BEAM BREAKUP WITH FINITE BUNCH LENGTH

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Abstract

Cumulative beam breakup in linear accelerators has been studied extensively for the case of infinitely short bunches. While this model is appropriate for high energy accelerators, in high-current ion accelerators the bunch length can occupy a significant fraction of an rf period of the deflecting mode. A semi-analytic model has been developed to study and predict beam breakup in the case of nonzero bunch length and arbitrary bunch shape. Simulations of steady-state bunch distortion are presented, and the role of focusing in controlling beam breakup with bunches of nonzero length is illustrated.

I. INTRODUCTION

A deflecting mode which induces cumulative beam breakup (BBU) can cause degradation of beam quality and possible beam loss to the cavity walls. In low-β accelerators, the bunches have nonzero length and can occupy a significant fraction of the rf period of the deflecting mode. In turn, they will drive wakefields which differ from those set up by the nearly delta-function bunches characteristic of high-β accelerators. The wakefields will also be affected by the distribution of current within each bunch. Therefore, the effects of cumulative BBU in low-β ion accelerators will generally be quantitatively different than in high-β electron accelerators.

This problem is of particular interest in connection with superconducting linacs for high-current ion beams.[1] In these linacs, the constituent cavities will be short and independently phased, and cumulative BBU is therefore expected to be the dominant transverse instability. Because superconducting linacs run cw, the steady-state properties of their beams are fundamentally important. Steady-state BBU predominates after times long compared to Q/ω, where Q and ω refer to the deflecting mode and represent the cavity quality factor and the angular frequency of this mode, respectively. The transient BBU which occurs earlier can be countered by slowly increasing the current during turn-on,[2] and the steady-state case is therefore of most interest in connection with linacs which run cw.

In this paper, we calculate cumulative BBU in a linac with smoothly varying parameters and arbitrary β. In particular, we calculate the effects of nonzero bunch length and arbitrary current distribution within each bunch on the steady-state BBU of a coasting beam. In choosing a coasting beam to model the problem, we are ignoring the effects of longitudinal synchrotron oscillations within each bunch which occur in the presence of acceleration.

II. EQUATION OF TRANSVERSE MOTION WITH PERIODIC CURRENT

We investigate cumulative beam breakup for the case of a periodic current composed of an infinite series of bunches of identical but arbitrary shape. The assumptions and notation of Ref. 3 are used. We will assume that $\beta\gamma$, κ, and ϵ are all independent of σ (coasting beam). As given in Ref. 3, the equation of transverse motion is then

$$\left(\frac{d^2}{d\sigma^2}+\kappa^2\right)x(\sigma,\zeta) = \epsilon\int_{-\infty}^{\zeta}d\zeta'\, w(\zeta-\zeta')F(\zeta')x(\sigma,\zeta'), \tag{1}$$

with

$$F(\zeta) = \sum_{k=-\infty}^{+\infty} F_k e^{ik\frac{2\pi}{\omega\tau}\zeta}. \tag{2}$$

As also indicated in Ref. 3, after Fourier transformation eq. (1) becomes

$$\left(\frac{d^2}{d\sigma^2}+\kappa^2\right)\tilde{x}(\sigma,Z) = \epsilon\,\tilde{w}(Z)\sum_{k=-\infty}^{+\infty} F_k\,\tilde{x}(\sigma,Z-\frac{2\pi k}{\omega\tau}). \tag{3}$$

Equation (3) is a difference-differential equation for the Fourier transform of the displacement of periodic bunches of arbitrary shape. If the problem is completely periodic, i.e. steady state has been reached and the misalignment at the entrance to the linac has the same period as the bunches, then the displacement can be expanded in a Fourier series:

$$x(\sigma,\zeta) = \sum_{m=-\infty}^{+\infty} x_m(\sigma)e^{im\frac{2\pi}{\omega\tau}\zeta}. \tag{4}$$

In that case eq. (3) is replaced by

$$\left(\frac{d^2}{d\sigma^2}+\kappa^2\right)x_m(\sigma) = \epsilon\,\tilde{w}_m\sum_{k=-\infty}^{+\infty} F_k\, x_{m-k}(\sigma), \tag{5}$$

where

$$\tilde{w}_m = \int_0^{\infty}d\zeta\, w(\zeta)\, e^{-im\frac{2\pi}{\omega\tau}\zeta} = \frac{1}{1+\frac{1}{4Q^2}-\left(\frac{2\pi m}{\omega\tau}\right)^2+i\left(\frac{2\pi m}{Q\omega\tau}\right)}. \tag{6}$$

The BBU problem reduces to the calculation of the eigenvalues of the linear system associated with eq. (5).

