Diffusive Transport Enhancement by Isolated Resonances and Distribution Tails Growth in Hadronic Beams.

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Abstract

The escape rates and evolution of a distribution of particles are considered for a 2-D model of transverse motion of particles in hadronic storage rings, when nonlinear resonances and external diffusion are present. Dynamic enhancement of diffusion inside separatrices can develop under a certain geometry of resonance oscillations and relatively wide resonances, leading to the fast growth of distribution tails and escape rates . The phenomenon is absent in 1-D.

1 General description

In hadronic colliders, the escape of particles to large betatron amplitudes and associated growth of distribution tails due to the small random modulations of the lattice parameters (predominantly the RF power) is an important practical issue. Experimental evidence indicates that the escape rate has an appreciable magnitude only in the presence of the beam-beam interaction. However, to the present knowledge we cannot expect a fast escape of particles to originate from the beam-beam interaction alone. Therefore, it seems apparent that the external noise and the beam-beam nonlinear dynamics "interfere" somehow to efficiently magnify their respective effects. The present paper (a concize version of /1/) is devoted to the description of one particular mechanism of amplification.

We will discuss the effect of noise on Hamiltonian dynamics, more particularly the following system:

$$\begin{aligned} \dot{\vec{x}} &= \vec{p} \\ \dot{\vec{p}} &= -\frac{\partial U(\vec{x},t)}{\partial \vec{x}} + \sqrt{2\eta}\,\vec{\xi}(t) \end{aligned} \qquad (1)$$

where η is the diffusion intensity. $\xi_i(t)$ here is the white-noise vector process $\langle \xi_i(t)\xi_k(t+\tau)\rangle = \delta_{ik}\delta(\tau)$. The potential U is supposed to consist of an unperturbed time-dependent part $U_0(\vec{x})$ corresponding to exactly integrable motion and a small perturbation $U = U_0 + \epsilon\delta U(\vec{x},t)$, time-periodic with frequency Ω. Since in realistic situations, the beam is small relative to the aperture during the entire storage time, we will be concerned with the distribution tails only.

For this model, the situation will be quite different depending on what type of Hamiltonian dynamics one is considering. The simplest case is when the dynamics is exactly integrable. Then, one can envision it as (topologically equivalent to) the trajectotries spiralling around the constant-action tori with a simultaneous diffusion in both phases (position on the torus surface) and actions (tori radii). In many cases however, the diffusion intensity is small, so that the trajectory densly covers the tori surface before moving appreciably in tori radii. One can then perform a suitable averaging on before-said surface, as it is common in the conventional Fokker-Planck problems /2/, and reduce the evolution to a certain diffusion process in the space of actions only. In most cases the dependence of diffusion intensities on actions will be smooth and monotonous, so that the evolution of any initial δ-functional distribution will be at least qualitatively similar to a usual gaussian spreading for coordinate-independent diffusion, without any "structure" to be observed. The tails of distribution, which are responsible for the particle escape to the distant boundaries, can be described in the same way as in the weak-noise asymptotics (WNA) for conventional nonequilibrium systems (with damping) /3/ as $\rho = Z\exp(-\phi/\eta)$ where Z and ϕ are both functions of phase space variables and time while η is the general factor of diffusion intensity (small parameter). The function ϕ, defining the exponential smallness of the transition probability from given initial condition, can be easily shown to depend on time as $\phi = \varphi/t$. The (time-dependent) escape rate r from any given initial condi- tions can be found in weak-noise approximation as $r = R\exp(-G/\eta)$, where G is the minimum of ϕ on the boundary $G = \varphi_{min}/t$. Thus, time-dependent escape in purely integrable Hamiltonian systems under the influence of weak noise can be analytically described through the combined implementation of averaging along Hamiltonian trajectories and WNA, and does not show any pecularities.

