WAKEFIELD SUPPRESSION USING BEATWAVE STRUCTURES*

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Abstract

A proposed method of suppressing transverse wakefields in an accelerating structure makes use of the fact that superposition of long-range wakes excited by an electron bunch traversing a series of accelerating cells with different transverse frequencies can produce interference cancellation, thereby significantly reducing the magnitudes of the harmful wake potentials. Analytic calculations as well as time-domain and modal sum simulations are performed to study the beatwave effects produced by detuned, disk-loaded cavities as a function of their transverse frequency spread and the population density.

INTRODUCTION

A major concern in designing the high gradient accelerating structure for future high-luminosity linear colliders is how to minimize beam instability and energy spread of electron bunches caused by long-range wakefields generated by preceding bunches traversing the accelerating cavities. Several ideas have been proposed in the last few years to suppress the transverse wakefields. While in principle low-Q cavities coupled to external waveguides[1] can be designed to take out higher order modes (HOMs), these designs are quite complicated and can pose many manufacturing problems. Furthermore the problem of discarding the HOMs, once taken out of the accelerating structure, remains challenging. In parallel with this approach, one of us (DY) has proposed a different method of suppressing the long-range HOM wake potentials. In this method, accelerating cells with the same fundamental frequency are detuned for all HOMs. Interference of wakefields at different frequencies for each HOM, made possible by cell-to-cell structural variations, would considerably diminish the amplitudes of all transverse and higher order longitudinal wake potentials.

A simple way to implement this concept is to mix cavities with different iris diameters in a disk-loaded accelerating structure. By properly adjusting other cavity parameters (e.g. the cavity radius), the fundamental frequency of the accelerating mode is identical for all cavities, but the frequency of HOM varies from cavity to cavity. In an early illustration of this idea, the transverse wakes of a three-cell "beatwave" structure were calculated. Figure 1 shows the dipole wakes (from a 2/27/89 TBCI run) as a function of the distance behind the bunch, for a 3-cell, 2.856-GHz, $2\pi/3$-mode structure with an aperture radius to wavelength ratio (a/λ_0) of 0.20, 0.15 and 0.10. The corresponding spread in the lowest transverse frequency is from 3.70 to 4.35 GHz. Two effects were apparent from the results of this simulation. First the interference effects indeed produced a relatively broad null in the combined wake potential. Secondly there was a gradual damping of subsequent peaks of the wake potential.

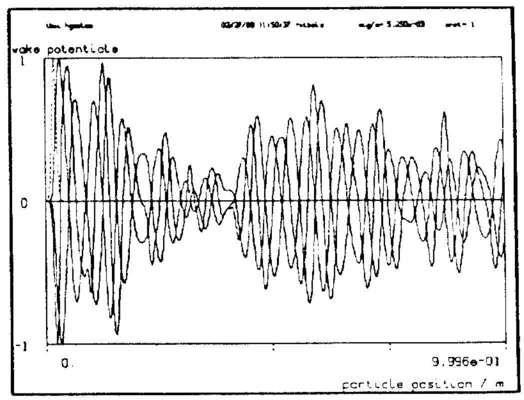

Fig. 1 TBCI dipole wakes for a 3-cell structure

The interference nulls in the wake potentials were further studied in detail by Yu and Wilson[2]. They showed by time and frequency domain calculations that using only two different sets of disk-loaded accelerator cells, relatively broad beating nulls in the transverse wakes could be produced. Since the position of the nulls varies as a function of the transverse frequency spread, it is possible to adjust the amount of detuning required so that the following bunch can be placed at the resulting interference nulls of the long-range transverse wake. For a beatwave structure with a/λ_0 = 0.20 and 0.15, resulting in a frequency spread of ≈8% for the dominant transverse mode, the interference effects reduced the amplitude of the transverse wake by an order of magnitude from its maximum at a distance of 48 cm for 2.865 GHz, or 12 cm for 11.424 GHz. For such a beatwave structure, the wakes are suppressed over a distance on the order of a wavelength.

