BEAM BREAKUP WITH LONGITUDINAL HALO

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Abstract

We have developed an analytical model of cumulative beam breakup in linear accelerators that predicts the displacement of particles between bunches. Beam breakup is assumed to be caused by a periodic current consisting of an infinite bunch train. The particles in the halo do not contribute to the breakup but experience the deflecting fields and are displaced by them. Under certain circumstances, the displacement of particles in the halo can be considerably larger than that of the bunches. This may have important consequences for the design of high-current cw accelerators where even a small flux of particles striking components of the accelerator cannot be tolerated because of activation.

I. INTRODUCTION

The cumulative beam breakup (BBU) instability results when the beam is offset transversely and couples to a deflecting mode in the accelerating structures, thereby enhancing it.[1,2] The mode deflects the trailing portion of the beam, which then couples more strongly to the mode in the next cavity, and the process evolves likewise along the length of the linac. This description of BBU focuses on the transverse dynamics of the bunches which comprise the beam, yet in the process wakefields are generated between the bunches. Any particles travelling between the bunches will respond to these wakefields. These particles may be viewed as constituting a diffuse "longitudinal halo". They respond to the deflecting fields but contribute negligibly to them.

The deflecting fields may drive the longitudinal halo into the cavity walls and thereby activate them. This is a particularly important concern in connection with the design of high-current linacs which are envisioned to run cw over long operational lifetimes. Examples include linacs envisioned for irradiation of fusion materials[3] and for tritium production.[4] The steady-state displacement of the halo is therefore of primary interest.

In this paper, we calculate the steady-state displacement of the longitudinal halo in the presence of cumulative BBU. We consider a linac with smoothly varying parameters and arbitrary β which drives a bunched beam. Focusing is included, and its role in mitigating activation caused by impingement of the longitudinal halo is quantified.

II. EQUATION OF TRANSVERSE MOTION WITH FOURIER TRANSFORMS

We calculate cumulative beam breakup within the framework of a model which incorporates a number of approximations. The cavities which comprise the linac are considered to have negligible length and to be the only source of deflecting fields. This approximation is, for example, justified for superconducting ion linacs which characteristically are comprised of short, independently phased cavities.[5] A single deflecting mode is assumed to be present, and its frequency is taken to be the same in every cavity. This corresponds to a worst-case analysis because a spread in deflecting-mode frequencies suppresses BBU,[6,7] but the worst case is of practical interest for the study of BBU control. A "continuum approximation" is used in which the discrete kicks in transverse momentum imparted by the cavities are considered to be smoothed along the linac. With this approximation, and allowing for arbitrary velocity, the equation of transverse motion of the beam in the presence of the single deflecting mode is:[8,9]

$$\left[\frac{1}{\beta\gamma}\frac{\partial}{\partial\sigma}\left(\beta\gamma\frac{\partial}{\partial\sigma}\right)+\kappa^2(\sigma)\right]x(\sigma,\zeta) = \epsilon(\sigma)\int_0^{\zeta}d\zeta'\,w(\zeta-\zeta')F(\zeta')x(\sigma,\zeta'). \quad (1)$$

Here, β and γ have their usual meanings; $\sigma = s/\mathcal{L}$ is a dimensionless spatial variable defined in terms of position along the linac, s, and total length of the linac, $\mathcal{L}$; $\zeta = \omega(t-\int ds/\beta c)$ is the time, made dimensionless by use of the angular frequency ω of the deflecting mode, measured after the arrival of the head of the beam at s; κ is the net transverse focusing wavenumber multiplied by $\mathcal{L}$; x is the transverse displacement of the beam centroid from the axis; $\epsilon(\sigma)$ is a dimensionless quantity which represents the strength of the BBU interaction; and $F(\zeta) = I(\zeta)/\langle I\rangle$ is the form factor for the current defined in terms of the beam current $I(\zeta)$ and average beam current $\langle I\rangle$. $w(\zeta)$ is a dimensionless "wake function" given by

$$w(\zeta) = e^{-\zeta/2Q}\sin\zeta\ \mathrm{u}(\zeta), \quad (2)$$

where $\mathrm{u}(\zeta)$ is the unit step or Heaviside function, and Q is the quality factor of the deflecting mode under consideration.

