The Self-Cooling of Charged Particle Beams in a Straight Line

Ya. S. Derbenev
Randall Laboratory of Physics, University of Michigan
Ann Arbor, Michigan 48109-1120 USA

Abstract

A method of decreasing the beam emittance is considered using intrabeam Coulomb scattering of particles during the processes of beam formation and acceleration. The necessary conditions for efficient heat transfer from transverse degrees of freedom into longitudinal ones are formulated, and the principal limitations on the rate of cooling and achievable temperature are established. An example of the self-cooling is given for the case of a beam formed and accelerated in the space charge regime and in the presence of external accompanying magnetic field.

Introduction

Cooling of charged particle beams is one of the important topics of modern accelerator physics. Four methods are known and are either used today or have good prospects to be applied for the cooling of circulating beams:

1. radiation damping of ultrarelativistic particles in synchrotrons;
2. electron cooling which is based on the use of a co-moving electron beam, straight or circulating, as in a thermostat;
3. stochastic cooling based on the use of RF feedback systems;
4. laser cooling of ion beams.

Among these methods, only the electron cooling is capable of cooling a straight low-energetic proton or ion beam within an acceptable length of cooling section; it is effective due to a very low value of the longitudinal electron temperature ($T_{\parallel}^{e} \sim 10^{-4}$ eV) and the transverse motion of electrons being bounded because of the magnetic field. However, in view of the longitudinal heating of electrons by the ions of the beam under cooling, the ratio between the electron's and the ion's current densities must be not less than $T_{\perp}^{i}/T_{\parallel}^{e}$. Therefore, the use of electron cooling for the ion beams with current density above $\sim$ 1 mA/cm^2 seems to be problematic.

In this report, we focus on the possibility of using intrabeam scattering of particles for the transverse cooling of a beam, with corresponding heating in the longitudinal direction; in this process, there are no external heat energy transfers from the beam. In such a situation, the total beam entropy is not decreased, but there is redistribution between the degrees of freedom of the beam. The decreasing of the transverse entropy then occurs under the condition $T_{\perp} > T_{\parallel}$; this condition can be maintained for a long time by longitudinally stretching the beam during acceleration and by transversely compressing the beam. The increase of the total entropy is small when these processes are performed slow with respect to the process of temperature relaxation. Note that the adiabatic process should start from a state of $T_{\perp} = T_{\parallel}$, in order to avoid a substantial increase of the entropy during the initial stage of the transverse cooling. Also, if we cannot exclude some intermediate states of the beam when $T_{\perp}$ is not close to $T_{\parallel}$, these states should be passed as fast as possible in order to minimize the resulting transverse phase space volume of the beam.

Self Cooling in the Adiabatic Limit

We describe evolution of the beam by the variables

$$\Gamma_{\perp} = T_{\perp} \cdot \pi a^2, \quad \Gamma_{\parallel} = \sqrt{T_{\parallel}} \cdot \frac{\gamma v e}{I},$$

where

$$T_{\perp} = \frac{<(\Delta \vec{p}_{\perp})^2>}{2m}; \qquad T_{\parallel} = \frac{<(\Delta p_{\parallel})^2>}{\gamma^2 m},$$

a is the beam radius, v is the average longitudinal velocity, $\gamma = 1/(1-\frac{v^2}{c^2})^{1/2}$, e is the particle charge, and I is the beam current. The variables $\Gamma_{\perp}$ and $\Gamma_{\parallel}$ are the dynamical invariants of the beam as an ensemble of particles; they can be considered as the transverse and longitudinal phase space volume of an element of beam length, respectively. The product $\Gamma = \Gamma_{\perp} \cdot \Gamma_{\parallel}$ is the corresponding 6-dimensional phase space volume of the element of beam length. Near the equilibrium state $T_{\perp} = T_{\parallel} = T$ we have

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$$\Gamma_\perp = \pi a^2 T, \quad \Gamma_\parallel = \frac{\gamma v}{I} e\sqrt{T}, \; \Gamma = \gamma \frac{T^{3/2}}{n},$$

where n is the particle concentration. During the adiabatic process, the parameters v, a, and I may change, but the volume Γ remains unchanged and we get the relation between values of $\Gamma_\perp$ at the end to that at the beginning of the adiabatic process as:

$$\frac{\Gamma_\perp}{\Gamma_{\perp o}} = [(\frac{Ia}{\gamma v})/(\frac{Ia}{\gamma v})_0]^{2/3} \tag{1}$$

The maximum cooling effect occurs when all the process of beam formation and acceleration is performed adiabatically. In this case, the value v_0 is related to the cathode temperature, T_c, ($T_c = mv_0^2$) and a_0 is the beam size at the cathode, a_c. Table 1 gives an illustration of the maximum cooling effect for a heavy particle beam assuming no beam bunching after acceleration.

