Dynamic Aperture of Low Beta Lattices at Tevatron Collider.

Vladimir Višnjić
*Fermi National Accelerator Laboratory**
Batavia, IL 60510, USA

Abstract

A comparative study of the dynamic aperture for several Tevatron lattices is presented with the special emphasis on the new low beta insertions. The results of tracking calculations indicate that the phase I ($\beta_* \approx 50$cm at both B0 and D0) lattice will have about twice the acceptance of the present machine, however the predicted acceptance for phase II ($\beta_* \approx 25$cm) is 40% below that of the present machine.

1 Computer Simulation

In order to increase the luminosity and at the same time provide another interaction region in Tevatron Collider, new low beta insertions are going to be installed in both D0 and B0 straight sections. This is expected to push the Tevatron performance to a new limit ($\beta_* \approx 25$cm, $\beta_{max} \approx 1500$m) and the question is what is the dynamic aperture of the machine going to be. This question is studied here by means of tracking and analytically.

The tracking is done using the code TEVLAT. The Tevatron is modelled as follows. First there are the linear elements of the lattice–the dipoles and the quadrupoles. Their strength is determined from the energy and the tune at which we want to do the tracking. Then the chromaticity sextupoles are turned on in order to compensate for the chromaticity of the linear lattice. Finally, the magnet errors in the form of higher multipole coefficient are read from the actual measurement data for each element.

In order to determine the dynamic aperture I scan a particular subspace of the four dimensional phase space (the xy plane) by tracking for 256 turns. The results of the numerical simulation given in Figs. 1 through 4. What is plotted here is the largest initial amplitude for which the particle survives 256 turns. The amplitude is obtained as $x_0\sqrt{1+\alpha^2(s_0)}$ and normalized as usual by $\sqrt{\beta(s_0)}$, s_0 the launching point. In other words, we consider the quantities

$$\rho_x = x(s)\sqrt{\gamma_x(s)} \quad \text{and} \quad \rho_y = y(s)\sqrt{\gamma_y(s)} \qquad (1)$$

which are independent of s and thus invariant. In this formula $\gamma_x(s) = \frac{1+\alpha_x(s)^2}{\beta_x(s)}$ and analog for γ_y.

I define dynamic apertures as maximal values of ρ_x and ρ_y. The dynamic aperture thus defined is the "radius" of the acceptance and is linear in the maximal amplitude the machine can take. In order to obtain the maximal amplitude in mm at a given point of the accelerator from ρ, Figs. 1 through 4, one has to multiply ρ expressed in (microns)$^{1/2}$ with $\sqrt{\frac{\beta[\text{meters}]}{(1+\alpha^2)}}$ at that point. The physical aperture (the beam pipe) is taken to be square with side 70 mm and is represented on the plots by a dotted line rectangle with dimensions $70\text{mm}/\sqrt{\beta_{xmax}}\times 70\text{mm}/\sqrt{\beta_{ymax}}$.

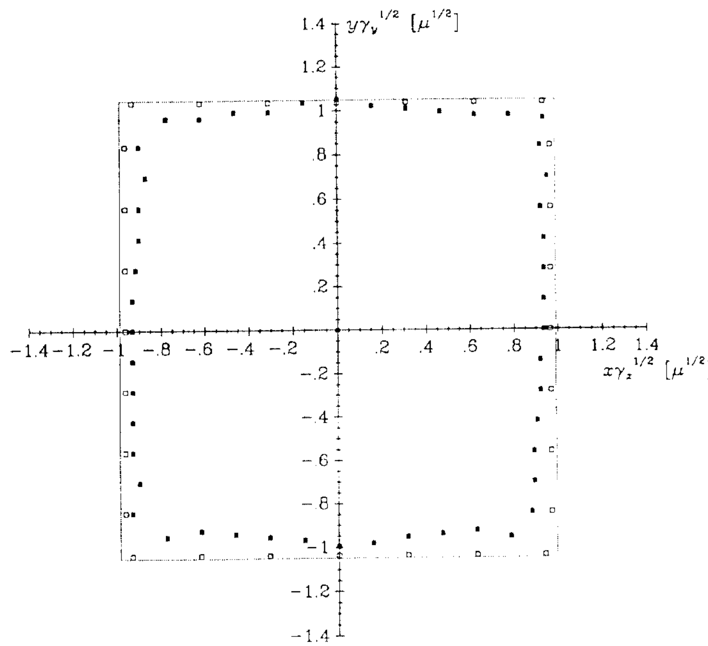

FIGURE 1 The dynamic aperture of the $\beta_* = 70$cm lattice. The open and solid squares denote the linear and the full lattice, respectively. The dotted rectangle is the physical aperture with the dimensions $70\text{mm}/\sqrt{\beta_{xmax}}\times 70\text{mm}/\sqrt{\beta_{ymax}}$.