We now investigate beam breakup for the case of a periodic current composed of an infinite series of bunches. We will assume that $\beta\gamma$, κ, and ϵ are independent of σ (coasting beam). When necessary, the results can be straightforwardly generalized using the WKBJ method.[10] The equation of transverse motion is then

$$\left(\frac{d^2}{d\sigma^2}+\kappa^2\right)x(\sigma,\zeta) = \epsilon\int_{-\infty}^{\zeta}d\zeta'\,w(\zeta-\zeta')F(\zeta')x(\sigma,\zeta'), \quad (3)$$

with

$$F(\zeta) = \sum_{k=-\infty}^{+\infty}F_k e^{ik\frac{2\pi}{\omega\tau}\zeta}. \quad (4)$$

Note that since the current is assumed to be periodic and started at $\zeta = -\infty$, the lower limit of the integral in eq. (3) is $-\infty$ and not 0 as it was in eq. (1). This does not restrict eq. (3) to the analysis of the steady state behavior since no assumption has been made about the initial conditions. For example, a beam which is turned on at $\zeta=0$ can be modelled by assuming that $x(0,\zeta<0)=0$ since no deflecting field will be generated as long as the current travels on axis.

Introducing the Fourier transforms of $x(\sigma,\zeta)$ and $w(\zeta)$,

$$\tilde{x}(\sigma,Z) = \int_{-\infty}^{+\infty} e^{-iZ\zeta} x(\sigma,\zeta)d\zeta\,,\ \ \tilde{w}(Z) = \int_{-\infty}^{+\infty} e^{-iZ\zeta} w(\zeta)d\zeta \ = \ \frac{1}{1-Z^2+\frac{1}{4Q^2}+\frac{iZ}{Q}},\qquad(5)$$

eq. (3) becomes

$$\left(\frac{d^2}{d\sigma^2}+\kappa^2\right)\tilde{x}(\sigma,Z) \ = \ \epsilon\,\tilde{w}(Z)\sum_{k=-\infty}^{+\infty} F_k\,\tilde{x}(\sigma,Z-\frac{2\pi k}{\omega\tau}).\qquad(6)$$

Equation (6) is a difference-differential equation for the Fourier transform of the displacement of periodic bunches of arbitrary shape. It replaces and is equivalent to the integro-differential equation (3) which is more commonly used. Equation (6) can be used to study a variety of transient and steady-state BBU problems. In this paper we will address the problem of the transverse displacement of particles located between bunches. The problem of the displacement and distortion of bunches of finite length is treated in a companion paper.[11]

III. Delta-Function Bunches

For the case of a beam composed of delta-function bunches, the Fourier components are $F_k = 1\ \forall\ k$, so that eq. (6) becomes

$$\left(\frac{d^2}{d\sigma^2}+\kappa^2\right)\tilde{x}(\sigma,Z) = \epsilon\,\tilde{w}(Z)\sum_{k=-\infty}^{+\infty}\tilde{x}(\sigma,Z-\frac{2\pi k}{\omega\tau}).\qquad(7)$$

If we define

$$\tilde{W}(Z) \equiv \sum_{k=-\infty}^{+\infty}\tilde{w}(Z-\frac{2\pi k}{\omega\tau}) = \frac{\omega\tau}{2}\,\frac{\sin\omega\tau}{\cosh[\omega\tau(1/2Q-iZ)]-\cos\omega\tau},\qquad(8)$$

and

$$\Lambda^2(Z) = \epsilon\,\tilde{W}(Z) - \kappa^2,\qquad(9)$$

the solution of eq. (7) is

$$\begin{aligned} x(\sigma,\zeta) = {} & x(0,\zeta)\cos(\kappa\sigma)+x'(0,\zeta)\,\frac{\sin(\kappa\sigma)}{\kappa} \\ & +\frac{1}{2\pi}\int_{-\infty}^{+\infty} dZ\,e^{iZ\zeta}\,\frac{\tilde{w}(Z)}{\tilde{W}(Z)}\,A(Z)\,\{\cosh[\Lambda(Z)\sigma]-\cos(\kappa\sigma)\} \\ & +\frac{1}{2\pi}\int_{-\infty}^{+\infty} dZ\,e^{iZ\zeta}\,\frac{\tilde{w}(Z)}{\tilde{W}(Z)}\,B(Z)\left\{\frac{\sinh[\Lambda(Z)\sigma]}{\Lambda(Z)}-\frac{\sin(\kappa\sigma)}{\kappa}\right\}, \end{aligned}\qquad(10)$$

with $A(Z)$ and $B(Z)$ determined from the initial conditions at $\sigma = 0$:

$$A(Z) = \omega\tau\sum_{k=-\infty}^{+\infty} x(0,k\omega\tau)\,e^{-ik\omega\tau};\ B(Z) = \omega\tau\sum_{k=-\infty}^{+\infty} x'(0,k\omega\tau)\,e^{-ik\omega\tau}.\ (11)$$