Equation (5) can also be solved via series expansion of the Fourier components of the transverse displacement:

$$x_m(\sigma) = \sum_{j=0}^{\infty}\frac{x_{m,j}\,\sigma^j}{j!}. \tag{7}$$

The following recurrence relation is obtained:

$$x_{m,j+2} = \epsilon\,\tilde{w}_m\sum_{k=-\infty}^{+\infty}F_k x_{m-k,j} - \kappa^2 x_{m,j}. \tag{8}$$

For the case of a misaligned beam with initial conditions $x(0,\zeta)=x_0$, $x'(0,\zeta)=0$, the first terms of the expansion are

$$x_{0,2} = \epsilon\, \tilde{w}_0 F_0 x_0 - \kappa^2 x_0, \qquad x_{m,2} = \epsilon\, \tilde{w}_m F_m x_0, \tag{9}$$

and the beam displacement over the interval $0 \le \zeta \le \omega\tau$ is given by

$$x(\sigma,\zeta) \approx x_0\left[1 + \frac{\sigma^2}{2}\left(\epsilon \sum_{m=-\infty}^{+\infty} \tilde{w}_m F_m e^{im\frac{2\pi}{\omega\tau}\zeta} - \kappa^2\right)\right]. \tag{10}$$

The displacement of the center of the bunch ($\zeta = 0$) is

$$x(\sigma,0) = \sum_{m=-\infty}^{+\infty} x_m(\sigma) \approx x_0\left[1 + \frac{\sigma^2}{2}(\Omega^2 - \kappa^2)\right], \tag{11}$$

with
$$\Omega^2 = \epsilon \sum_{m=-\infty}^{+\infty} \tilde{w}_m F_m = \epsilon \int_0^{+\infty} w(\zeta)\, F(-\zeta)\, d\zeta. \tag{12}$$

It is useful to define a "growth factor" $G(\sigma,\zeta;\kappa)$ as follows:

$$G^2(\sigma,\zeta;\kappa) \equiv \frac{1}{\epsilon\, x(\sigma,\zeta)} \frac{d^2x(\sigma,\zeta)}{d\sigma^2}. \tag{13}$$

The growth factor of the bunch centroid at the entrance of the linac is

$$G^2(0,0;\kappa) = \frac{1}{\epsilon x_0} \frac{d^2x(0,0)}{d\sigma^2} = \frac{1}{\epsilon}(\Omega^2 - \kappa^2). \tag{14}$$

The role of focusing is clear: $\kappa > \Omega$ generates an imaginary growth factor at the linac entrance which tends to stabilize the beam with respect to beam breakup.

III. Example: Uniform Bunches

In the case where each bunch has the uniform current distribution

$$F(\zeta) = \begin{cases} \omega\tau/\alpha & for \quad |\zeta| < \alpha/2, \\ 0 \quad for & \alpha/2 < |\zeta| < \omega\tau/2, \end{cases} \tag{15}$$

the growth factor at the linac entrance with zero focusing is

$$G^2(0,0;\kappa{=}0) = \frac{\omega\tau}{\alpha} \frac{4Q^2}{4Q^2+1} \times \left\{ \begin{aligned} &\left(2q + \frac{p}{Q}\right)\cos\left(\frac{\alpha}{2}\right)\sinh\left(\frac{\alpha}{4Q}\right) + \left(2p - \frac{q}{Q}\right)\sin\left(\frac{\alpha}{2}\right)\cosh\left(\frac{\alpha}{4Q}\right) \\ &+ 1 - \left[\cos\left(\frac{\alpha}{2}\right) + \frac{1}{2Q}\sin\left(\frac{\alpha}{2}\right)\right]\exp\left(-\frac{\alpha}{4Q}\right) \end{aligned} \right\} \tag{16}$$

where p and q are the resonance functions defined in Ref. 3. In Fig. 1, $G^2(0,0;\kappa{=}0)$ is plotted as a function of "filling factor" $f \equiv \alpha/\omega\tau$ for various values of $\omega\tau$. The figure shows that the bunch length is an important factor in determining the stability of the beam with respect to BBU. This observation is accentuated by the plots in Fig. 2. There, the growth factor is plotted for various values of filling factor in the vicinity of the resonance $\omega\tau = 4\pi(1+1/2Q)$ for $Q = 1000$. As the bunch length increases, the stability of the beam with respect to BBU qualitatively reverses.