For each lattice, the tracking is done with and without the nonlinear elements, in particular the higher multipoles in the dipoles and quadrupoles (magnet errors). The linear lattice contains only the ideal linear elements, *i. e.* dipoles

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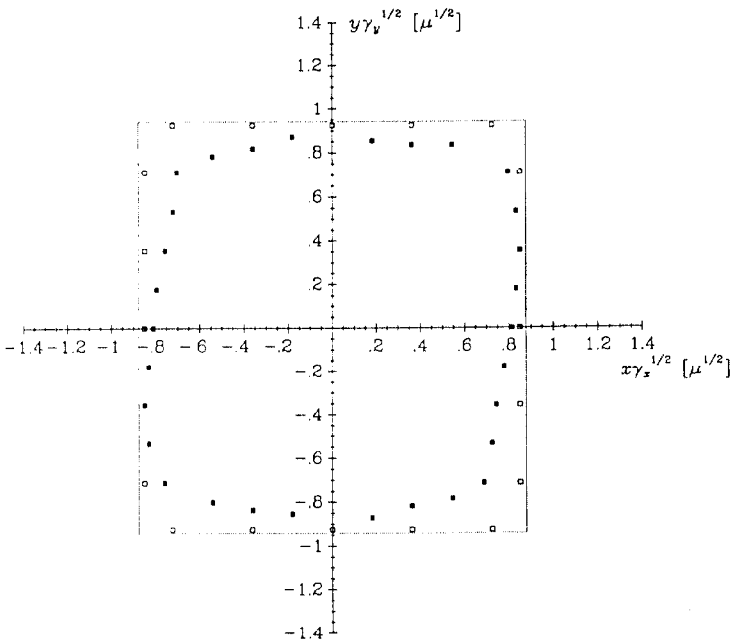

FIGURE 2 The dynamic aperture of the 1988/89 Collider run lattice.

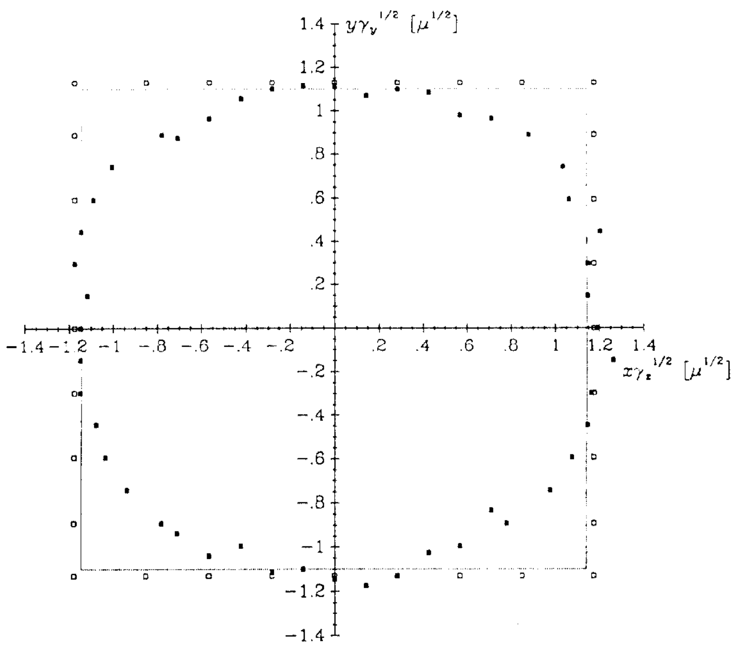

FIGURE 3 The dynamic aperture of the 1991 low beta lattice.

and quadrupoles and its dynamic aperture is identical with the physical one. Note that of the four lattices studied, only the phase II one has the dynamic aperture inside the physical one. Comparing the *horizontal* dynamic apertures of the four lattices normalized to the one used in the 1988/89 run we obtain the ratio

