Comments on the Behavior of α_1 in Main Injector γ_t Jump Schemes

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Abstract

Tracking studies of transition crossing in the Main Injector have shown that the Johnsen effect is the dominant cause of beam loss and emittance blow up. To suppress this effect one has to have control over α_1 (dispersion of the momentum compaction factor α). Various γ_t jump configurations are examined and the resulting changes in α_1 are assessed. These results are further validated by comparison between the simulation and simple analytic α_1–formulas derived for a model FODO lattice with full chromaticity compensation in the presence of an eddy current sextupole component. A scheme involving the introduction of a dispersion wave in the arcs of the Main Injector, around transition time, seems to be promising if one regards the strength of the eddy current sextupole family as an external "knob" to control values of α_1.

I. INTRODUCTION

Tracking studies of transition crossing in the Main Injector and other Fermilab accelerators, using the code ESME, have shown that the Johnsen effect is the dominant cause of beam loss and emittance blow up [1, 2]. This effect is rooted in the variation of γ_t the transition gamma, with the momentum offset. A useful parameter characterizing the strength of this effect [3] is the Johnsen time, T_J,

$$T_J = \frac{\gamma_t^o}{\dot{\gamma}} \left[\frac{3}{2} + \frac{\alpha_1}{\alpha_o} - \frac{\alpha_o}{2} \right] \frac{\sigma_p}{p} , \qquad (1)$$

where σ_p denotes the rms momentum spread. The Johnsen time, T_J, is directly related to the lattice parameter α_1, which is defined by the following equation

$$\frac{\Delta C}{C_o} = \alpha_o\, \delta + \alpha_1\, \delta^2 + ... , \qquad \delta = \frac{\Delta p}{p_o} , \qquad (2)$$

where C_o is the nominal closed orbit path length, and ΔC is the increase in path length for an off momentum particle. The coefficients α_o and α_1 are geometrical properties of the lattice, given by

$$\alpha_o = \frac{2\pi}{C_o} \langle \eta_o \rangle , \quad \alpha_1 = \frac{2\pi}{C_o} \langle \eta_1 \rangle , \qquad (3)$$

where angle brackets $\langle ... \rangle$ denote averaging weighted by bend angle. The quantities being averaged are component dispersions in a momentum expansion of the total dispersion. That is,

$$\eta(s) = \eta_o(s) + \eta_1(s)\, \delta + ... \qquad (4)$$

explicitly showing the dependence of the dispersion on s, the accelerator azimuth.

Clearly, if it is possible to measure and control α_1, then it should be possible to make $T_J = 0$, and ameliorate the damage done by the Johnsen effect [4], by setting

$$\alpha_1 = -\frac{3}{2}\alpha_o + \frac{1}{2}\alpha_o^2 . \qquad (5)$$

This may be true especially if α_1 control is combined with RF gymnastic tricks, such as the use of a synchronous phase of 90^0 and a second harmonic cavity, as now being discussed elsewhere [5].

II. DISPERSION, η_1 – DIFFERENTIAL EQUATION

The horizontal closed orbit h(s) is found by solving the differential equation

$$h'' + \frac{K(s)}{1+\delta} h + \frac{S(s)}{1+\delta} h^2 = G\left(1 - \frac{1}{1+\delta}\right) \qquad (6)$$

with periodic boundary conditions. A prime indicates differentiation with respect to s, K is the quadrupole strength, S is the sextupole strength, and G is the dipole bending strength. Here, h is expanded in a dispersion function series

$$h = x_{co} + \eta_o + \eta_1\, \delta + ... \qquad (7)$$

The solution of the lowest order equation is trivial when there are no closed orbit perturbations, $x_{co} = 0$, so that the remaining two equations become

$$\eta_o'' + K\,\eta_o = G , \qquad (8a)$$

$$\eta_1'' + K\,\eta_1 = -G + K\,\eta_o - S\,\eta_o^2 . \qquad (8b)$$

According to Eq.(3), solution of the above equations yields a direct knowledge of α_1, and quantitatively identifies the major factors which affect α_1.

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III. FODO LATTICE WITH EDDY CURRENT SEXTUPOLES

Suppose that an accelerator like the Main Injector is represented as made up purely of FODO cells. The quadrupoles are thin, and there are no drift spaces. All of the half cell length L is filled with two identical dipoles of bend radius R, which are separated by a thin sextupole representing the field due to vacuum chamber eddy currents, induced during the ramp. There are also two thin chromatic correction sextupoles per half cell, immediately adjacent to the focusing and defocusing quadrupoles. The strength of the (half) quadrupoles is ±q, and of the sextupoles is g_F, g_D, and g_E, given by

