Sensitivity Reduction against Misalignment of Quadrupole and Sextupole Magnets

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Abstract

We studied a magnet alignment method and proposed a new method which treats the quadrupole and sextupole magnets between bending magnets as a unit and aligns their magnetic center precisely before setting it on a design orbit. Alignment between each unit is done by the usual way. The equation of closed orbit distortion was derived and the expected closed orbit distortion was calculated. The results showed that the closed orbit distortion is less than 50 % that of the usual alignment method for the case of SPring-8 storage ring.

I. INTRODUCTION

The emittance goal of third-generation synchrotron radiation sources are ~10^{-9} m•rad [1][2]. Electron beams in such a low emittance storage ring are strongly focussed by quadrupole magnets. These strong field quadrupole magnets generate a large chromaticity. To correct this large chromaticity, strong field sextupole magnets are needed, which makes the dynamic aperture a small one. If the quadrupole and sextupole magnets are misaligned, the closed orbit is significantly distorted and the small dynamic aperture is further reduced. The large closed orbit distortion (c.o.d.) and small dynamic aperture makes it difficult to commission a ring and the ring final performance is degraded substantially. To avoid this, precise alignment of quadrupole and sextupole magnets are needed.

Generally magnets are aligned one-by-one on a design orbit using a transit or the like. In this case, alignment errors of magnets are randomly distributed around the design orbit and the expected c.o.d. is the sum of the randomly generated c.o.d.'s which originate from each quadrupole magnet misalignment. Then, we propose a new alignment method in which the expected c.o.d. is not the simple sum of the randomly generated c.o.d.'s but the cancellation works between the c.o.d.'s generated from operation of each quadrupole magnet. The results of the c.o.d. calculation and other effects to the electron beams are described.

II. ALIGNMENT METHOD

The magnet lattice of third-generation synchrotron radiation sources is the Chasman-Green type [1] or the Triple Bend Achromat type [2]. In these magnet lattices, there are several quadrupole and sextupole magnets between the bending magnets. Our alignment method treats these magnets which are placed on a straight line as a unit and aligns their magnetic center precisely before setting it on a design orbit. Alignment between each unit is done by the usual way which results in the error of 0.1-0.3 mm. Then the kicks due to misalignment of the quadrupole magnets within a unit cancel each other and the expected c.o.d. is reduced substantially. Figure 1 shows a generation pattern of misalignment of a unit and magnets for our alignment method.

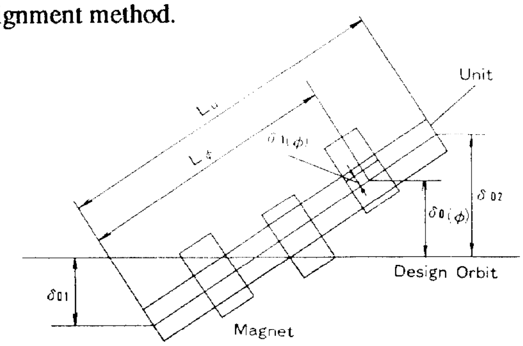


Fig. 1 A generation pattern of misalignment of a unit and magnets.

III. CLOSED ORBIT DISTORTION

A. *Equation for C.o.d. in the New Alignment Method*

The c.o.d. due to misalignment of quadrupole magnets is[3]

$$\eta(\phi) = \frac{\nu}{2\sin\pi\nu}\int_{\phi}^{\phi+2\pi} f(\xi)\cos\nu(\pi+\phi-\xi)\,d\xi,$$

$$\eta=\beta^{-1/2}y,\quad \phi=\int\frac{ds}{\nu\beta},\quad f(\xi)=\beta^{3/2}F(\xi),\quad F(\xi)=\frac{\Delta B_0+\Delta B_1}{B\rho},\qquad (1)$$

where ΔB_0 : error field due to misalignment of a unit; ΔB_1 : error field due to misalignment from the unit center. The quantity V (which corresponds to the Courant Snyder invariant for c.o.d.) is