$$1.15 : 1 : 1.46 : 0.82. \tag{3}$$

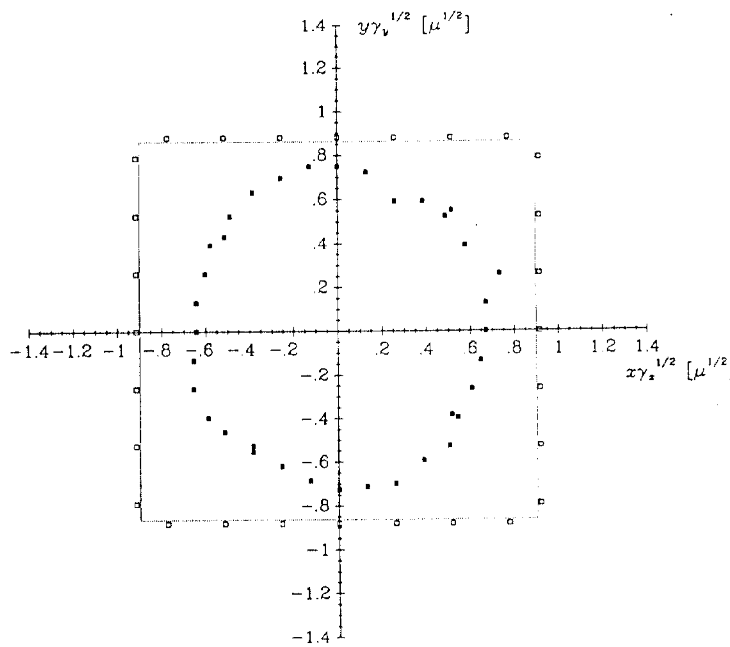

FIGURE 4 The dynamic aperture of the phase II low beta ($\beta_* =$ 25cm) lattice.

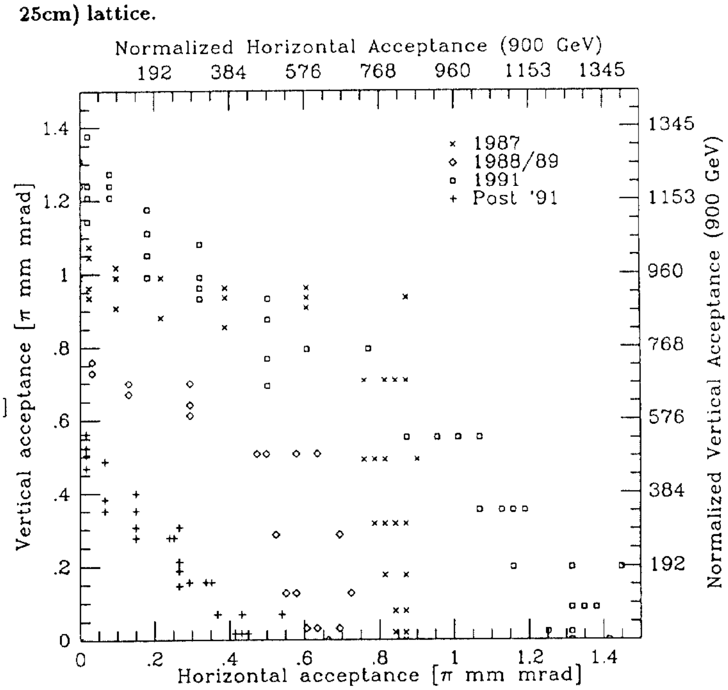

FIGURE 5 Vertical *vs.* horizontal acceptance for the four Tevatron lattices studied here.

The acceptance is obtained by taking the square of the dynamic aperture multiplied by π:

$$A_x = \pi\rho_x^2 \quad \text{and} \quad A_y = \pi\rho_y^2. \tag{4}$$

In terms of the acceptance, the ratios of Eq. (3) read

$$1.32 : 1 : 2.13 : 0.67. \tag{5}$$

The graphs A_x *vs.* A_y are shown in Fig. 5.

2 Tune *vs.* Amplitude Calculation

I will use the Hamiltonian approach. The Hamiltonian due to higher multipoles is

$$H = -\frac{RA_3}{B\rho},$$

where

$$A_3 = -B_0 \sum_{m,n}^{\infty} c_{mn} x^m y^n,$$

and the coefficients c_{mn} are given by

$$c_{mn} = \frac{1}{m+n} \begin{pmatrix} m+n \\ n \end{pmatrix} \begin{cases} (-1)^{n/2} & b_{m+n-1}, \text{ even } n \\ (-1)^{(n+1)/2} & a_{m+n-1}, \text{ odd } n \end{cases}$$

Consider now the *normal dodecapole, i.e.* b_5 which gives the dominant contribution to $\Delta\nu$. The only nonvanishing c_{mn} are

$$c_{06} = -c_{60} = -\frac{1}{6} b_5 \text{ and } c_{24} = -c_{42} = -\frac{5}{2} b_5$$

which gives

$$A_3 = \frac{1}{6} B_1 b_5 (x^6 - 15x^4y^2 + 15x^2y^4 - y^6).$$

The amplitudes

$$x = \sqrt{2\beta_x I_x} \sin\phi_x \quad \text{and} \quad y = \sqrt{2\beta_y I_y} \sin\phi_y,$$

where $I = \frac{\epsilon}{2\pi} = \frac{\epsilon_N}{2\pi\gamma}$. Averaging over ϕ gives for $< A_3 >$

$$\frac{5}{12} B_1 b_5 [(\beta_x I_x)^3 - 9(\beta_x I_x)^2 \beta_y I_y + 9\beta_x I_x (\beta_y I_y)^2 - (\beta_y I_y)^3]$$

and for the averaged Hamiltonian

$$< H > = \frac{5}{12} \frac{1}{2\pi} \times$$

$$\sum_{i=1}^{12} \frac{B_1 \tilde{b}_5}{B\rho} [(\beta_x I_x)^3 - 9(\beta_x I_x)^2 \beta_y I_y + 9\beta_x I_x (\beta_y I_y)^2 - (\beta_y I_y)^3].$$

Here the integration over the arc length s is implicit–this is the origin of the summation sign and $\tilde{b}_5^i$ here is $\int b_5 dl$ over the i-th magnet, the actual number given by the measurement. The β functions here are replaced by their average values in given magnets.

