Beam Breakup in Recirculating Linacs*

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ABSTRACT

In general, a recirculation path length in a recirculating accelerator has to be an integer multiple of RF wavelength if recirculations are all in the same direction. However, it is not necessary to require such a relation to be satisfied with respect to the bunching frequency, when the bunch repetition rate is different from the RF frequency. An analytic model of multipass beam breakup(BBU) in recirculating linacs studied by Bisognano and Gluckstern[1] has been generalized to include the case of a subharmonic bunching scheme in the operation of such linacs.

INTRODUCTION

Currently at CEBAF, we are undertaking a feasibility study to accommodate possible two color FELs (IR and UV) within the existing CEBAF structure, taking advantage of the excellent beam quality expected with a superconducting RF accelerator.[2] The envisioned infra-red FEL will use once-recirculated electron beam of 85 MeV from the CEBAF injector. The repetition frequency of FEL bunches is chosen at 7.5 MHz, which produces the peak current required for high gain within the limiting RF-power available. As a result, the bunching is at the 200-th subharmonic of the operating cavity mode of 1500 MHz. The recirculation set-up, which will serve as a test bed for beam dynamics calculations in coming months, has a path length which is 307 RF-period long. Since 307 is obviously not an integer multiple of 200, it has been necessary to extend the work of reference [1].

BBU EQUATIONS

An analytic model of multipass beam breakup in recirculating linacs has previously been studied by Bisognano and Gluckstern, whose notation we will closely follow for the convenience of readers. In this note, a generalization of the model is presented to include the case of a subharmonic bunching scheme in the operation of such linacs.

Consider a bunch(say, the M-th one) entering the n-th cavity at its p-th pass through the linac with its motion represented by a two component column matrix, $U_p(n, M)$ of x and p_x. While traversing the cavity, the bunch will get a momentum kick due to transverse wakes excited in the cavity by preceded bunches. The bunch will then proceed to the next cavity. The equation of transverse bunch motion described in this physical picture is

$$U_p(n+1, M) = T^{p,p}_{n+1,n} U_p(n, M)$$
$$+ I T^{p,p}_{n+1,n} G \sum_{q=1}^{n_p} \sum_{L=1}^{M+S(p,q)} U_q(n, L) s^{pq}_L(\omega_n, \tau) \; , \qquad (1a)$$

and

$$U_p(1, M) = T^{p,p-1}_{1,n_c} U_{p-1}(n_c, M)$$
$$+ I T^{p,p-1}_{1,n_c} G \sum_{q=1}^{n_p} \sum_{L=1}^{M+S(p-1,q)} U_q(n_c, L) s^{p-1q}_L(\omega_{n_c}, \tau) \; , \qquad (1b)$$

where

$$s^{pq}_L(\omega_n, \tau) = Z_n e^{-\frac{\omega_n}{2Q_n}((M-L)\tau+\tau_p-\tau_q)}$$
$$\times \sin \omega_n((M-L)\tau + \tau_p - \tau_q).$$

$T^{p,q}_{n,m}$ is the transfer matrix from the m-th cavity site at the q-th pass to the n-th one at the p-th pass. G is a 2×2 matrix with all elements equal to zero except $G_{21} = 1$. n_p and n_c are the number of passes and cavities, respectively. τ_p is the summation over recirculation times up to the p-th pass, and $\tau_1 = 0$. In general, τ_p has to be an integer multiple of RF period, τ_{rf} if recirculations are all in the same direction. However, it is not necessary to be an integer multiple of the bunch spacing τ, when subharmonically bunched. To illustrate a complication which may arise, let us consider a two pass system with only one cavity,

$$\tau_2 = 25 \; \tau_{rf}$$
$$\tau = 3 \; \tau_{rf} \; .$$

When a bunch, say the 55-th one arrives at the cavity site on its first pass, the 46-th bunch just crossed the cavity $2\tau_{rf}$ ago. On the other hand, the same bunch on its second pass gets kicked by the 63-rd bunch crossed the cavity one τ_{rf} ahead.