It is worth noting that the beam displacement is defined for all values of ζ, and not only for $\zeta = m\omega\tau$ which represents bunch m. In particular, eq. (10) can be used to calculate the transverse displacement of particles located outside the delta-function bunches, i.e. particles that do not contribute to the beam breakup but experience the transverse fields generated by the bunches. These particles comprise the "longitudinal halo". The transient displacement resulting from turn-on at $\zeta = 0$ of a misaligned beam may be obtained from the initial conditions $x(0,\zeta) = x_0\,u(\zeta)$ and $x'(0,\zeta) = 0$.

Steady-state BBU is of greater concern than transient BBU in connection with long-term cw operation. We now calculate the steady-state displacement induced by a misaligned beam for which $x(0,\zeta) = x_0$ and $x'(0,\zeta) = x_0'$. According to eq. (11), these initial conditions yield

$$A(Z) = 2\pi x_0\sum_{k=-\infty}^{+\infty}\delta(Z-\frac{2\pi k}{\omega\tau});\ \ B(Z) = 2\pi x_0'\sum_{k=-\infty}^{+\infty}\delta(Z-\frac{2\pi k}{\omega\tau}).\qquad(12)$$

After integrating over Z, the beam displacement over the interval $0 \le \zeta \le \omega\tau$ is given by

$$\begin{aligned} x(\sigma,\zeta) = {} & x_0\cos(\kappa\sigma) + x_0'\,\frac{\sin(\kappa\sigma)}{\kappa} \\ & + x_0\,e^{-\zeta/2Q}\left(\frac{1+q}{p}\sin\zeta+\cos\zeta\right)\left[\cosh[\Lambda(0)\sigma]-\cos(\kappa\sigma)\right] \\ & + x_0'\,e^{-\zeta/2Q}\left(\frac{1+q}{p}\sin\zeta+\cos\zeta\right)\left[\frac{\sinh[\Lambda(0)\sigma]}{\Lambda(0)}-\frac{\sin(\kappa\sigma)}{\kappa}\right] \end{aligned}\qquad(13)$$

with

$$\Lambda(0) = \sqrt{\epsilon\,\omega\,\tau\,p-\kappa^2}.\qquad(14)$$

The functions p and q include the resonances between the frequencies $1/\tau$ of the accelerating mode and $\omega/2\pi$ of the deflecting mode:

$$p = p(\omega\tau,Q) = \sum_{k=1}^{\infty} e^{-\frac{k\omega\tau}{2Q}}\sin k\omega\tau \ = \ \frac{\sin\omega\tau}{4\left[\sinh^2\left(\frac{\omega\tau}{4Q}\right)+\sin^2\left(\frac{\omega\tau}{2}\right)\right]};$$

$$q = q(\omega\tau,Q) = \sum_{k=1}^{\infty} e^{-\frac{k\omega\tau}{2Q}}\cos k\omega\tau \ = \ \frac{\cos\omega\tau - e^{-\omega\tau/2Q}}{4\left[\sinh^2\left(\frac{\omega\tau}{4Q}\right)+\sin^2\left(\frac{\omega\tau}{2}\right)\right]}.\qquad(15)$$

To find the displacement of the bunches themselves, we set $\zeta = 0$. The result is

$$x(\sigma,0) = x_0\,\cosh[\Lambda(0)\,\sigma] \ + \ x_0'\,\frac{\sinh[\Lambda(0)\,\sigma]}{\Lambda(0)}.\qquad(16)$$

IV. Transverse Displacement of Longitudinal Halo

Particles located outside the bunches can be deflected more than the bunches themselves. This is implied by Fig. 1, which provides example plots of the amplitude function $\chi = e^{-\zeta/2Q}\{[(1+q)/p]\sin\zeta + \cos\zeta\}$ with $Q = 1000$. As specific examples, we list three special cases where we assume $x_0' = 0$ for simplicity:

Case 1 -- $\omega\tau = 2n\pi(1 + 1/2Q)$ (resonance):

$$\frac{x(\sigma,\zeta)}{x_0} = \cos(\kappa\sigma)+(\sin\zeta+\cos\zeta)\left[\cosh\left(\sigma\sqrt{\epsilon Q-\kappa^2}\right)-\cos(\kappa\sigma)\right].\ (17)$$