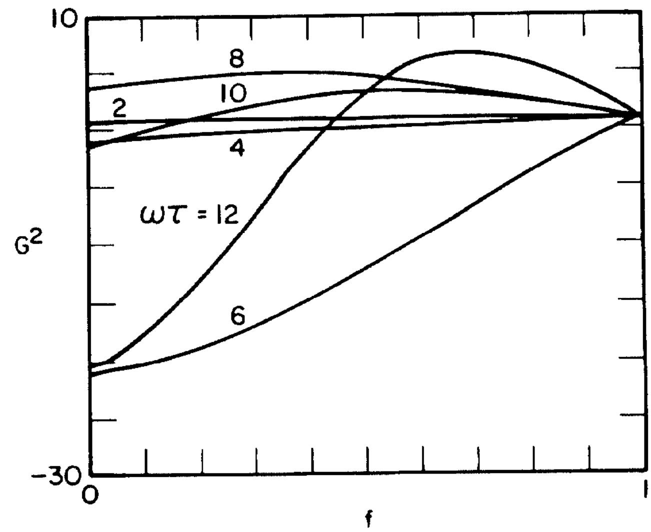

Fig.1. Growth factor $G^2(0,0;\kappa{=}0)$ vs. filling factor $f = \alpha/\omega\tau$ for $Q = 1000$ and for assorted values of $\omega\tau$. $f = 0$ corresponds to delta-function bunches, and $f = 1$ corresponds to the dc beam.

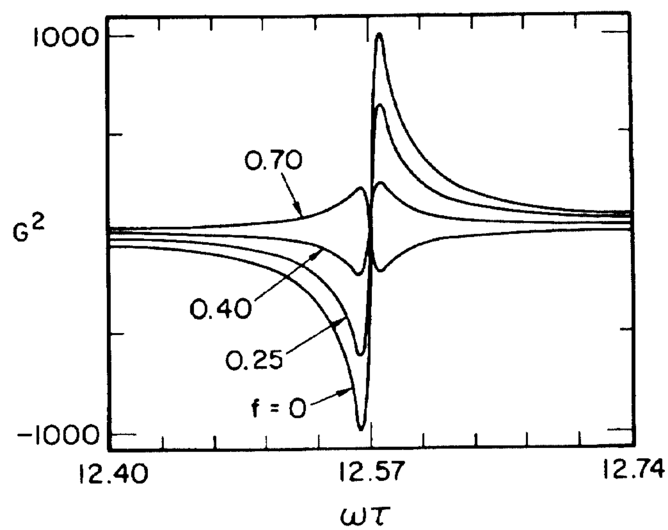

Fig. 2. Growth factor $G^2(0,0;\kappa{=}0)$ vs. $\omega\tau$ in the vicinity of the resonance $\omega\tau = 4\pi(1+1/2Q)$ for $Q = 1000$ and for assorted values of filling factor $f = \alpha/\omega\tau$.

In the case of finite but short bunches, we find

$$G^2(0,0;\kappa) \approx \frac{1}{\epsilon}\left[\epsilon\,\omega\tau\left(p + \frac{\alpha}{8}\right) - \kappa^2\right]. \tag{17}$$

Thus, we find that a slight spread of the charge distribution within bunches exacerbates beam breakup, independently of the focusing strength κ. This is also true for arbitrary current distributions.

