# Non-Linear Resonance Studies at the Synchrotron Radiation Center, Stoughton, Wisconsin* 

E. Crosbie, J. Bridges, Y. Cho, D. Ciarlette, R. Kustom, Y. Liu, K. Symon, ${ }^{1}$ L. Teng, W. Trzeciak ${ }^{1}$<br>Argonne National Laboratory<br>9700 South Cass Avenue<br>Argonne, IL 60439

## Abstract

A single bunch is stored in ALADDIN at 800 MeV . The coherent oscillations produced by a fast kicker are recorded for each turn at two horizontal position detectors separated by $\pi / 2$ in phase. Displaying the recorded positions on horizontal and vertical axis produces a phase plot of the coherent motion. Resonances are produced by adding a controlled amount of sextupole field. The results are compared with computer simulations. To adequately describe the behavior near unstable fixed points the finite size of the beam must be taken into account.

## I. INTRODUCTION

Non-lincar resonance studies are being conducted at the ALADDIN $800-\mathrm{MeV}$ Electron Storage Ring located at the University of Wisconsin Synchrotron Radiation Center in Stoughton, Wisconsin [1]. A fast kicker produces coherent oscillation of a single bunch stored in the ring. The turn to turn positions are recorded at two sets of horizontal and two sets of vertical pickup electrodes spaced at $90^{\circ}$ in each plane. The motion is recorded for 4096 turns for each experimental run.

The stored data is used to construct horizontal and vertical phase space representations of the motion. The experimental results are compared with results from the computer tracking program PACMAN. Since the bunch in the storage ring has a finite size ( $\sigma_{\mathrm{x}}=0.8 \mathrm{~mm}$ ), the tracking program has been modified to follow the average phase space motion of several particles. In this paper we describe only the horizontal phase space motion near the third integral resonance created by sextupole magnets.

## II. DATA ACQUSITION CAPABILITIES FOR SRC

The data acquisition system consists of circuitry for timing, pulse stretching, analog-to-digital converters, data display, and data storage. It is initiated by a trigger pulse at a selected time prior to firing the fast kicker. The A/O conver-

[^0]ters acquire and store the data at the beam revolution frequency. Eight channels are used to monitor the signals from 2 sets of horizontal and 2 sets of vertical stripline position pickups. The 1 nsec signals from the stripline are stretched to 100 nsec to match the $\mathrm{A} / \mathrm{O}$ acquisition times.

The data is then down-loaded into an IBM PC compatible computer and the difference divided by the sum of each pair of signals is calculated. A look-up table can be used to cancel the nonlinear transfer function of the voltage versus position signal. Position as a function of time (turn) can be displayed along with phase-space diagrams of both horizontal and vertical data. Once the experiment is finished the data is transferred to a VAX computer for further analysis.

## III. CORRECTED PHASE SPACE PLOTS

The phase difference between the beta-functions at the two pickups depends on the tune of the ring. Choosing $\Delta x_{1}(\mathrm{n})$ as one phase component, the other component is

$$
\begin{equation*}
\operatorname{Px}(\mathrm{n})=\left(\mathrm{R} \Delta \mathrm{x}_{2}(\mathrm{n})-\Delta \mathrm{x}_{1}(\mathrm{n}) \cos \Delta \phi\right) / \sin \Delta \phi \tag{1}
\end{equation*}
$$

where $\quad \Delta x_{j}(n)=x_{i}(n)-c_{i}$,
$\mathrm{c}_{\mathrm{i}}=$ orbit positions at i ,
$\Delta \phi=$ phase separations between 1 and 2, and $\quad R^{2}=\beta x_{1} / \beta x_{2}$.

The necessary corrections are obtained from linear oscillation data. A small amplitude oscillation is produced, and a least squares fit is made to the turn by turn position data.

$$
\begin{gather*}
x_{1}(n)=c_{1}+A \cos \left(2 \pi n v_{x}+\phi_{0}\right)  \tag{2}\\
x_{2}(n)=c_{2}+A / R \cos \left(2 \pi n v_{x}+\phi_{0}+\Delta \phi\right) \tag{3}
\end{gather*}
$$

The fitting parameters are $\mathrm{c}_{1}, \mathrm{c}_{2}, \mathrm{~A}, \mathrm{R}, v_{\mathrm{x}}, \phi_{0}, \Delta \phi$.
Figure 1 shows coherent linear oscillation monitored at position 1. Figure 2 shows the corrected phase space plot. The plots are presented in BPM units. 1 BPM unit $\approx 18 \mathrm{~mm}$.

## IV. THIRD INTEGRAL RESONANCE STUDIES

For resonance studies, the ALADDIN ring is tuned to near $v_{x}=7.33$. A single sextupole is powered to produce the 22 nd harmonic. Computer simulations predict the separatrices shown in Figures 4,6 , and $8{ }^{2}$.


Figure 1. Coherent oscillation at pick up 1.


Figure 2. Phase plot derived from pickups 1 and 2.
Figure 3 shows a typical experimental phase plot for a small amplitude oscillation. Because of the finite beam size and because of the proximity of portions of the bunch to the separatrix, there is an apparent damping of the oscillations due to loss of coherence. Figure 4 is a computer simulation of the same behavior. The figure shows the center of gravity motion of 5 particles starting on the vertical spin with initial amplitudes spread over 2 mm starting at about 1 mm from the separatrix.

Figure 5 illustrates the behavior for a larger initial kick. The kick put most, but not all, the beam into the "lobe". In the computer simulation, Figure 6, 4 of the 5 initial amplitudes are beyond the separatrix. The total spread is 2 mm .

When the kick is large enough the whole team oscillates about the stable fixed points. The behavior is illustrated in Figure 7. Again because of the finite size of the beam, the individual electrons get out of phase and produce an apparent damping of the beam to the three fixed points. (The real damping time constant is about 27 ms or 100,000 turns.) The corresponding computer simulation shown in Figure 8 shows the average of 5 particles spread initially over 2 mm .


Figure 3. Experimental phase plot for small amplitude oscillation near the resonance $3 v_{\mathrm{x}}=22$.


Figure 4. Computer simulation corresponding to Figure 3.


Figure 5. Experimental phase plot with large amplitude oscillations for the same conditions as Figure 3.


Figure 6. Computer simulation corresponding to Figure 5.


Figure 7. Experimental phase plot with total beam kicked to the stable fixed points.


Figure 8. Computer simulations corresponding to Figure 7.
Once the beam is in the "lobes" it stays there with no apparent loss in lifetime. The position monitors show the bunch moving from fixed point to fixed point. To return the beam to the center it is necessary to switch off the sextupole magnet.

## V. REFERENCES

[1] J. Bridges, Y. Cho, W. Chou, E. Crosbie, S. Kramer, R. Kustom, K. Kleman, R. Otte, W. Trzeciak, and K. Symon, "Dynamic Aperture Measurements on ALADDIN," Particle Accelerators, vol. 28-29, issues 14, pp. 479-487.


[^0]:    *Work supported by U.S. Department of Energy, Office of Basic Energy Sciences under Contract No. W-31-109-ENG-38.
    ${ }^{1}$ At the Synchrotron Radiation Center, Stoughton, Wisconsin.
    ${ }^{2}$ In order to get agreement with the experimental results it was necessary to add an average octupole tune shifting component. Such a component can come from quadrupole edge fields which are not included in our tracking program.