Case 2 -- $\omega\tau = 2n\pi$:

$$\frac{x(\sigma,\zeta)}{x_0} = \cos(\kappa\sigma)+\frac{\epsilon Q}{\kappa}\,\sigma\,\sin(\kappa\sigma)\sin\zeta.\qquad(18)$$

Case 3 -- $\omega\tau = (2n + 1)\pi$:

$$\frac{x(\sigma,\zeta)}{x_0} = \cos(\kappa\sigma)+\frac{\epsilon}{\kappa}\,\frac{(2n+1)\pi}{4}\,\sigma\,\sin(\kappa\sigma)\sin\zeta.\qquad(19)$$

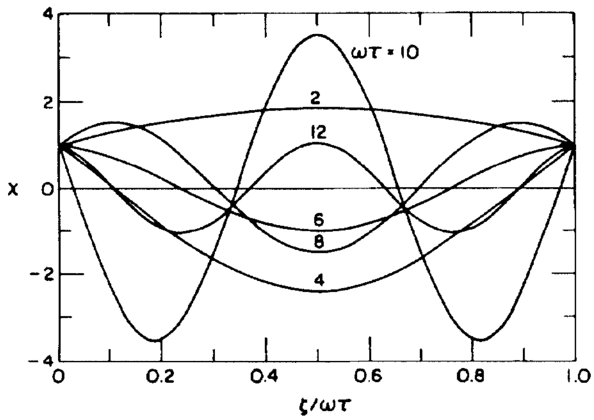

Figure 1. Plot of amplitude function χ vs. $\zeta/\omega\tau$ for delta-function bunches and for various values of $\omega\tau$. The bunches are located at $\zeta/\omega\tau = 0,1$.

These expressions illustrate clearly the role of focusing in controlling the transverse displacement of the longitudinal halo. For example, Case 2 requires strong focusing, $\kappa \gg \epsilon Q$, to confine the halo. Considerations of focusing for halo control are potentially important in connection with the suppression of activation of the cavity walls due to particle impingement over the long-term cw operation of a high-current linac.

V. Uniform Bunches of Arbitrary Length

Lengthening the bunches can significantly alter the transverse dynamics of the longitudinal halo. To demonstrate this, it is useful to define a generalized amplitude function as follows, assuming again that $x_0' = 0$:

$$\chi \equiv \frac{x(\sigma,\zeta) - x_0 \cos\kappa\sigma}{x(\sigma,0) - x_0 \cos\kappa\sigma}. \qquad (20)$$

For delta-function bunches in the steady state, we see from eq. (13) that χ depends only on ζ provided Q is large, and we plotted χ for this case in Fig. 1. Using the formalism of Ref. 11, we have numerically calculated χ for uniform bunches of arbitrary length. Results are given in Fig. 2 for the case $\omega\tau = 10$. This case exemplifies how a modest change of bunch length will sometimes cause a pronounced change in the amplitude function which, in turn, affects the dynamics of the longitudinal halo.

VI. Conclusions

The transverse displacement of particles comprising a diffuse longitudinal halo between bunches was calculated for the case of steady-state cumulative beam breakup. If the displacement were large enough to drive these particles into the cavity walls, then activation could become a problem due to the particulate flux accumulated over a long operational lifetime. It was seen that the longitudinal halo can sometimes be displaced much more than the bunches themselves. However, sufficiently strong focusing can be used as a cure.

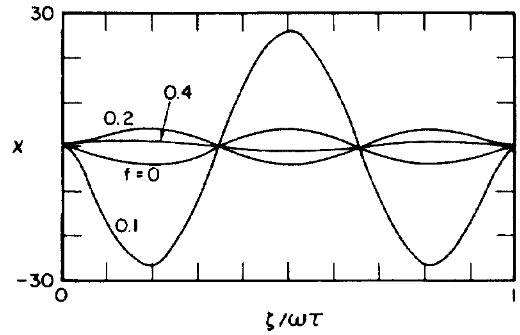

Fig.2. Generalized amplitude function χ vs. $\zeta/\omega\tau$ for $\omega\tau = 10$, $\epsilon = 0.2$, $Q = 1000$, $\kappa = 0$, $\sigma = 1$, and for assorted values of the filling factor f of the bunches (cf. Ref. 11). The bunch centers are located at $\zeta/\omega\tau = 0,1$.

VII. Acknowledgements

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