Table 1	
Maximum Cooling Effect	
Initial beam parameters	
Cathode temperature, eV	0.1
Beam radius at the cathode, cm	0.5
Parameters after acceleration	
Top energy after acceleration, MeV	100
Beam radius after acceleration, cm	0.02
Self-cooling effect	
Decreasing of beam emittance, times	100
Decreasing of transverse phase space volume, times	10^4

The adiabatic relation (Equation 1) and the conservation of Γ are valid under the condition

$$\lambda_d^{-1} \equiv 2\,(\frac{\gamma v}{Ia})' \cdot \frac{Ia}{\gamma v} << \lambda_{st}^{-1}. \tag{2}$$

where $'$ denotes change with respect to the longitudinal direction and λ_{st}^{-1} is the collision relaxation parameter defined from

$$(T_\perp - T_\parallel)'_{st} = -\lambda_{st}^{-1}(T_\perp - T_\parallel). \tag{3}$$

Near the equilibrium state $T_\perp = T_\parallel$, the parameter λ_{st}^{-1} is equal to

$$\lambda_{st}^{-1} = \lambda_{eq}^{-1} = g \cdot \frac{8\sqrt{\pi}ne^4L}{5\sqrt{m}T^{3/2}\cdot\gamma^2 v} = g\frac{8\sqrt{\pi}Le^4}{5\sqrt{m}\gamma v}\cdot\frac{1}{\Gamma}, \tag{4}$$

where g is a numerical factor with an order of value of about 1, and L is the Coulomb parameter

$$L = \frac{1}{2}\ln(\frac{\gamma T^3}{4\pi ne^6}), \tag{5}$$

with an order of value of about 3–5.

We can see from formula (4) that it is very important to have as small a value of Γ as possible in order to get a maximum rate of temperature relaxation and satisfy the adiabatic condition (2).

Self-cooling of an Actual Beam

When accelerating an actual beam, the adiabatic condition cannot be satisfied in the region near the cathode, because the characteristic time of acceleration there is about equal to the inverse plasma parameter:

$$\omega_p^{-1} = \sqrt{m/4\pi ne^2},$$

which is small in comparison to the temperature relaxation time λ_{st}/v. With the acceleration, the longitudinal temperature goes down very fast, and one must take into account intra-beam scattering which can limit its decrease, i.e. the longitudinal entropy will increase with collisions between particles (see the Appendix). After a distance of about a_c from the cathode, we can equalize the transverse and longitudinal temperatures by having the transverse expansion of the beam and, if necessary, by deacceleration of the beam. In this state, we obtain an intermediate energy W_0 such that $T_c << W_0 << W_{max}$, with initial (maximum) radius a_0 and initial value of relaxation parameter λ_{st}^{-1}. With these parameters, we can start the adiabatic cooling process (1). In view of the presence of the non-adiabatic stage at the beginning of the beam evolution, the self-cooling effect will be less than potentially possible as was presented in Table 1 (see Table 2).

In addition, we should note the following conditions for the beam dynamics in focusing and accelerating:

1. A solenoidal magnetic field can be used in order to keep an intensive low energy beam from repulsion by the space charge effect.
2. The current distribution at the cathode and the accelerating electric field should have axial symmetry.

3. Electric and magnetic fields have to be matched in the region of the beam injection into the solenoid, in order to avoid radial beam excitation inside the solenoid, i.e., to reach the Brilluen's beam state.