A proper counting of bunches is assured by having $S(p, q)$ defined as

$$S(p,q) = \begin{cases} int.(\frac{\tau_p - \tau_q}{\tau}), & \text{if } p \geq q \\ int.(\frac{\tau_p - \tau_q}{\tau}) - 1, & \text{otherwise.} \end{cases} \qquad (2)$$

It is also necessary to introduce τ_{pq}, R_{pq}, and M_p

$$\tau_{pq} = \tau_p - \tau_q - S(p,q)\tau \; ,$$
$$R_{pq} = S(p,q) - M_p + M_q \; , \qquad (3)$$
$$M_p = int.(\frac{\tau_p}{\tau}) \; .$$

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We look for a steady state solution in the asymptotic region where $M \to \infty$. Assuming that such a solution exists, and we write

$$U_p(n, M) = e^{i\Omega M\tau} V_p(n) \qquad (4)$$

A negative imaginary part of Ω would mean an instability. One obtains from equations (1a) and (1b)

$$V_p(n+1) = T^{p,p}_{n+1,n} V_p(n)$$

$$+ I T^{p,p}_{n+1,n} G \sum_{q=1}^{n_p} e^{i(M_p - M_q)\Omega\tau} w_n^{pq}(\Omega) F_n(\Omega) V_q(n) \ , \qquad (5a)$$

and

$$V_p(1) = T^{p,p-1}_{1,n_c} V_{p-1}(n_c)$$

$$+ I T^{p,p-1}_{1,n_c} G \sum_{q=1}^{n_p} e^{i(M_{p-1} - M_q)\Omega\tau} w_{n_c}^{p-1 q}(\Omega) F_{n_c}(\Omega) V_q(n_c) \ , \qquad (5b)$$

where

$$F_n(\Omega) = \frac{Z_n}{1 + H_n^2(\Omega) - 2H_n(\Omega)\cos\omega_n\tau} \ ,$$
$$H_n(\Omega) = e^{-\frac{\omega_n\tau}{2Q_n}} e^{-i\Omega\tau} \ .$$

All the complications due to subharmonic bunching are now isolated in one place with $w_n^{pq}(\Omega)$, where

$$w_n^{pq}(\Omega) = e^{-\frac{\omega_n \tau_{pq}}{2Q_n}} e^{i\Omega R_{pq}\tau} \qquad (6)$$

$$\times(\sin\omega_n\tau^{pq} + H_n(\Omega)\sin\omega_n(\tau - \tau^{pq})) \ .$$

By iterating equations (5a) and (5b), an eigenvalue problem for the threshold current of multipass beam breakup emerges. In the most general circumstance where none of τ_{pr} is zero, we must deal with a $n_c \times n_p$-1 dimensional matrix:

$$\frac{1}{I} G\bar{V}_p(m) = \sum_{q=1}^{n_p} \sum_{n=1}^{m-1} w_m^{pq}(\Omega) F_n(\Omega) G T^{qq}_{m,n} G\bar{V}_q(n)$$

$$+ \sum_{q=2}^{n_p} \sum_{r=1}^{q-1} \sum_{n=1}^{n_c} e^{-i(M_q - M_r)\Omega\tau} w_m^{pq}(\Omega) F_n(\Omega) G T^{qr}_{m,n} G\bar{V}_r(n) \qquad (7)$$

where

$$\bar{V}_p(m) = \sum_{q=1}^{n_p} e^{-iM_q\Omega\tau} w_m^{pq}(\Omega) V_q(m) \ .$$

The first term on the right hand side of equation (7) is understood to vanish in the case of a single cavity system. We also notice that $\bar{V}_p(m) = \bar{V}_r(m)$ if $\tau_{pr} = 0$. As a result, depending on the relation between recirculation lengths and the bunching frequency, the matrix dimension of this eigenvalue problem can be smaller. Therefore, caution is required in setting up the matrix for a numerical study. When all $\tau_{pr} = 0$, equation (7) reduce to equation (4) of reference [1].

