Instability Calculations for the MIT-Bates South Hall Ring *

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Abstract

Instability growth rates and thresholds have been calculated for the MIT-Bates South Hall Ring. Both single bunch and coupled bunch instabilities have been investigated. For single bunch effects, a broad-band impedance budget has been developed. Numerical estimates of the impedances of ring components were made, and required to be within the budget. As part of this, the difficulty of fitting computed loss parameters to those derived from the usual broad band impedance model were studied. We conclude single bunch instabilities should not be a problem. However, coupled bunch instabilities are a serious concern, since all 1812 of the 2856 MHz RF buckets around the ring are filled. A variety of numerical and analytic methods have been used to calculate growth rates and thresholds, and thereby determine acceptable impedance limits. Ring components are being designed to meet these limits. Details of these calculations are presented.

I. Introduction

The MIT Bates Linear Accelerator Center is building an electron storage ring, known as the South Hall Ring (SHR), for medium energy nuclear physics experiments. The ring will have two modes of operation. In pulse stretcher mode, electrons from a 1% duty factor linac will be injected into the ring, and then extracted uniformly over 1 ms to provide a CW beam to external target experiments. In internal target mode, the beam will be stored in the ring for a longer time (tens of milliseconds, or longer), providing high average currents for targets internal to the ring.

The SHR has a 190 m circumference, and is designed to store currents up to 80 mA at energies from 300 MeV to 1 GeV. The harmonic number is 1812, (ring RF frequency 2856 MHz), and every bucket will contain electrons. In order to achieve the desired beam storage times, with no significant degradation of beam quality, it is necessary to avoid instabilities of the electron beam. Both single bunch and coupled bunch instabilities are of concern. In this paper, we describe the analytic and numerical calculations which have been done regarding beam instabilities in the SHR.

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II. Single Bunch Instabilities

A. Thresholds

The single bunch instabilities of greatest concern for the SHR are turbulent bunch lengthening in the longitudinal direction, and the transverse fast blowup instability. To avoid these instabilities, the broad band impedance of the ring must be kept below specified limits. Using analytic calculations, as well as the computer program ZAP [1], we have determined that the longitudinal broad band impedance limit is 25.7 Ω, and the transverse broad band impedance limit is 56 MΩ/m. As long as the total ring broad band impedances are below these limits, single bunch instabilities should not interfere with storage of a 300 MeV, 80 mA beam. (These parameters represent the most severe operating conditions with respect to instabilities.) These impedance limits are fairly large, since the charge in each bunch in the ring is small (1.75×10^8 electrons per bunch.)

B. Impedance determination

To determine the contribution of each ring component to the total ring broad band impedance, we used the computer programs TBCI [2] and MAFIA [3]. The procedure used was to numerically calculate the longitudinal loss parameter $k_{\|}$ for each component as a function of the bunch RMS length σ_l. This was then compared with the $k_{\|}$ *vs.* σ_l expected theoretically for a bunch traversing a device with some impedance, and from this the impedance was determined.

In order to find the theoretical loss parameter as a function of bunch length, it is necessary to assume some functional form for the impedance. The usual form taken is

$$Z_{\|}(\omega) = \frac{R}{1 + iQ(\omega_r/\omega - \omega/\omega_r)} \qquad (1)$$

for the longitudinal case. Here R is the shunt impedance, ω_r is the characteristic resonant frequency, and Q is the usual quality factor.

For a Gaussian bunch of RMS length σ_l, the loss factor is given by

$$k_{\|}(\sigma_l) = \frac{1}{\pi}\int_0^\infty \mathrm{Re}[Z_{\|}(\omega)]e^{-(\omega\sigma_l/c)^2}d\omega. \qquad (2)$$

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Using the expression for $Z_{\parallel}(\omega)$ in Eq. (1), the above integral can be evaluated to give

$$k_{\parallel}(\sigma_l) = \frac{R\omega_r}{4QS}\left[(S-1)w(i\frac{\omega_1\sigma_l}{c}) + (S+1)w(i\frac{\omega_2\sigma_l}{c})\right] \quad (3)$$

for $0 < Q < 1/2$,

$$k_{\parallel}(\sigma_l) = R\omega_r\left\{e^{\alpha^2}(1+2\alpha^2)[1-\mathrm{erf}(\alpha)] - \frac{2\alpha}{\sqrt{\pi}}\right\} \quad (4)$$

for $Q = 1/2$, and

$$k_{\parallel}(\sigma_l) = \frac{R\omega_r}{2Q\alpha}\left(1-\frac{1}{4Q^2}\right)^{-1/2}\mathrm{Re}[zw(z)] \quad (5)$$

for $Q > 1/2$. In these equations, $S = \sqrt{1-4Q^2}$, $\omega_1 = -\omega_r/2Q(S-1)$, $\omega_2 = \omega_r/2Q(S+1)$, w is the exponentially scaled complex error function, erf is the error function, $\alpha = \omega_r\sigma_l/c$, and $z = (\sqrt{1-1/4Q^2} + i/2Q)\alpha$.

