Evaluation of the Synchrotron Close Orbit.

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Abstract.

The computation of the close orbit is reduced to a determination at an arbitrarily chosen azimuth of the eigenvector of the total transfer matrix of the synchrotron ring and to tracing with this vector desired orbit. The eigenvector is found as a result of an iteration.

Introduction.

The knowledge of the closed orbit position is an essential condition for the effective work of any accelerator. Therefore questions of calculations, measurements and controls have great importance [1]. For example, during injection of particles into a synchrotron, the amplitudes of their betatron oscillations may become commensurable with the working region of the synchrotron. This makes us pay attention at the problem of formation of the optimum orbit with use of correcting optical elements. In addition, it is often necessary to calculate such an orbit at the end of the acceleration cycle when particles are deposited at internal targets or removed from the synchrotron. In dedicated synchrotron radiation sources, large e^+e^- colliders or sperconducting accelerators non-linear forces act on the particle and may lead to unstable motion [2]. In these maschines the dynamic aperture is small and the closed orbit distortion may excite losses of particles. It may lead to the transition of superconducting elements in the heating condition [3]. The correction of the closed orbit and of trajectories of particles is require for decreasing of blackgrounds in the experimental detectors.

Calculations of the closed orbit.

In matrix calculations, the result of passing through a periodic element is that initial vector $\vec{Y}_o$ is linearly transformed into vector $\vec{Y}$. This transformation is described by the corresponding transfer matrix M [4]. For the matrix M can be written:

$$M\vec{Y}_\lambda=\lambda\vec{Y}_\lambda \qquad (1)$$

where λ - the eigenvalue of the matrix M and $\vec{Y}_\lambda$ - eigenvector of the matrix M corresponding to λ. To particles moving on the closed orbit meet following conditions:

$$\vec{Y}_{co}(\vartheta+2\pi)=\vec{Y}_{co}(\vartheta), \qquad (2)$$

or:

$$M_t\vec{Y}_{co}(\vartheta)=\vec{Y}_{co}(\vartheta), \qquad (3)$$

where Y_{co} - the vector describing the particle position and M_t - the transfer matrix of one turn. If compare (1) and (3) one can see that the vector $\vec{Y}_{co}$ is eigenvector of matrix M_t for eigenvalue $\lambda=1$.

In principle, calculations of a transient equilibrium orbit reduce to determination of the eigenvector of the complete transition matrix that corresponds to a given azimuth and to construction of the sought orbit [5].

The analyses was done in the Poincare map corresponding to the given form of the motion (radial or vertical). The eigenvector was determined in the following iteration procedure. First, the initial position of a particle is arbitrarily specified, and its motion in the given azimuth is followed during a certain number of revolutions. If the motion of the particle is determinated by linear forces, the phase space vectors are found on the boundary off an ellipse for the motion in one degree of freedom. If a small nonlinearity is turned on, the ellipses will be distored but will still enclose the same area. The number of revolutions depends on the frequency of betatron oscillations, it is chosen so that the phase-plane points, which image the particle's motion in the given azimuth, fulfill the phase ellipse almost completely.

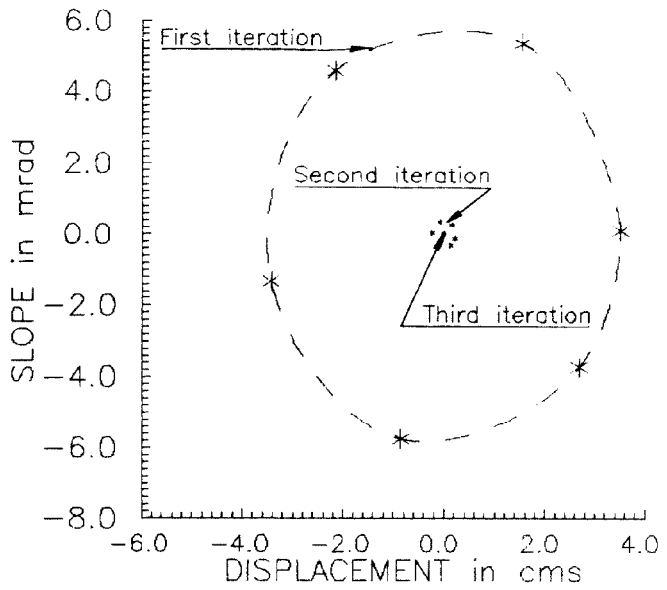

Fig.1. Graphic representation of the iteration process of searching for the closed orbit.

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The number of revolutions is given by the following expression:

$$N=E\left(\frac{1}{1-(\nu_{x,z}-E(\nu_{x,z}))}\right) \qquad (4)$$

where $\nu_{x,z}$ is the radial betatron tune or the vertical betatron tune. The function E calculate integer part of the argument.After calculations we obtain a set of N vectors corresponding to passing through the given azimuth in each revolution. Then we find the components of vector which are the arithmetic averaged of the corresponding components

$$X_{km}=\frac{1}{N}\sum_{k=1}^{N}X_k \qquad X'_{km}=\frac{1}{N}\sum_{k=1}^{N}X'_k$$

$$Z_{km}=\frac{1}{N}\sum_{k=1}^{N}Z_k \qquad Z'_{km}=\frac{1}{N}\sum_{k=1}^{N}Z'_k \qquad (5)$$

Components of the obtained vector $\vec{Y}_{km}$ are consecutively compared with those of $\vec{Y}_k$ If at least one difference between components of and exceeds the tolerance specified for the given calculation, then the calculation is run again but the initial vector is taken to be equal to $\vec{Y}_{km}$. Calculations are repeated until the eigenvector is determined with necessary accuracy. On fig.1 is shown the Poincoare map for three iteration procedure. So we determine the eigen vector of the complete transition matrix at the given azimuth. Finding of the transient equilibrium orbit reduces to determination of the trajectory of a particle whose initial state is described by the eigen vector of the complete transition matrix.