ANALYSIS

For the simplest example, let us consider a one cavity–two pass system. In this case, we have a one dimensional eigenvalue problem

$$\frac{1}{I} G\bar{V}_1(1) = e^{-iM_2\Omega\tau} w_1^{12}(\Omega) F_1(\Omega) G T^{21}_{1,1} G\bar{V}_1(1) \ . \qquad (8)$$

If $\tau_{21} = 0$, which will be the case when every bucket is filled, $w_1^{12}(\Omega)$ and $F_1(\Omega)$ combine to produce a function

$$\frac{Z_1 H_1(\Omega)\sin\omega_1\tau}{1 + H_1^2(\Omega) - 2H_1(\Omega)\cos\omega_1\tau} \ .$$

This is the function $Z_1 h_1(\Omega)$ defined in reference [1]. For a high Q mode such that $\frac{\omega_1\tau}{2Q} \ll 1$, a good approximation for the threshold current, when the mode is not near at a harmonic of the bunching, is obtained from

$$I_{th} = \frac{2\omega_1}{eZ'' T^2 T^{21}_{1,1} \sin M_2(\omega_1\tau + K)}$$

where K is the solution of

$$K = \frac{\omega_1\tau}{2Q} \cot M_2(K + \omega_1\tau)$$

It is interesting to study how the threshold current depends on the bunch repetition frequency for a given higher order mode(HOM). We have looked at this problem and others by solving equation (8) numerically. The specific parameters chosen for the input are

$$\text{recirculation path length} = 300\ \tau_{rf}$$
$$\text{HOM frequency} = 1890.35\ MHz$$
$$\frac{Z'' T^2}{Q} = 1.57 \times 10^5\ \Omega/m^3$$
$$\text{Q of the mode} = 32000$$

Figure 1 is the plot of the threshold current vs. the bunching subharmonic. For a given deflecting HOM, the BBU limit is quite sensitive to the relation between the three frequencies involved; the HOM frequency, the bunching frequency and the recirculation frequency determined by the path length. Let us take a 9-th subharmonic bunching for an example. We read from Figure 1 that BBU instability would not be seen for a beam current less than 38.5 mA. However, this will change if the mode frequency were shifted to a nearby value due to manufacturing tolerances of cavities or for many other reasons. In Figure 2, we see the effect of HOM frequency shifts on the threshold current. Minimum threshold is at $1.06mA$.

We have also scanned in frequency a 31–dimensional matrix obtained by modeling CEBAF recirculating injector linac with 16 cavities. The acceleration takes place in two cryomodules, each module consisted of 8 superconducting 5–cell cavities. The dipole mode excited is the

1890 mode described in the previous paragraph, and the recirculation path length is adjusted to 307 τ_{rf}. It turns out that this mode produces the lowest threshold. Figure 3 is the result of such a scan showing the threshold current at 4.74 mA for the 200-th subharmonic bunching scheme envisioned. This has been compared with the prediction of a multipass beam breakup simulation code TDBBU developed at CEBAF[3]. Threshold displacement of bunches at the 16-th cavity site on their 2-nd pass is shown in Figure 4 when the current is 4.85 mA. The agreement between these two are quite encouraging.

CONCLUSION

Formulation of multipass BBU as a matrix eigenvalue problem has been properly extended to accommodate general subharmonic bunching schemes. Comparison of results with a bunch to bunch simulation based on equations (1a) and (1b) confirms also the consistency of our formulation, which is generally less time consuming in searching for a threshold and thus provides a useful complementary tool to the simulation.