Figure 1 shows an example of fitting the loss parameter $k_{\parallel}$ determined by TBCI, as a function of RMS bunch length. Using the parameters of the fit, the broad band impedance can then be calculated.

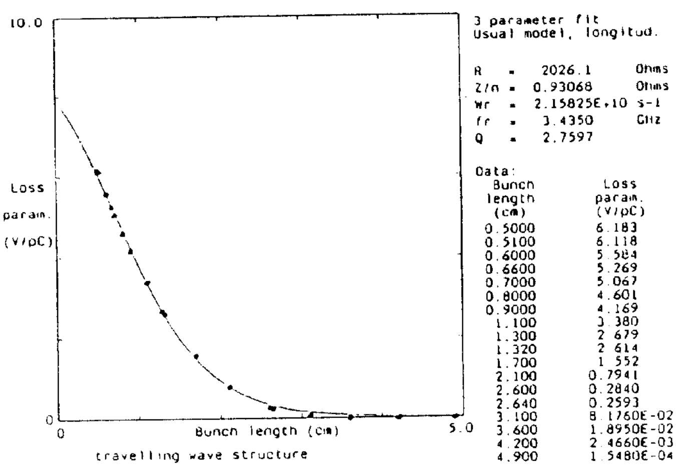

Fig. 1: Longitudinal loss parameter as a function of RMS bunch length for a travelling wave structure. The points are the result of TBCI calculations, and the solid line is a best fit of Eqs. 3–5 to the points.

C. Limitations

The fit of the loss parameter as a function of bunch length can produce erroneous results. In searching through parameter space to do the least squares fit, it is possible to become caught in some incorrect minimum. The incorrect minimum may have parameters (R, ω_r, and Q) significantly different from the true minimum, but nonetheless produce a fit of comparable quality. In order to understand this, a map of parameter space was made. The goodness of the fit was measured by the unnormalized χ^2, equal to the sum of the squares of the deviation of the fit curve from the data points generated by TBCI and MAFIA. All points were weighted equally. The parameters ω_r (the resonant frequency) and Q (the quality factor) were varied over a wide range. At each ω_r and Q, the shunt impedance R was adjusted to obtain the minimum χ^2. The χ^2 obtained in this fashion can then be plotted as a function of ω_r and Q. The results are shown in Fig. 2. From this we can see that it may be difficult to guarantee that the results of the fit are in the proper minimum.