The code structure.

In order to make calculations we have developed a computer code to calculate both trajectories of individual particles and closed transient orbits. The method in question is based on the fact that the entire magnetic structure of a synchrotron may be represented as a series of discrete elements. All such elements may be classified into three groups:

a) free rectilinear segments,

b) rectilinear segments with strayed magnetic field,

c) magnetic sectors,

d) rectilinear segments with accelerating electric field.

Even element has its own index depending onthe element's position, this allowing information related to an element to be easily filed or retrieved. Thus the parameters of the elements needed for calculations were filed in computer with use of the said notation. This splitting of magnetic structure into homogeneous elements gives us opportunity to make calculation in the matrix form [4], hence compilation of the computer code and its usage become more convenient. Also, it is easy to modify the code if necessary.

In our calculations particle conditions are specified by vectors in following forms:

$$\vec{Y}=(x,x',z,z',L,E) \qquad (6)$$

where x,x',z,z' - parameters of transversal motion, L - the lengh of the particle path and E - its energy.

The six-dimensional transition matrix may be represented as:

$$M=\begin{pmatrix} M(4,4) & M(4,2) \\ \cdots & \cdots \\ M(2,4) & M(2,2) \end{pmatrix} \qquad (7)$$

where matrix M(4,4) is well-known matrix of transition in transversal motion. Matrix M(2,4) and M(4,2) are zero matrices. Matrix M(2,2) describes the linear transformation of the trajectory length of a particle and its energy when passing through an element. This matrix has the following form:

$$M(2,2)=\begin{pmatrix} 1+\Delta L/L & 0 \\ 0 & 1+\Delta E/E \end{pmatrix} \qquad (8)$$

where ΔL and ΔE are changes in the trajectory length of the particle and its energy in the lattice element. L and E are the path length and the energy of the particle before entering this element. ΔL and ΔE are calculated by corresponding analitical expressions truncated at the second order in small quantities.

Matrix M(2,2) is introduced because it seems to be of interest to study the effect of betatron oscillations on the process of phase oscillations [6,7] which is due to the difference in lengths of the trajectories of particles having different amplitudes of betatron oscillations. It proves to be necessary to introduce the element of matrix M(2,2) describing transformation of energy because the changes in energy due to, say, induction acceleration or synchrotron radiation, must be taken into account when passing through an element.

In our calculations we use a series of subroutines which evaluate the components of the transition matrix for a certain group of elements. Calculations of matrix components is done with due regard for the code of the parameters.

Thus, the code for numeric calculations allows us to determine the coordinates of a particle , the length of its path, and its energy before and after passing through each element. The results of calculations may be presented in graphics, this being very convenient for analyzing the motion of individual particles in an accelerator.

Rezults of calculations.

The method and the code described above have been used for calculations of the closed

orbit in the "Pakchra" synchrotron. It was found out that for determining the closed orbit to the accuracy of 10^{-2} cm only two or three iteration steps are needed. The code used includes several orbit correction algorithms. It allowes to make calculations for real magnet lattice including various corrections. The resulting closed orbit is presented in Fig.2.

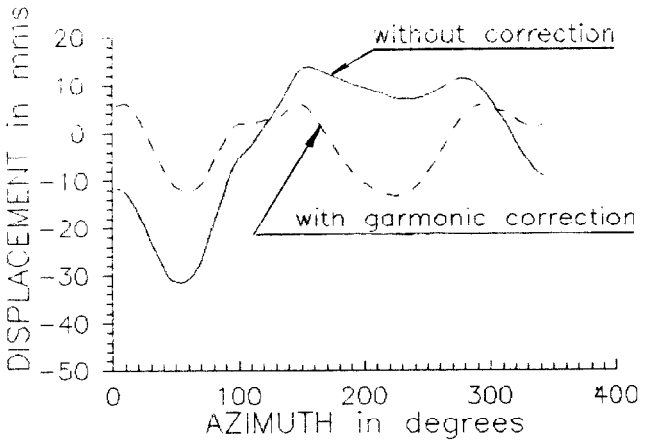

Fig.2. Azimuthal dependence of the radial deviation of the closed orbit.

Also, the orbit obtained after introducing corrections is shown. The data obtained are in reasonable agreement with experiments.

This method may be realized with any program used for particle tracking [7]. One can note that the method may be used for solutions of others problems concerned with finding eigenvectors of a matrix.

References

[1] J.-P. Koutchouk, "Trajectory and closed orbit correction", Frontiers of particle beams; Observation, diagnosis and Correction, Lectures Notes in Phisics 343, Springer-Verlag, 1989,p. 46.

[2] A. Ropert, "Dynamic aperture", CERN Accelerator School,Uppsala University. Sweden, 1989, cern 90-04, p. 26.

[3] M. Harisson, "Resonant extraction at the Tevatron", Fermilab summer Sch. 1987, Cornell summer Sch. 1988, vol.2, New York 1989, p 2009.

[4] H. Bruck, "Acceleraturs circullaires de particules", Paris, 1966.

[5] Yu. A. Bashmakov, V. A. Karpov, "On formation of the instantaneous orbit in the synchrotron", Proc. IX-th All-Union conf. on Charged Particle Acceleratos, Dubna, 198 , p. 190.

[6] D. C. Carey, R. V. Servranckh. K. L. Brown, "Second-order path length terms for a general bending magnet", Particle Accelerators, 1983, v.13 No.3-4, p.199-207.

[7] F. Willeke, "Analysis of particle tracking data", CERN Accelerator School,Uppsala University. Sweden, 1989, cern 90-04, p. 26.