DAMPED BEATWAVE STRUCTURE

The second effect observed in Figure 1, i.e. damping of the long-range wake and broadening of the region of wake suppression, becomes increasingly pronounced as the number of transverse frequencies participating in the interference beating increases. As shown below, the damping rate of the transverse wake and the length of the wake suppression region are proportional to the population density (i.e. number of different frequencies) in a given frequency spread. The damping effect was recognized recently also in accelerator structure work at SLAC[3], where it was shown that substantial damping of the transverse

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wake could be sustained over a long distance behind the bunch provided a very large number of cavities with a Gaussian distribution of transverse frequencies were used.

We will first demonstrate the damping effect analytically in the continuum limit. Although the principle of wakefield suppression by interference of detuned cavities applies to any three-dimensional accelerating structure, it is straightforward to illustrate the effect using cylindrically symmetric disk-loaded structures. The dipole wake potential for such a structure is given by[4]:

$$W_d(s) = \frac{2r_0}{a}\sum \frac{k_{1n}}{\omega_{1n}a/c}\, e^{-\omega_{1n}^2\sigma^2/2c^2} \sin(\omega_{1n}s/c)$$

where s is the distance behind the center of the bunch, r_0 is the offset of the driving charge and a is the aperture of the cavity. ω_{1n} and k_{1n} are respectively the angular frequencies and the loss factors of the nth transverse mode of the cavity. σ is the bunch length, and c is the speed of light. Consider now only the lowest, dominant dipole mode for an infinite number of detuned cavities with a frequency spread of $\Delta\omega = \omega_2 - \omega_1$. Superposition of the transverse dipole wakes gives

$$W_d \propto \int_{\omega_1}^{\omega_2} d\omega \left(\frac{k_1 c}{\omega a^2}\right) e^{-\omega^2\sigma^2/2c^2} \sin(\omega s/c)$$

for a continuous range of equally weighted frequencies between ω_1 and ω_2; and

$$W_d \propto \int_{\omega_1}^{\omega_2} d\omega\, e^{-\left(\frac{\omega-\omega_a}{\Delta\omega}\right)^2}\left(\frac{k_1 c}{\omega a^2}\right) e^{-\omega^2\sigma^2/2c^2} \sin(\omega s/c)$$

for a Gaussian frequency distribution centered at ω_a = $(\omega_1+\omega_2)/2$ with a half width of $\Delta\omega$. Define the scaleable dimensionless parameters: $x \equiv \omega/\omega_0$ and $\bar{s} \equiv s/\lambda_0$, where the subscript 0 refers to the fundamental mode. We have calculated values of k_1, ω_0 and ω using the TRANSVRS program for selected values of a/λ_0 ranging from 0.10 to 0.20. For this range of parameters, the combination $k_1c/\omega a^2$ (for the dominant transverse mode) can be expressed as a linear function in x, i.e. C_1+C_2x. Substituting into the above equations, we find, for $\sigma/\lambda_0 \ll 1$,

$$W_d \propto \frac{1}{\pi\bar{s}}\left[(C_1+C_2x_1)\sin(\pi\bar{s}x_+)\sin(\pi\bar{s}x_-) - \frac{C_2}{2}x_-\cos(2\pi\bar{s}x_2)\right] + \frac{C_2}{2\pi^2\bar{s}^2}\sin(\pi\bar{s}x_-)\cos(\pi\bar{s}x_+)$$

with $x_+ \equiv x_1+x_2$, and $x_- \equiv x_2-x_1$, for the case of flat distribution; and

$$W_d \propto (\Delta\sqrt{\pi})\, e^{-\Delta^2\pi^2\bar{s}^2}[(C_1+C_2x_a)\sin(2\pi\bar{s}x_a) + 2\pi^2\bar{s}\,\Delta^2 C_2\cos(2\pi\bar{s}x_a)]$$

+ terms containing complex Error functions,

with $\Delta \equiv \Delta\omega/\omega_0$ for the case of Gaussian distribution. At large distance, the wake potential falls off as $1/\bar{s}$ for the flat distribution, and as $\bar{s}\cdot\exp(-\Delta^2\pi^2\bar{s}^2)$ for the Gaussian distribution. These falloffs are further modulated by a sinusoidal function with a period inversely proportional to the frequency spread. The location of the first wake null for the flat distribution is also inversely proportional to the frequency spread. The dipole wake potentials for a flat, continuum, transverse frequency distribution is illustrated in Figure 2a for a frequency spread of 9% centered at $x_a = 1.4$. The wake potentials for truncated Gaussian and Gaussian distributions are shown, respectively, in Figures 2b and 2c. Contributions to the wake potentials from higher order modes can be similarly included by summing over these modes.