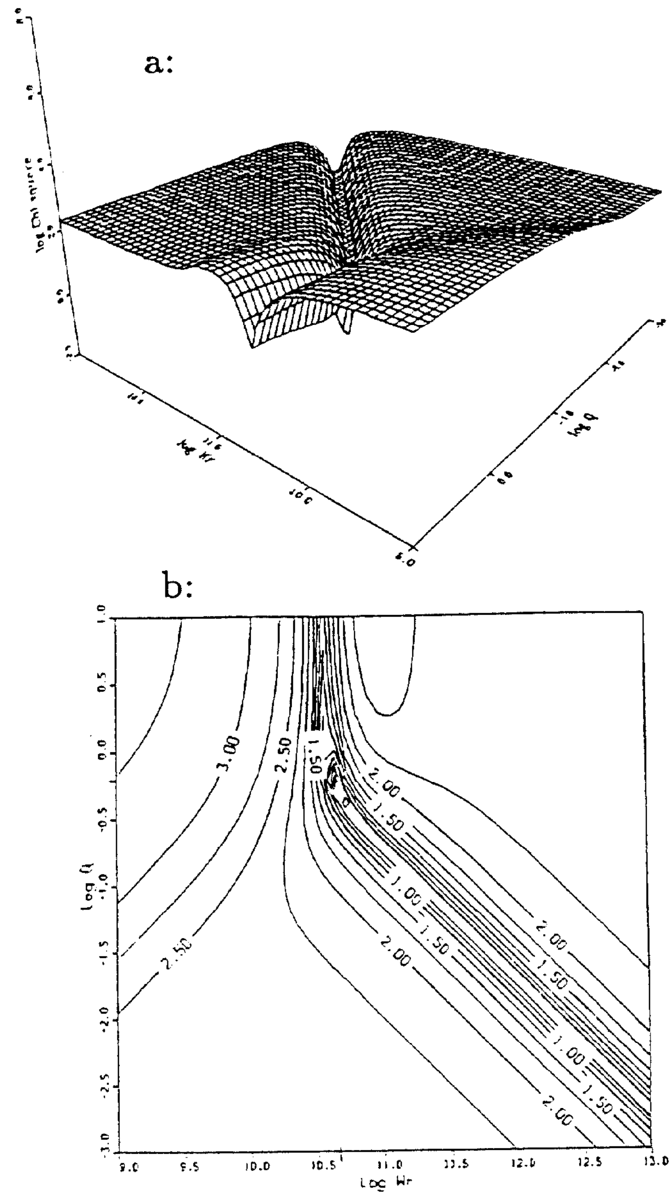

Fig. 2: **a:** Goodness of fit measured by χ^2, as a function of resonant frequency ω_r and quality factor Q. **b:** Contour plot showing the same results as the top three dimensional plot. Note that all scales are logarithmic.

In addition, the form of $Z(\omega)$ in Eq. (1) may not accurately describe the impedance of a ring component. Typically, in fact, any one component will have a more complicated dependence of the impedance $Z(\omega)$ on frequency.

Alternate forms of $Z(\omega)$ which may be more appropriate have been proposed [4], but we have not investigated them thoroughly.

D. Summary

Subject to the limitations described here, we have computed the broad band impedance of a variety of ring components. In general, it appears that it should not be too difficult to avoid single bunch instabilities, as long as reasonable care is taken in component designs. Because the 80 mA average current is spread among 1812 bunches in the ring, the charge per bunch is low, giving high impedance limits.

III. Coupled Bunch Instabilities

Although the large number of bunches around the ring helps reduce the problem of single bunch instabilities, there may be a serious problem with coupled bunch instabilities. Each bunch is separated from the next by 350 ps. As a bunch passes through a ring component with resonant modes, such as the RF cavity or an injection kicker tank, these modes may be excited. The fields of the excited modes will act on succeeding bunches, thus coupling the motion (both longitudinal and transverse) of all the bunches together. Instability may result.

The procedure used in analyzing the couple bunch problem was similar to that used for the single bunch case. A limit on the allowable impedances of the resonant modes was determined using analytic and numerical calculations. The impedances of all ring components were calculated using URMEL [5] and MAFIA [3], and compared to the impedance limits to see if they were below the instability thresholds.

The impedance limits found were 500 Ω for longitudinal modes, and 100 kΩ for transverse modes. These limits are such that the growth rates of any unstable beam modes should be less than the damping arising from finite momentum spread and betatron tune spread. The impedance limits were determined using several different means. These include the computer program ZAP [1], analytic calculations and a program developed by Balewski [6], and a numerical tracking program by Yu [7]. The results of all methods were in good agreement.

As part or our investigations, we have developed a simulation code [7] to model the transverse and longitudinal coupled bunch instabilities for a partially filled ring. For the transverse motion in a ring with a single RF cavity, where the length of the cavity is much smaller than the betatron wavelength, we can neglect the betatron phase advance within the RF cavity. The transport of the bunch from the output of the RF cavity to the input of the RF cavity is then given by the ring transport matrix. The kick received by the bunch within the cavity is determined by the wakefields of all bunches (including itself) that have previously passed through the cavity.

The code allows for an arbitrary number of particles in each bunch and various ring elements such as rf cavities, kickers, and internal target structures. For a uniform filling of the ring, the growth rate calculated from our code agrees with ZAP [1] to within 3%. The simulation extends the work of Thompson and Ruth [8] where bunches have identical charge, but are not uniformly distributed in the ring. By assigning a different charge to each bunch, the two turn injection used in the SHR, which results in a $\sim$ 15 ns of half filled bunches, can be examined. Preliminary calculations show that, for this loading, there are no significant differences in the growth rate from that of a uniformly filled ring.

A frequency domain formalism, which will be presented elsewhere, [7] has been used to find dispersion relations which yield the growth rates of both transverse and longitudinal instabilities for any distribution of bunch charge. For the case of the SHR, with 1812 bunches, the direct simulation technique is probably superior to an analytical solution which requires the inversion of a large matrix.

Further extensions of the simulation to include the slow extraction of the beam during the millisecond storage (in pulse stretcher operation) and the inclusion of internal bunch structure are planned.

IV. Conclusions

We have investigated both single bunch and coupled bunch instabilities in the MIT-Bates South Hall Ring. By calculating the broad band impedances of the various ring components, and comparing these to the allowable broad band impedance limits, we conclude that single bunch instabilities in the SHR should not be a significant problem, as long as reasonable care is taken in component design. This is due to the low current per bunch in the ring.

However, the impedance limits on resonant modes are stringent. This is due to the fact that there are 1812 electron bunches around the ring, each separated by 350 ps. Thus, coupled bunch instabilities can easily arise. By using a variety of design tools and instability calculations, we are working to minimize the potential severity of the problem.

V. References

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