More difficult case is when the Hamiltonian is not exactly integrable, but only nearly so, i.e. consists of an exactly integrable time-independent part and a small perturbatron (with periodic, if any, dependence on time). Such perturbations are known /4/ to drive nonlinear resonances, which constitute in their turn an everywhere dense net of progressively (higher orders)/(narrower widths). The question then is, first, how can each undividual resonance affect the evolution of distribution tails and, second, what is the combined effect of many resonances. From a qualitative considerations one can infer that the answer will be quite different in 1-D (one spatial coordinate) and higher dimensionality. Indeed, in 1-D the only spatial scale, associated with each resonance i is its width, which is proportional to the square root of perturbation strength ϵ. Choosing, to be more particular, the unperturbed action space I, one can also say that the sum of all widths of res-

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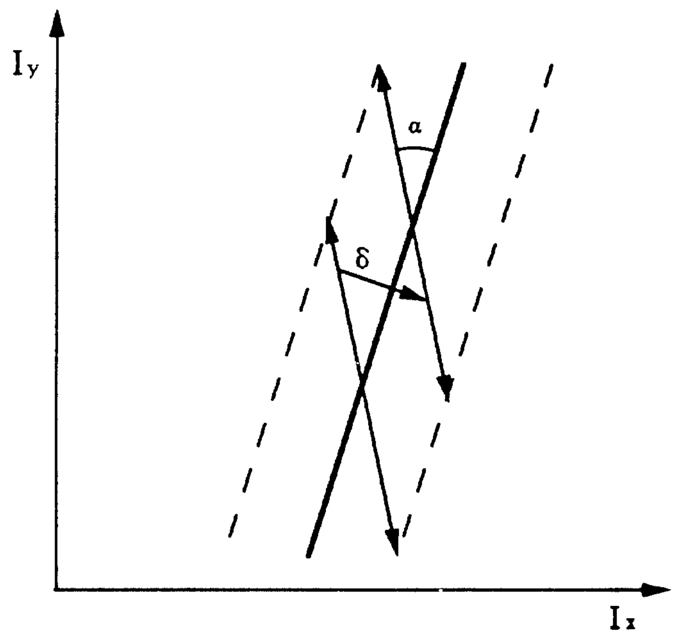

Figure 1: Displacement of resonance oscillation center by transverse kick. Thick solid line is the resonance line. Dashed lines are the separatrix.

onances $\Delta_{int} = \sum_{i=1}^{\infty} \Delta I_i$ is finite and proportional to $\sqrt{\epsilon}$. The smallness of Δ_{int} for small ϵ means, loosely speaking, that the resonances cover only a small submanifold of any given region of I axis. It is quite clear that the perturbation of ϕ by resonances $\Delta\phi = \phi - \phi_0$ (the difference of ϕ's with and without resonances, which we expect to be always negative) in 1-D should be of the order of Δ_{int}, and thus small for small ϵ. Otherwize put, in 1-D the effect of (Hamiltonian) perturbation on escape rate is small as long as its "mechanical" effect is small. In accelerator problems, in most cases the resonance widths are much smaller than the characteristic apertures where particle loss occurs; then in 1-D the transport ability of these resonances is minimal, since their influence is confined to a small region near their separatrices.

In 2 and higher dimensions, resonances are surfaces (or lines) in the action space, and the possibility of the particles to diffuse along them staying inside the separatrices changes the situation completely. The major reason for this is a certain "renormalization" of diffusion inside separatrices, leading to an increase of diffusion intensity along the surface. This can be explained, in 2-D for simplicity, as follows. In the plane of actions I_x, I_y, where resonances are lines, one can draw the arrow of separatrix oscillations, which shows the direction of trapped particle oscillations about the resonance line. Its length Δ is simply the width of the separatrix (or twice the maximum oscillation amplitude) and its center is the resonance line (see Fig. 1). Now consider a small kick δ applied to a trapped particle in the direction orthogonal to the resonance line; it is clear that the center of oscillations will be displaced a distance $\delta \cot(\alpha)$ along the resonance line. Similarly, if we introduce noise of intensity η in this direction, then the diffusion of the oscillation center along the resonance will have the diffusion coefficient $\eta \cot(\alpha)$. Thus, for small angles α between the resonance oscillations and resonance line, diffusion is enhanced inside the separatrix. This enhancement has been termed diffusive "resonance streaming" and is well known /5/. For escape rate and distribution tail problems, it leads to a very strong effect, since the particles can travel long (as compared to small resonance width) distances along the resonance lines while staying within the regions of enhanced diffusion intensity. The density in the distribution tails and associated escape rates increase exponentially strongly. More particularly, the decrease of the function ϕ by the effect of the resonances is of the order of unity $\phi - \phi_0 \sim \phi_0$ even for small perturbation strength ϵ (as long as $\epsilon \gg \eta$), which is a drastic exponentially strong amplification of the effect as compared to 1-D. The power of the exponential in the escape rate can increase thus several times, making the phenomenon spectacularly strong and supposedly important for applications. One can say that unlike in 1-D, the effect of (Hamiltonian) perturbation on escape rate in 2-D can be large even when its "mechanical" effect is small. The situation is somewhat similar to the escape rate and distribution function behaviour in oscillator with nonlinear resonances, damping and noise /6/, where both damping and diffusion are "renormalized" within the separatrices.

