_w(s)$, and can not usually be solved analytically except in the simplest cases such as the pure resistive wall. Approximations have to be made for proper interpretation of the physical consequences of Eq. (4). In the real frequency domain the wall impedance per unit length has the general form

$$Z_w(\omega) = R(\omega) + iX(\omega), \tag{5}$$

where $R(\omega)$ and $X(\omega)$ are the real and imaginary parts of the complex impedance. It is usually supposed that the zeros of

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Eq. (4) associated with the circuit $Z_w(s)$ are not of interest and that the approximation may be made by setting $s=-i\omega_0=-ikv_0$ in the last term of Eq. (4). Thus, one gets

$$(s+ikv_0)^2 - ik\eta\lambda_0 v_0 R(\omega_0) + k^2\eta\left(\frac{g}{4\pi\varepsilon_0\gamma^2} - v_0^2\frac{X(\omega_0)}{\omega_0}\right)\lambda_0 = 0 \quad . \quad (6)$$

In the above approximation we actually rewrite Eq. (3) as

$$E_w(k,s) \approx -v_0\lambda_1(k,s)\left[R(\omega_0) - iX(\omega_0)\right] = ikv_0^2\frac{X(\omega_0)}{\omega_0}\lambda_1(k,s) - R(\omega_0)v_0\lambda_1(k,s) \quad , (7)$$

where the perturbed beam current component $\lambda_0 v_1(k, s)$ in Eq. (3), which is usually much smaller than the term $v_0\lambda_1(k, s)$ in most applications, is neglected. This equation shows that the real part of the wall impedance produces the field in the opposite phase with respect to the perturbed line charge density, while the imaginary part of the wall impedance produces the field that is 90° out of phase with respect to the perturbed line charge density. Inverse Laplace-Fourier transform of Eq. (7) yields

$$E_w(z,t) = v_0^2\frac{X(\omega_0)}{\omega_0}\frac{\partial\lambda_1}{\partial z} - R(\omega_0)v_0\lambda_1 \quad . \quad (8)$$

Therefore, the total longitudinal field acting on the beam can be expressed by

$$E_z(z,t) \approx -\left(\frac{g}{4\pi\varepsilon_0\gamma^2} - v_0^2\frac{X(\omega_0)}{\omega_0}\right)\frac{\partial\lambda_1}{\partial z} - \lambda_1 v_0 R(\omega_0) \quad . \quad (9)$$

It is interest to note that the inductive impedance which has a positive imaginary part can reduce the space charge field and the capacitive impedance just acts in the opposite way.

III. DISCUSSION OF THE GROWTH RATES

The dispersion equation (6) has the general solution for the perturbed wave frequency and the temporal growth rate:

$$\begin{cases} \omega_r = kc\left\{\beta \pm \left[\frac{g}{2\gamma^5\beta}\frac{I}{I_0}\left(1 - \frac{2\beta\gamma^2}{g}\frac{X(\omega_0)\Lambda}{Z_0}\right)Y_+\right]^{1/2}\right\} \\ \omega_i = \mp kc\left[\frac{g}{2\gamma^5\beta}\frac{I}{I_0}\left(1 - \frac{2\beta\gamma^2}{g}\frac{X(\omega_0)\Lambda}{Z_0}\right)Y_-\right]^{1/2} \end{cases} \quad (10)$$

where I is the beam current, $I_0=4\pi\varepsilon_0 mc^3/q$ is the characteristic current of the charged particles, Λ is the wavelength of the growing waves, $Z_0=1/(\varepsilon_0 c)=377$ ohms is the characteristic impedance of free space, and $Y_\pm=(1+y^2)^{1/2}\pm1$ with y being determined by

$$y = \frac{2\beta\gamma^2}{g}\frac{R(\omega_0)\Lambda}{Z_0}\left(1 - \frac{2\beta\gamma^2}{g}\frac{X(\omega_0)\Lambda}{Z_0}\right)^{-1} \quad . \quad (11)$$

Equation (10) is valid for y>0. In the above equations the subscript 0 is dropped for β and γ for simplicity.

An interesting parameter range is $y<<1$, or

$$\frac{2\beta\gamma^2\left[R(\omega_0) + X(\omega_0)\right]\Lambda}{g\,Z_0} << 1 \quad , \quad (12)$$

which requires that the algebraic sum of the real and imaginary parts of the wall impedance per perturbation wavelength must be much smaller than the characteristic impedance of free space. In this case Eq. (10) can be approximated as

$$\begin{cases} \omega_r \approx kc\left[\beta \pm \left(\frac{g}{\gamma^5\beta}\frac{I}{I_0}\right)^{1/2}\left(1 - \frac{\beta\gamma^2}{g}\frac{X(\omega_0)\Lambda}{Z_0}\right)\right] \\ \omega_i \approx \mp kc\beta\frac{R(\omega_0)\Lambda}{Z_0}\left(\frac{1}{g\gamma\beta}\frac{I}{I_0}\right)^{1/2}\left(1 + \frac{\beta\gamma^2}{g}\frac{X(\omega_0)\Lambda}{Z_0}\right) \end{cases} \quad . \quad (13)$$

For a pure resistive-wall where $R(\omega_0)=R$ and $X(\omega_0)=0$, the slow-wave growth rate is

$$\omega_i \approx kc\beta\frac{R\Lambda}{Z_0}\left(\frac{1}{g\gamma\beta}\frac{I}{I_0}\right)^{1/2} \quad . \quad (14)$$