According to eqs. (12) and (14),

$$\Delta G^2 \equiv G^2(\alpha \neq 0) - G^2(\alpha = 0) = \sum_{m=-\infty}^{\infty} \tilde{w}_m (F_m - 1). \qquad (18)$$

For nearly delta-function bunches, the terms with large m predominate. From eq. (6), the corresponding $\tilde{w}_m$s are real and negative, so that

$$\Delta G^2 = 2\sum_{m=1}^{\infty} \tilde{w}_m [Re(F_m) - 1]. \qquad (19)$$

We now see that $\Delta G^2 > 0$ because $Re(F_m)$ gradually decreases from unity as m increases. Accordingly, slightly spreading the bunches always exacerbates beam breakup. This conclusion is valid only for small deviations from the delta-function case, i.e. for $\alpha \ll \omega\tau$ and $\alpha \ll 2\pi$. In the limit $\alpha \to \omega\tau$, we recover the dc case from eq. (16).

Even when the beam is inherently stable, the deflecting modes can distort each bunch and thereby degrade beam quality. An example is presented in Fig. 3a for an unfocused beam with parameters which are plausible for a superconducting linac.[4] These results, which were generated numerically from eq. (8), reveal that portions of the bunch can be deflected to transverse displacements exceeding the initial displacement even though the bunch centroids are deflected toward the axis. However, as shown in Fig. 3b, focusing can be used to suppress bunch distortion.

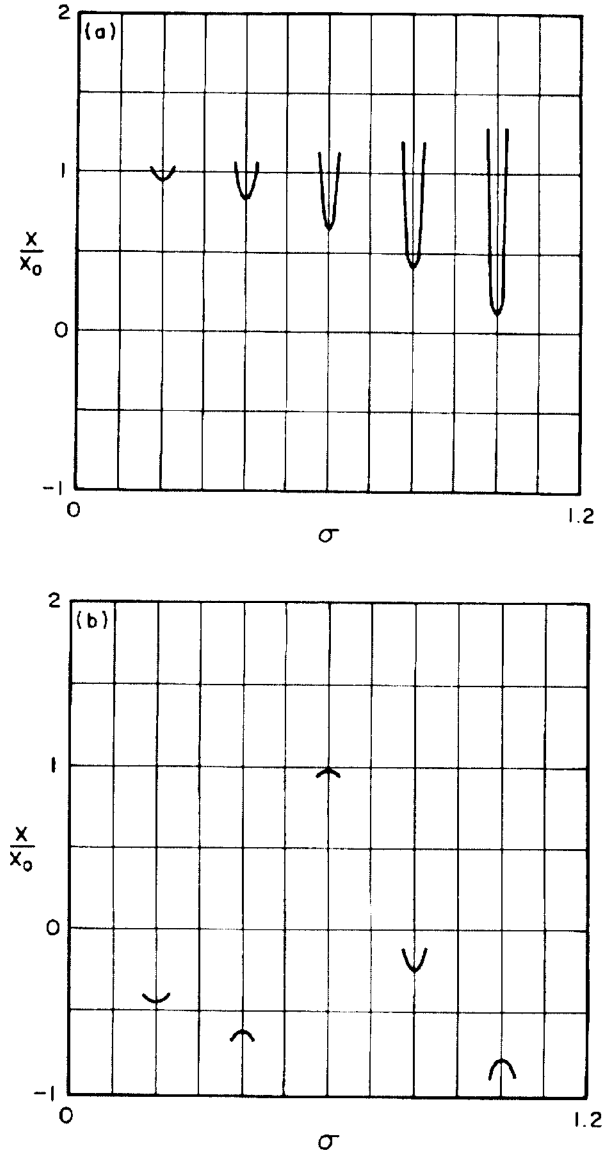

Fig. 3. Plots of bunch displacement and shape vs. position σ along the linac for $\omega\tau = 12$, $e = 0.2$, $Q = 10^6$, $f = 0.25$, and for (a) $\kappa = 0$ (no focusing) and (b) $\kappa = 10$. Transverse displacement, plotted as the ordinate, is normalized with respect to the initial displacement. $\sigma = 0$ denotes the entrance to the linac, and $\sigma = 1$ denotes the exit.

IV. Conclusions

A formalism which enables the calculation of steady-state cumulative beam breakup with bunches of arbitrary shape and nonzero length comprising a coasting beam was developed. As an example, the formalism was applied to uniform bunches. Bunches of short, but nonzero, length always exacerbate BBU relative to delta-function bunches. Bunches of finite length may be severely distorted by the deflecting modes, even in circumstances which would be stable were the bunches to have zero length. In each case, however, focusing can be used as a cure.

V. Acknowledgements

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VI. References

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