$$q = \frac{s}{L} = \frac{\sin(\phi_{1/2})}{L}, \tag{9a}$$

$$g_I = \frac{(S_I \Delta l)\, \eta_o{}^I}{q}, \quad I = F, D, E. \tag{9b}$$

Here, $\phi_{1/2}$ is the half cell phase advance, while S_F and $\eta_o{}^F$ (for example) are the sextupole gradient and the lowest order dispersion, at the F chromatic sextupole of thin length Δl. Using these convenient definitions, it can be shown by solving Eqs.(8) that the matched values of the dispersion at F, E, and D are

$$\eta_o{}^F = \frac{L^2}{R}\frac{2+s}{2s^2}, \tag{10a}$$

$$\eta_o{}^E = \frac{L^2}{R}\frac{8-s^2}{8s^2}, \tag{10b}$$

$$\eta_o{}^D = \frac{L^2}{R}\frac{2-s}{2s^2}. \tag{10c}$$

When only the F and D sextupoles are turned on, at a strength to correct for f times the natural chromaticity, it can easily be shown that

$$\alpha_o = \frac{L^2}{R^2}\frac{1}{s^2}\left[\, 1 - \frac{s^2}{12} \right], \tag{11a}$$

$$\alpha_1 = \frac{L^2}{R^2}\frac{1}{s^2}\left[\, 1 - f + \frac{s^2}{12} \right]. \tag{11b}$$

IV. CONTROLLING α_1 WITH SEXTUPOLES

The middle sextupole, of strength g_E, can be thought of in (at least) two ways. In the first point of view, it represents the sextupole field caused by eddy currents induced in the vacuum chamber of the dipoles. In the second point of view, g_E represents a free knob with which α_1 can be controlled. For the sake of a semi-quantitative interpretation, suppose that the F and D sextupoles have their strengths set to compensate for the sum of the chromaticity induced by an eddy current sextupole of strength g_E, plus f times the natural chromaticity. In this case it is readily shown that the F and D sextupole strengths are given by

$$g_F = \frac{f}{2} - \frac{2-s^2}{4}\, g_E, \tag{12a}$$

$$g_D = -\frac{f}{2} - \frac{2-s^2}{4}\, g_E. \tag{12b}$$

Carrying out similar analysis as in the previous section one can generalize Eq.(11b) as follows

$$\alpha_1 = \frac{L^2}{R^2}\frac{1}{s^2}\left[\, (1 - f + \frac{s^2}{12}) - \left(\frac{3}{8}\right)^2 s^3 g_E \right]. \tag{13}$$

To test the results of Eq.(13) and to gain some insight into the prospect of controlling α_1 in the Main Injector, consider a lattice made up of 80 simple FODO cells. The lattice design code MAD was used to study the variation in closed orbit path-length as a function of Δp/p, over a range from −0.003 to +0.003. Table 1 shows good qualitative agreement between the analytic predictions and simulated values of α_o and α_1.

(f,g_E)	α_o ($\times 10^{-3}$) predicted	simulated	α_1 ($\times 10^{-3}$) predicted	simulated
(0,0)	2.956	2.956	3.213	3.332
(1,0)	2.956	2.956	0.129	0.0244
(1,5)	2.956	2.956	−0.638	−0.512

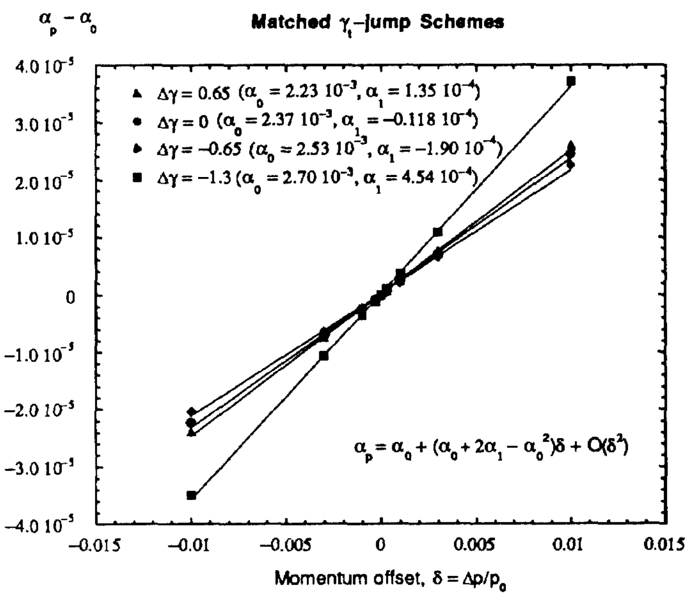
Table 1. Comparison of predicted and simulated α_o and α_1.

If the sextupole family strength parameter g_E is employed to control values of α_1, the inevitable conclusion is that the sensitivity to the family is too weak to reduce the Johnsen time to zero. Investigations are currently under way to find an optical configuration that will significantly increase the orthogonality of the three families beyond the unfortunate results of the FODO lattice. An apparently promising candidate involves the introduction of a dispersion wave in the arcs of the Main Injector, around transition time. This begins to resemble an unmatched γ_t jump scheme [6] – except that the lattice perturbation can be introduced slowly, and that the needed size of the dispersion wave is expected to be relatively modest.