$$V(\phi) = \frac{\nu^2}{4\sin^2\pi\nu}\int_{\phi}^{\phi+2\pi}\int_{\phi}^{\phi+2\pi} f(\xi)f(\chi)\cos\nu(\chi-\xi)\,d\xi\,d\chi$$

$$= \frac{\nu^2}{4\sin^2\pi\nu}\int_{\phi}^{\phi+2\pi}\int_{\phi}^{\phi+2\pi}\beta^{3/2}(\xi)\beta^{3/2}(\chi)\cos\nu(\chi-\xi)\cdot$$

$$\left[\frac{\Delta B_0(\xi)\Delta B_0(\chi)+\Delta B_1(\xi)\Delta B_0(\chi)+\Delta B_0(\xi)\Delta B_1(\chi)+\Delta B_1(\xi)\Delta B_1(\chi)}{(B\rho)^2}\right]d\xi\,d\chi \qquad (2)$$

We separate the contribution of errors into two cases.

(1) Contribution from the magnets in different units.

Since errors are uncorrelated, the expectation value $\langle f(\xi)f(\chi)\rangle=0$.

(2) Contribution from the magnets placed in the same unit.

Since ΔB_1 is randomly distributed, the expectation value of the second and third terms of eq. (2) is zero. Thus, the expectation value of the quantity V is

$$\langle V(\phi)\rangle = \frac{\nu^2}{4\sin^2\pi\nu}\int_{\phi}^{\phi+2\pi}\int_{\phi}^{\phi+2\pi}\beta^{3/2}(\xi)\beta^{3/2}(\chi)\cos\nu(\chi-\xi)\cdot\left[\frac{\langle\Delta B_0(\xi)\Delta B_0(\chi)\rangle+\langle\Delta B_1(\xi)\Delta B_1(\chi)\rangle}{(B\rho)^2}\right]d\xi d\chi$$

$$= \frac{\nu^2}{4\sin^2\pi\nu}\left\{\sum_{i=1}^{m}\sum_{j=1}^{n}\beta^{3/2}(\xi_i)\beta^{3/2}(\chi_{ij})\cos\nu(\chi_{ij}-\xi_i)\cdot\left\langle\frac{\Delta B_0(\xi_i)\Delta\xi\,\Delta B_0(\chi_{ij})\Delta\chi}{(B\rho)^2}\right\rangle+\sum_{i=1}^{m}\beta^3(\xi_i)\left\langle\left(\frac{\Delta B_1(\xi_i)\Delta\xi}{B\rho}\right)^2\right\rangle\right\}. \quad (3)$$

where i is summed for all quadrupole magnets and j for the magnets in the same unit as i. Using the relation

$$K\delta = \frac{B'L_q\delta}{B\rho} = \frac{\Delta BL_q}{B\rho} = \frac{\Delta B\nu\beta\Delta\phi}{B\rho}, \quad (4)$$

$$\langle V(\phi)\rangle = \frac{1}{4\sin^2\pi\nu}\left\{\sum_{i=1}^{m}\sum_{j=1}^{n}\beta^{1/2}(\xi_i)\beta^{1/2}(\chi_{ij})\cos\nu(\chi_{ij}-\xi_i)\cdot\langle K(\xi_i)\delta_0(\xi_i)K(\chi_{ij})\delta_0(\chi_{ij})\rangle+\sum_{i=1}^{m}\beta(\xi_i)\langle(K(\xi_i)\delta_1)^2\rangle\right\}, \quad (5)$$

where K: magnet strength; L_q: magnet length; $\delta_0(\phi)$: displacement error of a unit; $\delta_1(\phi)$: displacement error of a magnet measured from the center of a unit. If we set the length of a unit as L_u, displacement errors at the entrance and exit of a unit δ_{01} and δ_{02} and the length from the entrance of a unit to a quadrupole magnet L_ϕ, $\delta_0(\xi_i)\delta_0(\chi_{ij})$ is

$$\delta_0(\xi_i)\delta_0(\chi_{ij}) = \delta_0(L_{\xi_i})\delta_0(L_{\chi_{ij}})$$
$$= \left[\frac{L_{\xi_i}}{L_u}\delta_{02} + \left(1-\frac{L_{\xi_i}}{L_u}\right)\delta_{01}\right]\left[\frac{L_{\chi_{ij}}}{L_u}\delta_{02} + \left(1-\frac{L_{\chi_{ij}}}{L_u}\right)\delta_{01}\right]. \quad (6)$$