Thus, the resistive instability growth rate is proportional to the resistance per unit length of the transport channel. This growth rate is also proportional to the wavelength of the perturbation, implying that the slow waves of perturbation with the fundamental frequency is more serious than the higher harmonics. Equation (14) also indicates that the resistive instability growth rate is higher for a beam with higher current and lower energy. This is the case in the induction linac as drivers for Heavy Ion Inertial Fusion. If the pipe wall is inductive, e.g. $Z=R+i\omega_0 L$, one gets

$$\omega_i \approx kc\beta\frac{R\Lambda}{Z_0}\left(\frac{1}{g\gamma\beta}\frac{I}{I_0}\right)^{1/2}\left(1 + \frac{\beta\gamma^2}{g}\frac{\omega_0 L\Lambda}{Z_0}\right) \quad . \quad (15)$$

Therefore, the addition of an inductive component increases the growth rate in comparison with Eq. (14). For a capacitive wall modeled by a circuit of R and C in parallel, Eq. (13) can be rewritten as

$$\omega_i \approx k c \beta \frac{R\Lambda}{Z_0}\left(\frac{1}{g\gamma\beta}\frac{I}{I_0}\right)^{1/2} \times$$
$$\times \frac{\left[1+(\omega_0 RC)^2 - \frac{\beta\gamma^2}{g}(\omega_0 RC)\frac{R\Lambda}{Z_0}\right]}{\left[1+(\omega_0 RC)^2\right]^2} \quad . \quad (16)$$

As expected, the growth rate of the slow wave is reduced when a capacitance is added. This result agrees with ref. [1] which states that in general the capacity reduces growth rates compared with the case of pure resistance by lowering the impedance as frequency increases.

Fig. 1 shows the relative growth rate versus the relative time constant of the wall impedance. In this example the growth rate for a pure resistive wall is set to unity. The lower branch of the plot is for the capacitive wall with $\tau/\tau_0=RC\omega_0$ while the upper branch is for the inductive wall with $\tau/\tau_0=\omega_0 L/R$. The curves A-1 and A-2 illustrate the results from Eqs. (14)-(16) where the parameters used are $\beta=0.1$, $R=100\ \Omega/m$ and $\omega_0=2\pi x 10^8\ s^{-1}$. The growth rate for the capacitive wall decreases rapidly when the capacitance is increased. By contrast, the growth rate for an inductive wall is slightly increased.

If the condition (12) is not satisfied, the results from Eqs. (13) to (16) are not valid any more. In another extreme case

$$\frac{2\beta\gamma^2}{g}\frac{\left[R(\omega_0) + X(\omega_0)\right]\Lambda}{Z_0} >> 1 \quad , \quad (17)$$

the perturbed frequency and growth rate can be expressed by

$$\begin{cases} \omega_r \approx kc\left\{\beta \pm \left[\frac{1}{\gamma^3}\frac{I}{I_0}\frac{R(\omega_0)\Lambda}{Z_0}\right]^{1/2}\right\} \\ \omega_i \approx \mp kc\left[\frac{1}{\gamma^3}\frac{I}{I_0}\frac{R(\omega_0)\Lambda}{Z_0}\right]^{1/2} \end{cases} \quad , \quad (18)$$

which shows that the growth rate is determined by the real parts $R(\omega_0)$ of the complex impedance. In this case a R, C circuit in parallel will still reduce the growth rate, but a R, L circuit in series would not increase the growth rate in comparison with a pure resistive case. The curves B-1 and B-2 in Fig. 1 illustrate such an example where the parameters used are $\beta=0.3$, R=300 ohms/m and $\omega_0=2\pi 10^6\ s^{-1}$.

IV. SUMMARY

The cold one-dimensional fluid model has been used to analyze the longitudinal instability in a beam transport channel with general wall impedances. The dispersion equation derived explicitly relates the growth rate of the slow waves to all the relevant parameters including the space charge, the real and imaginary parts of the complex wall impedances. The temporal growth rate of the instability can be dramatically reduced by a capacitive component of the wall impedance along the transport channel. By contrast, the inductive wall component could enhance the instability.

V. REFERENCES

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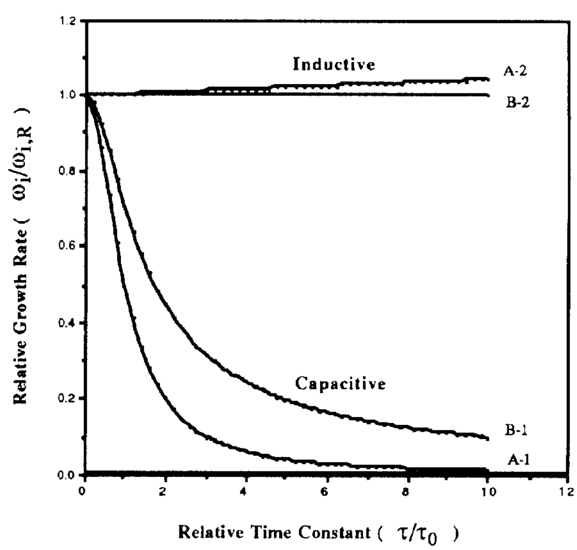

Fig. 1 . Relative growth rate $\omega_i/\omega_{i,R}$ vs. the relative time constant τ/τ_0 for a wall modeled by R, L in series or R, C in parallel, where ω_i is the growth rate for inductive or capacitive wall, $\omega_{i,R}$ is the growth rate for a pure resistive wall, τ=RC for the capacitive wall, or τ=L/R for the inductive wall, and $\tau_0=1/\omega_0$. The curves A-1 and A-2 illustrate the results from Eqs. (14)-(16) with the parameters $\beta=0.1$, $R=100\ \Omega/m$ and $\omega_0=2\pi x 10^8\ s^{-1}$. The curves B-1 and B-2 represent the results of Eq. (18) where the parameters are $\beta=0.3$, $R=300\ \Omega/m$ and $\omega_0=2\pi x 10^6\ s^{-1}$.