Particle Tracking and Map Analysis for Compact Storage Rings

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Abstract

Particle tracking codes for large storage rings approximate the Hamiltonian by neglecting terms of order x/ρ and higher. For storage rings with small bending radius magnets like the Super Conducting X-ray Lithography Source (SXLS) [1], which make use of combined function bending magnets, these approximations cannot be made. We use an explicit symplectic integrator [2] to construct a tracking code which uses the exact Hamiltonian for drifts and isomagnetic combined function bending magnets, in a manner similar to the Teapot code [3]. Hard edge fringe fields are included in a symplectic manner for dipoles and quadrupoles. The integrator is coupled to the DA package of Berz [4] to provide an arbitrary order map which can be analyzed using the tools of Forest and Irwin [5]. A discussion of the techniques and an application to the SXLS ring at Brookhaven is presented.

I. Introduction

The present note describes our modeling of a compact synchrotron xray lithography source, the SXLS device [1]. The first phase of this device, which incorporates conventional electromagnets, is in operation at Brookhaven National Laboratory and is described in a separate presentation at this meeting. SXLS, which is being developed at BNL in cooperation with Grumman Aerospace Corporation under the auspices of the Defense Advanced Research Projects Agency, [1] is intended to eventually serve as a commercial x-ray source for the lithographic production of computer chips.

The second phase of this device will incorporate 3.87 Tesla superconducting combined function bending magnets which are being constructed by General Dynamics Corporation. The design of these superconducting magnets and their effect on the optics of the phase II device is our principal concern.

SXLS has an 8.5 meter circumference a 60 centimeter bend radius and an unique gradient FODO-like lattice structure. The small size of the device implies that some approximations made in large-machine codes are invalid, in particular, the dropping of terms in the Hamiltonian of order x/ρ and higher, where x is horizontal deviation from the closed orbit and ρ is the bending radius of the design orbit. Analysis is further complicated by the strong effect of high-order magnetic field multipoles, the combined function nature of the dipole magnets and the relatively large fraction of the device circumference occupied by fringe fields.

We have not yet addressed all of these problems in our approach to modeling SXLS. For example, our analysis of non-isomagnetic effects is currently done through Marylie 3.0 and a Genmap-like code [6] tailored specifically for SXLS. However, we have begun to develop a computational tool, the Krakpot code, which, we believe, will eventually let us answer many of our unresolved questions.

The principal result of our work is that in devices of this small size it is important to make the high order field multipoles as small as possible. Failure to do so reduces the dynamic aperture. In order to determine this result, we have had to push the state-of-the-art in both magnetic field calculations and tracking code design.

II. Overview of Krakpot

Krakpot, so named because of both because of its origins in Teapot[3], is an analysis code, not a design code. At present, it incorporates • a drift, • a combined function straight element, • a combined function bend element, • a simple RF cavity kick, and • hard-edge fringe elements.

The combined function elements are all of the isomagnetic type. In each of the elements the Hamiltonian is split into one or two exactly soluable pieces as in Teapot [7], and the resulting equations of motion used to implement an explicit canonical integrator.

Quadratic, fourth order [2], and sixth order integrators are all implemented within the code, and higher order integrators are possible. Also present in the code are a routines for the tracking of orbits, determination of the dynamic aperture, and a six dimensional fixed point finder.

The code is intimately linked to the numerical differential-algebra (DA) package [4] of Berz. [2] A "DAified" version of each element and each canonical integrator exists. An arbitrary order Taylor map through a succession of elements can be generated and saved for later analysis.

[1] This work was performed under the auspices of the U.S. Department of Energy and funded by the U.S. Department of Defense

[2] This DA package was developed by Dr. Martin Berz at Lawrence Berkeley Laboratory in 1987 and 1988. It has been considerably modified by the LBL Exploratory Studies Group. The original author cannot be held responsible for its contents.

The map analysis is carried out through post-processor codes based on the LIELIB package of Forest [5]. These extract the normal form of the map, yielding the tunes, chromaticities, geometric tune shift with amplitude and other dynamical lattice parameters.