The expectation value $\langle\delta_{01}\delta_{02}\rangle=0$ and $\langle\delta_{01}^2\rangle=\langle\delta_{02}^2\rangle$. Putting $\langle\delta_{01}^2\rangle=\langle\delta_c^2\rangle$ and using these relationships, we have

$$\langle V(\phi)\rangle = \frac{1}{4\sin^2\pi\nu}\left\{\sum_{i=1}^{m}\sum_{j=1}^{n}\beta^{1/2}(\xi_i)\beta^{1/2}(\chi_{ij})\cos[\nu(\chi_{ij}-\xi_i)]K(\xi_i)K(\chi_{ij})\cdot\left(1+\frac{2L_{\xi_i}L_{\chi_{ij}}}{L_u^2}-\frac{L_{\xi_i}+L_{\chi_{ij}}}{L_u}\right)\langle\delta_c^2\rangle+\sum_{i=1}^{m}\beta(\xi_i)\langle(K(\xi_i)\delta_1)^2\rangle\right\}. \quad (7)$$

From eq. (7), we can obtain the expectation value of c.o.d. y_c as follows.

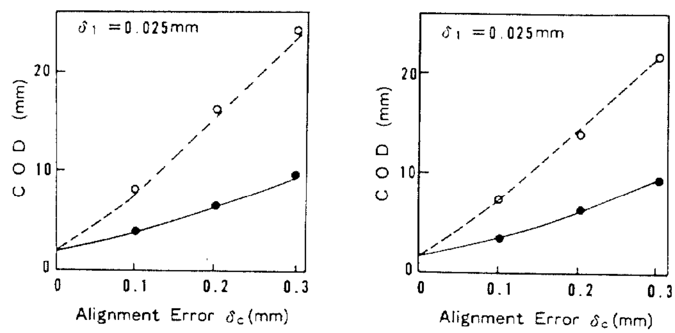
$$y_c = \frac{\sqrt{\beta}\sqrt{\langle V\rangle}}{\sqrt{2}} = \frac{\sqrt{\beta}}{2\sqrt{2}\sin\pi\nu}\left\{\sum_{i=1}^{m}\sum_{j=1}^{n}\beta^{1/2}(\xi_i)\beta^{1/2}(\chi_{ij})\cos[\nu(\chi_{ij}-\xi_i)]\cdot K(\xi_i)K(\chi_{ij})\left(1+\frac{2L_{\xi_i}L_{\chi_{ij}}}{L_u^2}-\frac{L_{\xi_i}+L_{\chi_{ij}}}{L_u}\right)\langle\delta_c^2\rangle+\sum_{i=1}^{m}\beta(\xi_i)\langle(K(\xi_i)\delta_1)^2\rangle\right\}^{1/2}. \quad (8)$$

In a special case where a unit is displaced parallel to the design orbit, c.o.d. becomes

$$y_c = \frac{\sqrt{\beta}}{2\sqrt{2}\sin\pi\nu}\left\{\sum_{i=1}^{m}\sum_{j=1}^{n}\beta^{1/2}(\xi_i)\beta^{1/2}(\chi_{ij})\cos\nu[(\chi_{ij}-\xi_i)]K(\xi_i)K(\chi_{ij})\langle\delta_c^2\rangle+\sum_{i=1}^{m}\beta(\xi_i)\langle(K(\xi_i)\delta_1)^2\rangle\right\}^{1/2}. \quad (9)$$