V. BEHAVIOR OF γ_t JUMP SCHEMES

The satisfactory agreement between MAD simulations and analytic predictions reported in the previous section encourages the use of the program to study the behavior of momentum compaction factors for more realistic Main Injector lattices, where analytic results are no longer tractable. Here we consider two families of Main Injector lattices representing matched and unmatched γ_t jump schemes. These schemes are described in detail elsewhere [6]. It is important to check that the resulting change of α_1 does not greatly affect the Johnsen time T_J, extending the variation of transition crossing time for different parts of a bunch.

The simulation places one thin eddy current sextupoles of strength g_E at the middle of each dipole, with a multipole strength [7] of $b_2 = 0.561\ m^{-2}$. Two families of chromatic sextupoles are used to compensate for both natural and eddy current chromaticities. It is assumed that the F and D sextupole strengths are not changed while jumping through transition.



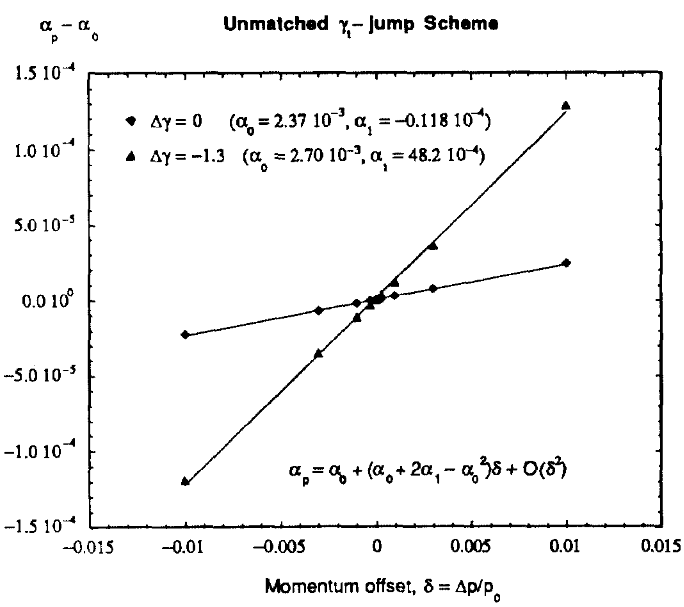


Figure 1. Numerical simulation of α_p versus δ carried out for various γ_t jump configurations.

Figure 1 summarize the behavior of the *matched* scheme with bipolar ($\Delta\gamma_t = \pm 0.65$) and unipolar ($\Delta\gamma_t = -1.3$) excitations as well as an *unmatched* unipolar excitation - a bipolar jump is not possible in this scheme. The linear character of $\alpha_p(\delta)$ in the realistic range $\delta = -0.01$ to $+0.01$ is apparent in all cases.

One can conclude that eddy current sextupoles are not expected to significantly affect transition crossing performance. Continuing investigations suggest that a modest dispersion wave significantly improves the orthogonality of three families of sextupoles.

VI. CONCLUSIONS

General analytical expressions make it possible to evaluate how the critical Johnsen time depends on effects like eddy current sextupoles in the Main Injector dipoles, or on transition jump configurations. In the simple case of a FODO lattice representation of the Main Injector, analytic results are in good quantitative agreement with a lattice design code.

Examination of nominal Main Injector transition jump schemes reveals that a *matched* scheme produces little change in the Johnsen time, but that T_J is more than doubled in the *unmatched* scheme.

A third family of sextupoles might be used to deliberately and practically control the Johnsen time, without modifying the nominal transition momentum. Two other sextupole families are used to achieve the desired net chromaticities. Such control, especially when used in conjunction with RF gymnastics, may make transition crossing so innocuous that it becomes unnecessary to include a transition jump in Main Injector designs.

VII. REFERENCES

[1] S.A. Bogacz, "Transition crossing in the Main Injector - ESME simulation", Proceedings of the F III Instability Workshop, Fermilab, July 1990.

[2] I. Kourbanis and K.Y. Ng, "Main Ring transition crossing simulations", Proceedings of the F III Instability Workshop, Fermilab, July 1990.

[3] K. Johnsen, "Effects of non-linearities on the phase transition", CERN symposium on high energy accelerators and pion physics, Vol. 1, p. 106, 1956.

[4] A. Sorenssen, "Crossing the phase transition in strong focusing proton synchrotrons", Particle Accelerators Vol. 6, pp. 141-165, 1975.

[5] J. Griffin and J. MacLachlan, "Synchrotron phase transition crossing using an RF harmonics", Fermilab, TM-1734, March 1991.

[6] S.A. Bogacz, F. Harfoush, and S. Peggs, A comparison of matched and unmatched γ_t jump schemes in the Main Injector, These Proceedings.

[7] J-F Ostiguy, Eddy current induced multipoles in the Main Injector, Fermilab, MI-0037, October 1990.