Table 2	
Self-cooling of an Actual Beam	
Beam current I, A	1
Beam radius at cathode a_c, cm	0.5
Cathode temperature T_c, eV	0.1
Anode voltage V_A, kV	10
Longitudinal length of expansion section cm	6
Beam radius after expansion a_0, cm	3
Initial energy of the adiabatic process W_0, keV	10
Initial relaxation length λ_0, m	0.3
Final energy W_f, MeV	100
Maximum value of solenoidal field B_f, Tesla	10
Final beam radius in the solenoid a_f, cm	0.02
Final relaxation length λ_f, m	15
Cooling effect on beam emittance, times	25
Cooling effect on beam brightness, times	600

Appendix

To describe the evolution of a beam, we use the equations as follows:

$$\Gamma'_\perp = \pi a^2 (T'_\perp)_{st}, \quad \Gamma'_\parallel = \gamma v e (T'_\parallel)_{st} / 2I\sqrt{T_\parallel} \qquad (A1)$$

where $(T'_\perp)_{st}$, $(T'_\parallel)_{st}$ are the rates of temperature change under intrabeam collisions. With the definition (3) and taking into account heat energy conservation for collisions, we obtain:

$$(T'_\parallel)_{st} = -2(T'_\perp)_{st} = 2\lambda_{st}^{-1}(T_\perp - T_\parallel)/3 \qquad (A2)$$

From $(A1)$ and $(A2)$, we get immediately:

$$\Gamma' = \Gamma'_\perp \Gamma_\parallel + \Gamma_\perp \Gamma'_\parallel = \lambda_{st}^{-1} \cdot (T_\perp - T_\parallel)^2 \Gamma / 3T_\perp T_\parallel \quad (A3)$$

It is not difficult to calculate λ_{st}^{-1} in the cases of $T_\perp >> T_\parallel$ and $T_\perp << T_\parallel$, using the Landau integral [1]:

$$\lambda_{st}^{-1} = \frac{2Ie^3}{\sqrt{m}v^2a^2} \cdot \begin{pmatrix} \frac{\sqrt{\pi}}{T_\perp^{3/2}} L(T_\perp), & \text{at } T_\perp >> T_\parallel \\ \frac{L^2(T_\parallel) - L^2(T_\perp)}{\sqrt{\pi} T_\parallel^{3/2}} & \text{at } T_\perp << T_\parallel \end{pmatrix},$$

where $L(T_\parallel)$, $L(T_\perp)$ are functions like (5) with effective temperatures $T_\parallel$ and $T_\perp$, respectively. In order to calculate the numerical factor in (4) in the case when $T_\perp \approx T_\parallel$, we use the model with Gaussian distribution in temperatures $T_\perp$ and $T_\parallel$; then we get $g = 1$.

We can calculate the increase of Γ in the quasiadiabatic case. Taking into account that $\lambda_d >> \lambda_{st}$, we find from the equations $(A1)$ and $(A2)$ that

$$\Delta T \equiv T_\perp - T_\parallel \approx T\lambda_{eq}/\lambda_d;$$

then, after substituting ΔT in (A3), we get:

$$\Gamma' \approx (\lambda_{eq}/3\lambda_d^2)\Gamma$$

Solving this equation, one can establish the boundaries of stability of the quasiadiabatic process and calculate non-adiabatic effects.

The relaxation equations $(A1)$ and $(A2)$ can be used also to calculate collision effects when one of two temperatures, $T_\parallel$ or $T_\perp$, becomes small in relation to the other due to fast non-adiabatic space expansion of the beam. We consider two cases as follows:

1. $T_\parallel << T_\perp$; this case corresponds to non-adiabatic acceleration of a beam from the cathode. Then, we have from equations $(A1)$:

$$(v^2 T_\parallel)' = 2\sqrt{\frac{\pi}{T_\perp m}} \frac{Ie^3 L(T_\perp)}{a^2}.$$

We can integrate this equation from the cathode surface to the state $T_\parallel = T_\perp = T_0$ to logarithmic accuracy taking into account, that $T_\perp a^2 = \text{const}$.

2. $T_\perp << T_\parallel$; we get this situation when the beam is expanded fast transversely after the adiabatic acceleration to maximum energy. Then $\Gamma_\perp$ increases with collisions as:

$$\Gamma'_\perp = 2\sqrt{\pi} Ie^3 [L^2(T_f) - L^2(T_\perp)] / v_f^2 \sqrt{T_f m},$$

with $T_\perp = \Gamma_\perp / \pi a^2$.

References

[1] L. D. Landau, Zh. Eskp. Teor. Fiz. **7**, 203 (1937).