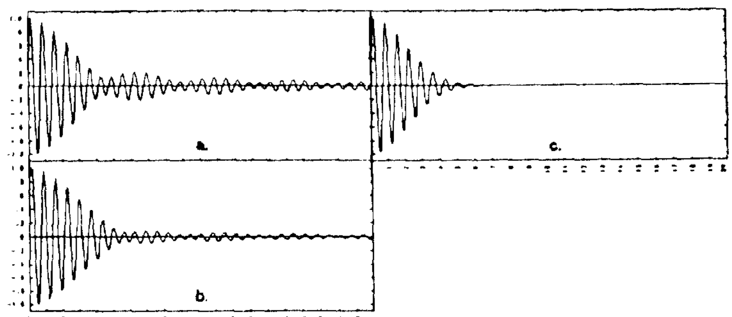

Fig. 2 Transverse wakes from analytic calculations

WAKEFIELD SIMULATIONS

The transverse wake damping rate and the locations of the interference nulls which characterize a beatwave structure with a finite number of discretely detuned cavities are determined with numerical simulations. Using a relatively small number of these cavities, it is possible to design a beatwave structure which provides a suppressed transverse wake profile tailored to match the multibunch requirements of a linear collider.

Extending the work of Yu and Wilson[2], we calculate the wake potential for a train of $2\pi/3$-mode, disk-loaded cells with different irises, using both the time-domain TBCI code, and the modal summation method in the frequency domain. By fixing the fundamental frequency, the cavity radius, b, is calculated for nine values (a/λ_0 = 0.10 to 0.20, in steps of .0125) of the beam-hole radius, a, using the KN7C program. With no loss of generality, an S-band frequency of 2.856 GHz is used as the basis for scaling. The cavity parameters and the HOM frequencies are completely scaleable with the fundamental frequency of any disk-loaded structure, i.e. S-band, X-band, etc. The corresponding transverse frequencies for the first 25 HOMs are calculated with the TRANSVRS program. Scaling of the wake potentials vs fundamental frequency is given in reference 2.

To study the variation of the wake potential as a function of the population density and the frequency spread, we use TBCI to calculate the combined wakes for several beatwave structures:

case	$(a/\lambda_0)_{min}$	$(a/\lambda_0)_{max}$	no. of cavities
a	0.100	0.200	3
b	0.100	0.200	5
c	0.100	0.200	9
d	0.125	0.175	5
e	0.125	0.175	20

A long beam pipe is appended to the structure in the TBCI models in order to avoid the problem of reflections at the boundaries. The mesh size is chosen so that the bunch length (σ/λ_0 =0.025) is represented by at least five mesh points. Figures 3a-d show the TBCI transverse wakes (m=1) for cases a-d. The wake potential is normalized to its maximum value, and plotted as a function of the dimensionless distance, s/λ_0. Comparison of Figures 3a, b and c shows that for a given frequency spread, the position of the first null remains the same, while the damping improves as the number of cavities increases. The residual wake is damped to 55%, 33% and 25% of the initial peak for 3, 5 and 9 cavities, respectively. Comparison of Figures 3b and 3d shows the relationship between the null position and the frequency spread. For a frequency spread of 16.2%, the first null occurs at s/λ_0 = 3.5. For a frequency spread of 8.6%, the null occurs at s/λ_0 = 6.5. The distance of the first null from the center of the leading bunch is thus inversely proportional to the frequency spread. TBCI calculations for the monopole (m=0) mode show that the longitudial wakes are well preserved for the accelerating mode. Higher-order longitudinal wakes, as well as quadrupole (m=2) transverse wakes are found to be effectively damped.