It should be stressed that the WNA description of the system, with the "resonance streaming" emerging as it's ingredient is essentially relying on the condition $\epsilon \gg \eta$ of the resonance being wide enough relative to the noise intensity. When this condition is violated, the situation is getting more involved The basic dynamic process to take into account to evaluate the "macroscopic"(i.e. involving distances much larger then the resonance width) transport rate along the resonance line is the diffusion of particles in transverse direction, so that because of the different longitudinal diffusion intensities inside and outside of the separatrix, the transverse diffusion modulates the longitudinal one. This makes the effective one-dimensional random walk along the resonance line a more complex stochastic process, in fact not even describable by any sort of diffusion process.

When trying to apply the WNA to a generic noisy nearly-integrable oscillator, which posesses an infinite hierarchy of arbitrarily narrow resonances, one immedeately runs into a major difficulty. Indeed, the condition of the resonance being wide enough in respect to noise $\epsilon V_m \gg \eta$ (V_m here is the resonance harmonic amplitude), will break down for all resonances of high enough order. Therefore, the "resonance streaming" diffusion enhancement scenario /5/, implicitly relying on this condition, is essentially incomplete. The fuller description, carried out in /1/ in a certain phenomenological approach gives the overall effect of the resonance as dependent on the width $\Delta \sim \sqrt{\epsilon V_m}$ of the resonance, going to zero as Δ tends to zero. This introduces a certain cutoff of narrow enough resonances and "regularizes" the problem.

2 Concluding remarks

Few observations are in order about the more technical (though very important) question under what conditions can it manifest itself in the real hadronic colliders. The necessary condition of resonance-induced enhancement is the condition (54) of Ref./1/, requiring smallness of angle between the resonance line and resonant oscillations direction. The question then is when this small angle can appear. The important point is that this angle is determined only by the nonlinear tune shifts $\delta\nu_x$, $\delta\nu_y$ dependencies on betatron amplitudes A_x, A_y, and not by the harmonic amplitudes (defining the resonance width). The tune shifts are created by both the multipole components of magnetic fields and the nonlinear beam-beam interaction field. Since the hadronic beams are usually round , the beam-beam interaction is symmetric and preliminary numerical evidence is that the resonant oscillations are always nearly orthogonal to the resonance line. Thus it does not look likely that the phenomenon can manifest itself in the absence of multipole components. Superimposing the latter on the top of beam-beam interactions can however change the situation.

Consider now the effect of the multipoles in the absence of beam-beam force. The lowest order multipole tune-shifts come either from the first-order perturbation term of the octupole component or the second-order one of the sextupole component and have the same functional forms:

$$\begin{aligned}\delta\nu_x &= C_1 A_x^2 + C_2 A_y^2 \\ \delta\nu_y &= C_2 A_x^2 + C_3 A_y^2\end{aligned} \quad (2)$$

where A_x, A_y are the betatron amplitudes and the coefficients C_1, C_2, C_3 are the integrals of the multipole amplitudes along the ring and can vary in respect to each other in the wide range. It is easy to see that the resonance line is straight in the action variables $J_x = A_x^2$, $J_y = A_y^2$ and that the angle between this line and the resonant oscillations can be varied arbitrarily by varying the constants C_1, C_2, C_3. Thus the phenomenon of the resonance-enhanced diffusion can be more easily observed in the absence of beam-beam interaction and is conceivable when both beam-beam interaction and lattice nonlinearities affect the tune shifts.

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