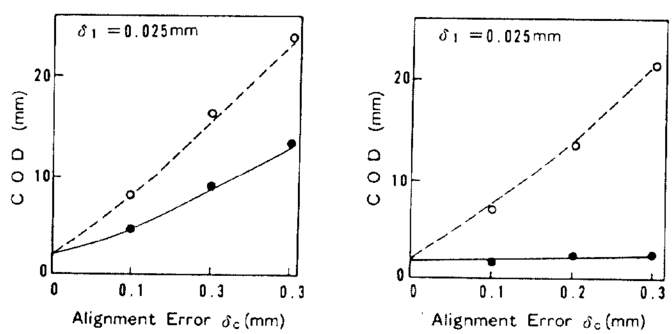
B. Numerical Examples

As an example, we applied our method to the SPring-8 storage ring [4]. Assuming δ_1=0.025 mm, the numerical calculation was done using eq. (8). Simulation was also done for 20 machines using the computer code RACETRACK [5]. Evaluation points of c.o.d. are the monitor positions. Results are shown in Fig. 2. For comparison c.o.d. for a case where the quadrupole magnets are aligned one-by-one by the usual method are shown. The expected c.o.d. by our method is reduced to about 40 % that of the usual method if $\delta_c \gg \delta_1$. Numerical calculation and simulation for a special case where a unit is displaced parallel to the design orbit is also shown in Fig. 3. Vertical c.o.d. is reduced drastically than that of the usual method.



(a) Horizontal (b) Vertical

Fig. 2 Expectation value of closed orbit distortion (—: this method (theory), • : this method (simulation), - - -: usual method (theory), ∘ : usual method (simulation)).



(a) Horizontal (b) Vertical

Fig. 3 Expectation value of closed orbit distortion for a special case where a unit is displaced parallel to the design orbit.

The reason for the c.o.d. reduction is as follows. For simplicity we assume that $\delta_c \gg \delta_1$ and the phase difference in a unit is small. Thus c.o.d. by our method is

$$y_c = \frac{\sqrt{\beta}}{2\sqrt{2}\sin\pi\nu}\left[\sum\left(\sum_{j=1}^{n}\beta^{1/2}(\xi_j)K(\xi_j)\delta(\xi_j)\right)^2\right]^{1/2}. \quad (10)$$

The summation for j is done for a magnet in the same unit. On the other hand, c.o.d. by the usual method is

$$y_c=\frac{\sqrt{\beta}}{2\sqrt{2}\sin\pi\nu}\left[\sum\left(\beta^{1/2}(\xi_j)K(\xi_j)\delta(\xi_j)\right)^2\right]^{1/2}. \tag{11}$$

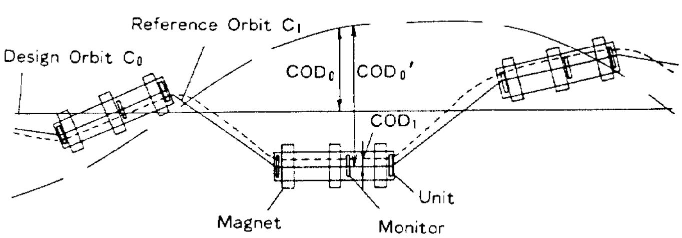
We can find from eqs. (10) and (11) that the c.o.d. by our alignment method is determined by $(\Sigma\beta^{1/2}K\delta)^2$ and that of the usual method is determined by $\Sigma(\beta^{1/2}K\delta)^2$. Accordingly c.o.d. in our case is less than that of the usual method. The reason that the vertical c.o.d. for the special case is reduced drastically is as follows. If we set K_{Fi} and K_{Di} as the strengths of focussing and defocussing magnets, respectively, the horizontal and vertical c.o.d's are

$$x_c\propto\Sigma(\sqrt{\beta_x}K_{Fi}-\sqrt{\beta_x}K_{Di}),\quad z_c\propto\Sigma(\sqrt{\beta_z}K_{Fi}-\sqrt{\beta_z}K_{Di}). \tag{12}$$