III. Dodecapole Order Combined Function Dipole

The Hamiltonian for an isomagnetic combined function bending magnet can be written in cylindrical coordinates, with longitudinal distance as the independent variable of integration, as a sum of two explicitly soluable pieces, a drift-like term

$$H_d = -(1 + x/\rho)\sqrt{(1+\delta)^2 - P_x^2 - P_y^2} - \frac{P_t}{\beta_0}, \quad (1)$$

and a kick

$$H_k = -(1 + x/\rho)\,\frac{A_s(x,y)}{B_0\rho}. \quad (2)$$

In Krakpot, the midplane field expansion for A_s has been carried out to dodecapole order (x^5) with MACSYMA using a stream function method [8] developed for the treatment of non-isomagnetic bends.

An exact solution of the equations of motion of each of these Hamiltonians is straightforward and given by Forest [7]. Incorporation of the solutions into an explicit canonical integrator is straightforward. A quadratic integrator results by applying the drift transformation for half a time step, the kick transformation for a full time step, and then the drift transformation again for half a time step.

One minor difficulty of this method in the combined function bend is that the closed orbit, for which initially x, y, t, P_x, P_y, P_t are all zero and $P_s = 1$, is not sent into the closed orbit. The size of the closed orbit error is given by the order of the integration method. A quadratic integrator applied to the isomagnetic combined function bend yields, to lowest order, closed orbit errors of

$$P_x(s) \approx \frac{s^3}{6\rho^3}, \; x(s) \approx \frac{s^4}{12\rho^3}. \quad (3)$$

A simple subtractive procedure is adopted in Krakpot to avoid buildup of the closed orbit error over many timesteps.

For a given element, the closed orbit errors from a single time step are calculated and saved. Thereafter, these saved errors are subtracted from the result of every time step for a general orbit. This subtraction procedure is symplectic, since it is only a translation of the origin of phase space, retains the order of accuracy of the integration method, and preserves the closed orbit to machine precision.

In the DAified versions of these transformations, the six canonical dependent variables (x, y, t, P_x, P_y, P_t) are declared as independent DA variables. Arbitrary order derivatives of the canonical transforms with respect to these variables can then be manufactured for the whole lattice by concatenation of transforms from single time steps. An Nth order Taylor map generated in this fashion is automatically canonical up to terms of order $2N + 1$, which is the order of the error term in the Poisson brackets of the Taylor map.

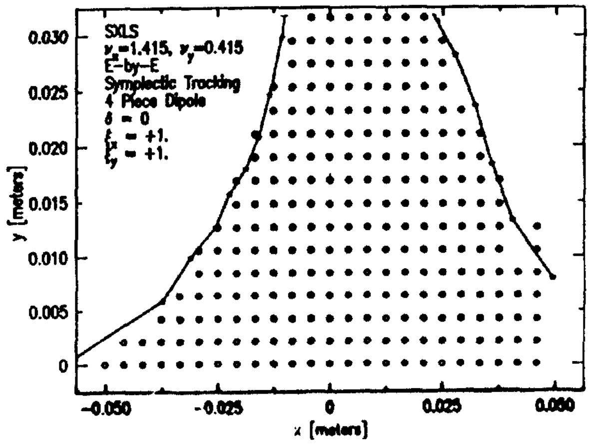

Figure 1: Comparison of Krakpot and Marylie Isomagnetic Apertures

IV. Comparison to Marylie

Several codes have been applied to SXLS and disagree in their predictions of even the first-order chromaticity. The Marylie 3.0 code makes minimal approximations and is well suited for comparison with Krakpot.

Figure 1 overlays dynamic apertures found using isomagnetic elements with Marylie and Krakpot. Horizontal and vertical tunes were parked at the design values of 1.415 and 0.415 respectively. Sextupole elements in the lattice were adjusted to yield a chromaticity of 1.0 in both planes. On energy orbits were tracked symplectically for 1000 turns through the lattice. Tracking was done on an element by element basis in Marylie using the circulate command. The dipole was split into four pieces to improve accuracy. Each element in the Krakpot code was divided into as many as one hundred explicit canonical transformations. Even though the tracking methods used differ considerably, the apertures agree. This agreement extends to the values of nonlinear lattice parameters found by a normal form transformation of the one turn maps, which can be carried out to only third order with Marylie.

V. Results for SXLS

The chief impetus for the development of Krakpot was to obtain a reliable tool for the modeling of high order effects not treatable with the Marylie 3.0 code. Based on previous code results, we expected that bending magnet multipoles, of octupole order and above, would have a significant effect on the SXLS dynamic aperture. This has turned out to be the case, and has a considerable impact on the design of the bending magnet.