The tune shifts are

$$\Delta\nu_x = \frac{\partial < H >}{\partial I_x} \quad \text{and} \quad \Delta\nu_y = \frac{\partial < H >}{\partial I_y},$$

i.e.

$$\Delta\nu_x = A_1 (\frac{\epsilon_x}{\pi})^2 + 2A_2 \frac{\epsilon_x}{\pi} \frac{\epsilon_y}{\pi} + A_3 (\frac{\epsilon_y}{\pi})^2$$

and

$$\Delta\nu_y = A_2 (\frac{\epsilon_x}{\pi})^2 + 2A_3 \frac{\epsilon_x}{\pi} \frac{\epsilon_y}{\pi} + A_4 (\frac{\epsilon_y}{\pi})^2,$$

where

$$A_1 = \frac{5}{8\pi} \frac{1}{(2\gamma)^2} \frac{1}{B\rho} \sum_{i=1}^{12} B_1^{(i)} \tilde{b}_5^{(i)} \beta_x^{(i)3}$$

$$A_2 = -\frac{15}{8\pi} \frac{1}{(2\gamma)^2} \frac{1}{B\rho} \sum_{i=1}^{12} B_1^{(i)} \tilde{b}_5^{(i)} \beta_x^{(i)2} \beta_y^{(i)}$$

$$A_3 = \frac{15}{8\pi} \frac{1}{(2\gamma)^2} \frac{1}{B\rho} \sum_{i=1}^{12} B_1^{(i)} \tilde{b}_5^{(i)} \beta_x^{(i)} \beta_y^{(i)2}$$

$$A_4 = -\frac{5}{8\pi} \frac{1}{(2\gamma)^2} \frac{1}{B\rho} \sum_{i=1}^{12} B_1^{(i)} \tilde{b}_5^{(i)} \beta_y^{(i)3}.$$

At the maximal current $\tilde{b}_5$ is –1.74 and –2.9 in^{-3} for the 230 and 130 inch quadrupoles, respectively. The values of γ and $B\rho$ at E = 900GeV are 959 and 3×10^4 kGm, respectively. The gradients and the beta functions are obtained from SYNCH and we obtain

$$A_1 = 9.31 \times 10^{-7} \text{mm}^{-2} \quad A_2 = -3.79 \times 10^{-7} \text{mm}^{-2}$$

$$A_3 = -5.21 \times 10^{-7} \text{mm}^{-2} \quad A_4 = 1.16 \times 10^{-6} \text{mm}^{-2},$$

and, finally

$$\Delta\nu_x = 9.31 \times 10^{-7} (\frac{\epsilon_x}{\pi})^2 - 7.59 \times 10^{-7} \frac{\epsilon_x}{\pi} \frac{\epsilon_y}{\pi} - 5.21 \times 10^{-7} (\frac{\epsilon_y}{\pi})^2$$

and

$$\Delta\nu_y = -3.79 \times 10^{-7} (\frac{\epsilon_x}{\pi})^2 - 1.04 \times 10^{-6} \frac{\epsilon_x}{\pi} \frac{\epsilon_y}{\pi} + 1.16 \times 10^{-6} (\frac{\epsilon_y}{\pi})^2.$$

The values of $\Delta\nu_x$ and $\Delta\nu_y$ for ϵ_x and ϵ_y in the range between 10 and 50 π mm mrad are less than 10^{-6}.

3 Concluding remarks

Empirically, the tracking results tend to overestimate the acceptance of the machine by approximately a factor of two as compared with the measurement. If we revise the predictions of Fig. 7 in view of this uncertainty, we conclude that the normalized acceptance at 900 GeV of the phase I lattice is expected to be 700 π mm mrad and that of the phase II lattice 200 π mm mrad. The tracking results assert that the most important single factor limiting the dynamic aperture is the physical aperture at the point where $\beta = \beta_{max}$ (*i. e.* in low beta quadrupoles), rather than the field quality of those quadrupoles.

The analytical calculation of tune *vs.* amplitude confirms the conclusion from the computer simulation that the new low beta lattice is expected to perform very well.