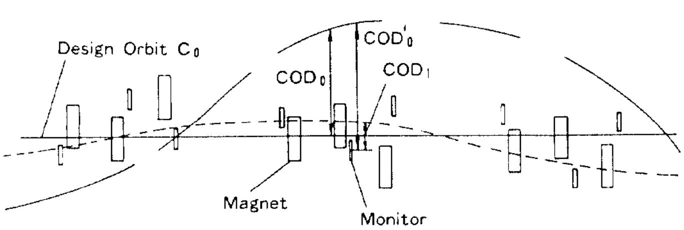
In the low emittance ring, K_F is larger than K_D. The betatron function β_x at the focussing magnet is larger than β_x at the defocussing magnet and β_z at the focussing magnet is smaller than β_z at the defocussing magnet, which means $\Sigma(\sqrt{\beta_x}K_{Fi}-\sqrt{\beta_x}K_{Di})$ has a larger value than zero but $\Sigma(\sqrt{\beta_z}K_{Fi}-\sqrt{\beta_z}K_{Di})$ has a value which is almost equal to zero.

C. C.o.d. Before and After C.o.d. Correction

Figure 4(a) shows the concept of c.o.d. for our alignment method before and after c.o.d. correction. Before c.o.d. correction, COD_0 is the orbit distortion from a design orbit C_0. If we measure the c.o.d. by monitors, COD_0' is obtained. Assuming that the position monitor error is the same order as the quadrupole magnet alignment error δ_1, the new reference orbit C_1 which is the line connected monitor-to-monitor can be obtained. After correction of c.o.d., residual COD_1 which is the orbit distortion from the reference orbit C_1 is obtained. Thus the alignment error of a unit after c.o.d. correction has no influence on a beam.



(a) This method



(b) Usual method

Fig. 4 The concept of c.o.d. before and after c.o.d.correction.

IV. OTHER EFFECTS

A. Commissioning

Closed orbit distortion of a low emittance storage ring before c.o.d. correction is so large that the commissioning of such a ring is expected to be difficult. However, since the c.o.d. generated by our alignment method is less than 50 % that of the usual one, application of our alignment method makes the commissioning of a low emittance ring easier.

B. Strength of Correction Magnets

Correction magnet strength is expected to be reduced according to the reduction of the natural c.o.d..

C. Dynamic Aperture

In a low emittance storage ring a dynamic aperture of the ring with errors is much smaller than that of the ideal ring. After c.o.d. correction the dynamic aperture recovers to some extent, but does not recover 100 %. The reason of this is that the small amount of the residual c.o.d. still remains and field error of the quadrupole magnet and misalignment of the sextupole magnet exist. If the error of the quadrupole magnet strength $\Delta K/K$ is about 5×10^{-4} and the misalignment of the sextupole is about 0.1mm, the quadrupole component due to misalignment of the sextupole magnet is three to four times larger than the error of the quadrupole magnet strength in the SPring-8 storage ring. Thus if the misalignment of the sextupole magnet is reduced to 0.025 mm, the dynamic aperture is expected to recover to that of the ideal ring.

D. Spurious Dispersion and Residual C.o.d.

Spurious dispersion and the residual c.o.d. is also expected to be reduced to the ratio of misalignment of the new and usual methods.

V. CONCLUSION

A new alignment method of quadrupole and sextupole magnets was proposed. The method treats the quadrupole and sextupole magnets between the bending magnets as a unit and aligns their magnetic center precisely before setting it on a design orbit. Alignment between each unit is done by the usual way which results in the error of 0.1 to 0.3 mm. We derived the equation for closed orbit distortion in the proposed alignment method and applied it to the SPring-8 storage ring. We found that the closed orbit distortion was reduced to less than 50 %.

Detailed discussion will be published in reference [6].

VI. REFERENCES

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