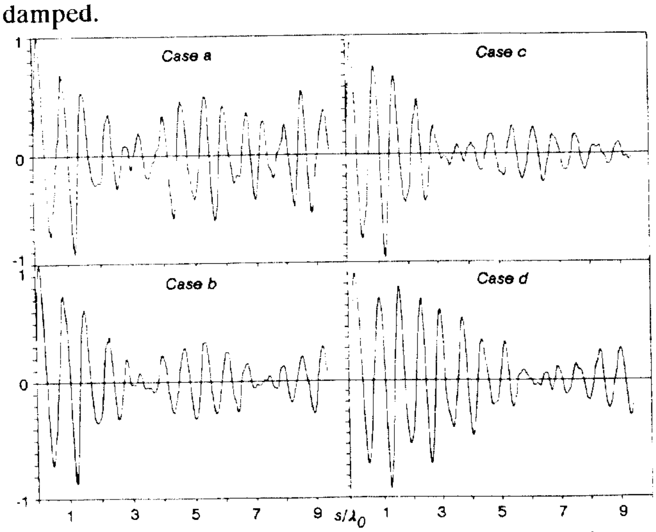

Fig. 3 TBCI transverse wakes for cases a-d

Longer-range transverse wakes are calculated with the modal summation method. Figures 4a-c show the beatwave effects due to the lowest transverse mode for cases b-d. Figures 4e-g include 25 transverse modes with σ/λ_0=0.025 for the same cases. The effectiveness of long range damping is further enhanced by the inclusion of higher order modes. These figures also show clearly the range broadening of the wakefield suppression as the number of cavity increases. It is possible to construct a region of suppressed wakes broad enough to accommodate the entire train of multibunches (say, about $100\lambda_0$). Figure 4d shows the transverse wake potential from 20 different frequencies with a frequency spread of 9% (case e). For a larger frequency spread of 16% (as in cases a-c), about 30 different frequencies would be needed in order to suppress recurring peaks in the wake potential within a distance of $100\lambda_0$. The amplitudes of the residual wakes decrease exponentially as the number of beating frequencies increases. However unless the number of frequencies is extremely large, a small residual wake persists. The cumulative residual transverse wakes in the beatwave structure can be taken out using waveguide coupled damping structures. Because of the effectiveness of the beatwave method to suppress long range wakes, such damping structure is not required for every cavity. This will simplify the manufacturing process considerably.

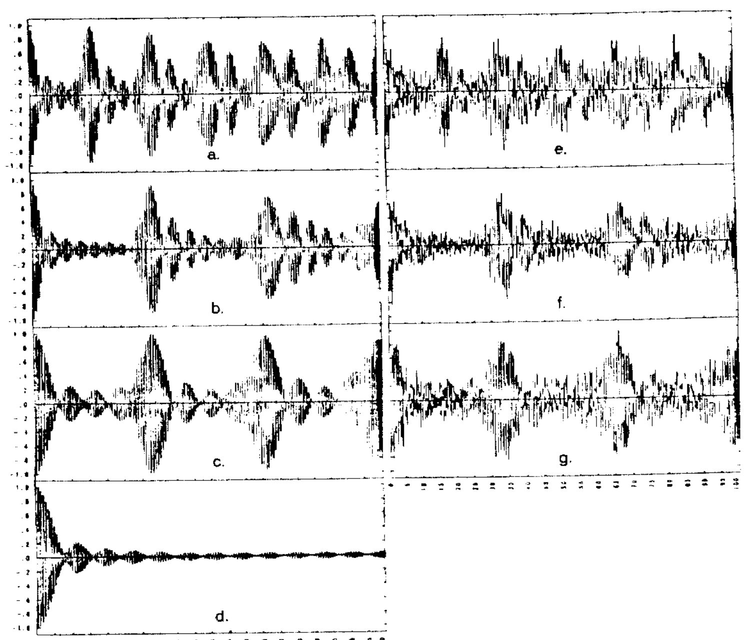

Fig. 4 Transverse wakes from modal sum method

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[2]D. Yu and P. Wilson, SLAC-PUB-5062, September (1989); and Particle Accelerators 30, pp.65-72 (1990).

[3]H. Deruyter, et al., Proceedings of 1990 Linear Accelerator Conference, September 10-14 (1990) p. 132.

[4]See, for example, P. Wilson, SLAC-PUB-4547, January (1989).