The results presented below were all generated with Krakpot using isomagnetic elements and hard edge fringes. Since tracking was carried out using explicit canonical integration rather than a taylor map, the only approximations in order are made in the expressions for the vector potential.

Figure 2 gives the variation of on energy dynamic aperture as a function of the magnitude of octupole (B_3) in the 180 degree bending magnet.

The improvement of aperture with the addition of a small amount of octupole shown in Fig. 2 may explained

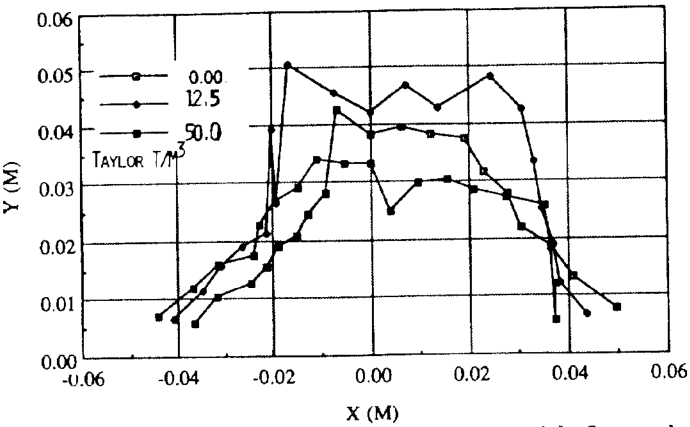

Figure 2: Variation of SXLS Aperture with Octupole

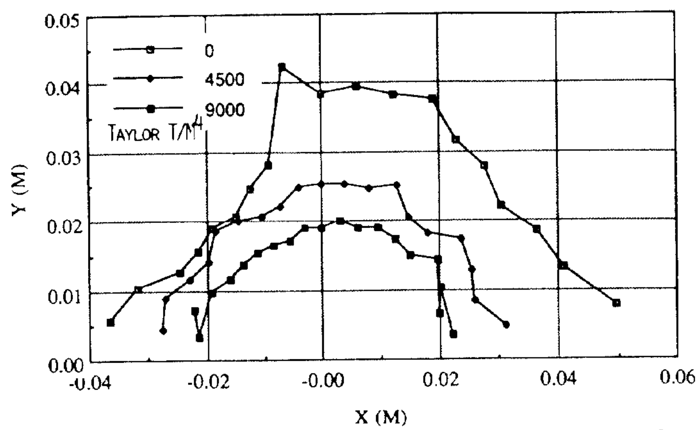

Figure 3: Variation of SXLS Aperture with Decapole

by the form of the vertical tune shifts with amplitude as a function of the octupole strength, $\Pi_{yy} = -28.16 + 7.08\,B_3/B_0$. Vertical aperture can be increased by reducing Π_{yy} to zero with an appropriate choice of $B_3/B_0 \approx 4$ with relatively little effect on the horizontal aperture.

Aperture variation with decapole is given in figure 3. Decapole components on the order of $10^4\ T/M^4$ are quite likely in small radius, combined function, bending magnets designed without regard for higher order multipoles.

Trim windings, mainly for controlling the quadrupole and sextupole moments, have been included in the design of the SXLS phase II magnet because of our concern about some of these effects.

VI. DA Applied to Magnetic Field Calculations

We have written a new code which applies the DA package to the Biot Savart law, in essence, differentiating under the integral sign. The output of the code is not only the three components of the magnetic field at a point but also the arbitrary order spatial derivatives of the field components at the point, that is, the field moments. This procedure gives a more accurate local estimate of the moments than may be obtained from the alternative procedure of fitting a polynomial to the field at several points. We plan to apply this code to fringe regions in the Krakpot code.

A bonus of this approach is that the conductor positions, or combinations of them, may be declared as DA variables, yielding the derivatives of the various moments with respect to conductor position. Mechanical tolerances are immediately established based on the maximum tolerable amplitude of moments. Precise combinations of moments may be sought by automated iterative adjusment of the conductor positions.

VII. Conclusions

Compact electron storage rings, like the BNL Superconducting X-ray Lithography Source, present a significant challenge to the accelerator designer. A novel "Krakpot" code has been described, which has been developed to overcome some of the difficulties presented by these rings. Some of the techniques employed in this code may find an application in other accelerators.

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