ACKNOWLEDGEMENT

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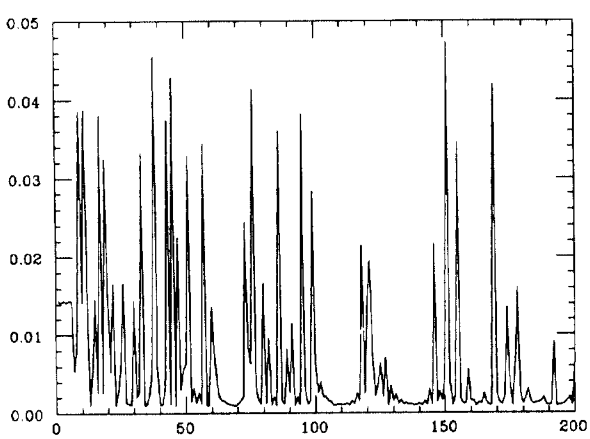

Figure 1. Threshold current of a single cavity–two pass system *vs.* bunching subharmonic. Sensitivity and degree of fluctuation decreases as the Q of offending mode reduces.

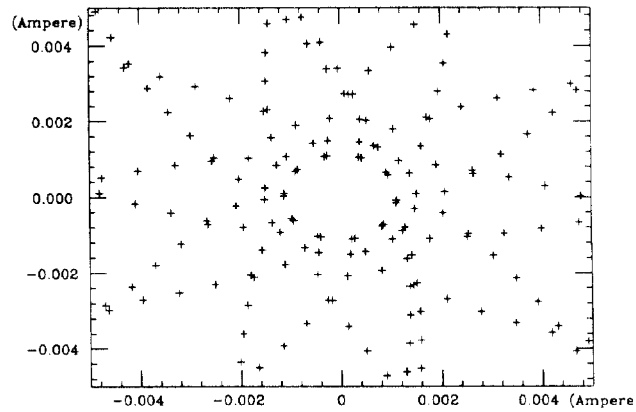

Figure 2. Plot of complex eigenvalues near origin for the case of the 9-th subharmonic bunching. The mode frequency is distributed uniformly with a spread of 2.9 MHz. Minimum threshold current is shown to occur at 1.06 mA.

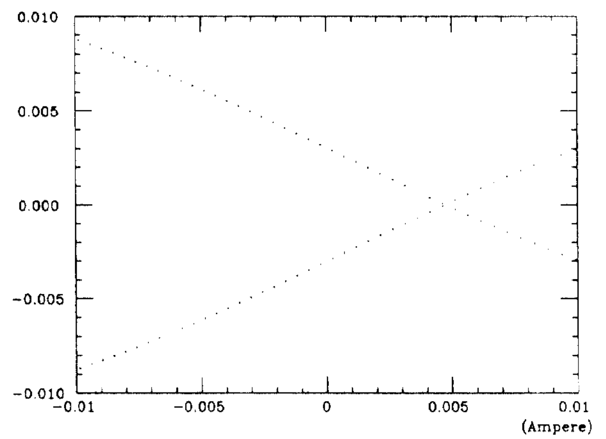

Figure 3. Scan of eigenvalues to locate the threshold current of 16–cavity model of recirculating CEBAF injector in a FEL operating mode.

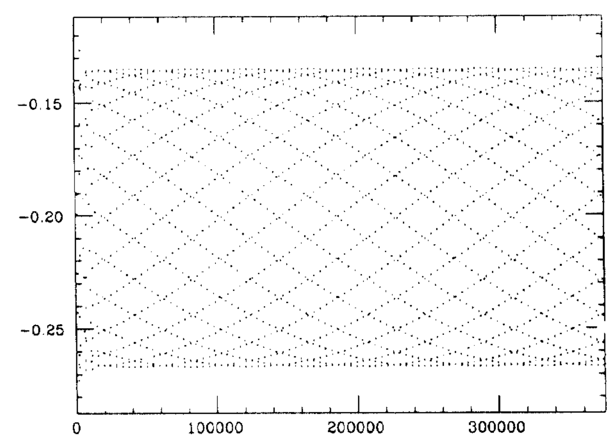

Figure 4. Transverse coordinates of successive bunches passing the last cavity. Bunch motion was tracked for 375,000 τ_{rf} long.