# Suppression of Single Bunch Beam Breakup by Autophasing*

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## I. INTRODUCTION

The increased interest in using intense, narrow, short bunches in accelerators has triggered an interest in the phenomenon of single bunch beam breakup[1]. Several analyses of this phenomenon have been made[2-4], including the effects of energy spread in the bunch. In a recent paper[4] we developed an analysis of single bunch beam breakup, including the effect of a linear variation of the transverse focussing force with longitudinal position within the bunch (leading to what is known as BNS damping[5]). A subsequent suggestion by Balakin[6] to shape the variation of transverse force with position within the bunching, called autophasing, makes it possible, in principle, to eliminate growth due to beam breakup.

The physical picture of autophasing is easy to understand and leads to a rough approximation for the necessary force gradient. But the required shaping is technically very difficult and one will probably have to be satisfied with approximate suppression.

In Section II we review the theory of single bunch beam breakup with BNS damping for a coasting beam and in Section III show the implications of autophasing. In Section IV we present a reliable way to estimate the growth due to beam breakup with linear variation of the transverse force in terms of two universal parameters, and include comparison with simulations.

## II. SINGLE BUNCH BEAM BREAKUP

The equations which govern the displacement ($\xi(N,M)$) and excitation of the deflecting mode (proportional to $z(N,M)$) for the $M^{th}$ "macroparticle" of the bunch as it enters the $N^{th}$ cavity are

$$\frac{\partial^2\xi(N,M)}{\partial N^2}+\mu^2(M)\xi(N,M)=z(M,N) \qquad (1)$$

and

$$z(N,M)=\omega\tau\int_o^M d\ell(M-\ell)\; r(\ell)\;\xi(N,\ell), \qquad (2)$$

*Work supported by the Department of Energy.

where $\mu(M)$ is the phase advance of the transverse oscillation per cavity. We have approximated the cavity wakefield, which is proportional to
$r(\ell)\; sin(M-\ell)\;\omega\tau$, by its linear approximation for a bunch whose length is short compared to an r.f. wave length. Here

$$r(\ell)=\frac{e\sigma(\ell)}{2W}\frac{Z_\perp T^2}{Q}\;L, \qquad (3)$$

where $r(\ell)$ is a measure of the charge in the $\ell^{th}$ macroparticle and its influence on the transverse motion. The bunch has an energy $W$ and a total charge $N_pe=\int d\ell\sigma(\ell)$. The cavities, separated from each other by a distance $L$, have a transverse mode frequency $\omega/2\pi$, a quality factor $Q$, and a shunt impedance parameter $Z_\perp T^2/Q$, where $T$ is transit time factor. For uniform longitudinal bunch density we have $\sigma(\ell)=N_pe/M_o$, where the bunch is divided into $M_o$ macroparticles. One readily includes several modes by the replacement

$$\omega(Z_\perp T^2/Q)\;\rightarrow\sum_j\omega_j(Z_\perp T^2/Q)_j. \qquad (4)$$

Two derivatives of Eq. (2) with respect to $M$ lead to

$$\frac{\partial^2 z(N,M)}{\partial M^2}=\omega\tau\; r(M)\;\xi(N,\ell). \qquad (5)$$

Equations (1) and (5) are the starting point for our analysis. Our notation is very similar to that used in an earlier formulation of cumulative beam breakup.[7]

## III. AUTOPHASING

In autophasing[6], one looks for a solution in which $\xi(N,M)=\xi(N)$ is a function only of $N$, that is, the bunch moves as a rigid body. This is accomplished by writing

$$\mu^2(M)=\mu_o^2+\delta\mu^2(M), \qquad (6)$$

where

$$\frac{\partial^2\xi}{\partial N^2}+\mu_o^2\xi=0, \qquad (7)$$

leading to

$$\delta\mu^2(M)\xi(N)=z(M,N). \qquad (8)$$

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From Eq. (5) we obtain the autophasing condition

$$\frac{d^2}{dM^2}[\delta\mu^2(M)] = \omega\tau r(M), \tag{9}$$

that is, the second derivative of the focussing force with respect to position within the bunch must match the transverse wakefield.

If we assume a symmetric Gaussian-like bunch with 90% of the bunch between $M = M_1$ and $M = M_2$, Eq. (9) can be integrated to obtain

$$\delta\mu^2(M_2) - \delta\mu^2(M_1) \cong \frac{\omega\tau(M_2 - M_1)}{2}\int_{-\infty}^{\infty} d\ell\ r(\ell). \tag{10}$$

This is then the change of the focussing force from one "end" of the bunch to the other required to suppress beam breakup.

## IV. UNIVERSAL PARAMETERS

In view of the difficulty in shaping the variation of the transverse force with position within the bunch, we shall treat the case of a linear variation for a coasting beam bunch of uniform density. Equations (1) and (5), for constant $\mu(M) \equiv \mu$ and $r(M) \equiv r$, have the approximate solution[4]

$$\frac{\xi(N,M)}{\xi_o} \cong \frac{1}{\sqrt{4\pi}} Re\left(\frac{e^{i\mu N + ue^{-i\pi/6}}}{\sqrt{ue^{-i\pi/6}}}\right), \tag{11}$$

where $\xi_o$ is the initial bunch offset, and where

$$u = \frac{3}{2}\left(\frac{\omega\tau}{\mu}\right)^{1/3} M^{2/3}\ N^{1/3} \tag{12}$$

is a universal parameter assumed to be large compared to 1 in Eq. (11). Beam breakup is associated primarily with the term in $u$ in the exponential.

In Fig. 1 we plot ln $(\eta^s) \equiv \ln(\xi_{max}/\xi_o)$, obtained from numerical simulations of the original difference equations[7], as a function of $u$, and compare it with the analytic value predicted by Eq. (11):

$$\ln(\eta^a) \equiv \ln(\xi_{max}/\xi_o) = \frac{u\sqrt{3}}{2} - \frac{\ln\ 4\pi u}{2}. \tag{13}$$

Here $\xi_{max}$ is the amplitude of the oscillation with respect to $N$. Specifically, we show $\ln(\eta^s)$ and $\ln(\eta^s) - \ln(\eta^a)$ in Fig. 1. The agreement between $\eta^s$ and $\eta^a$ is excellent (within ±4%) for values of $u > 3$.

If we now allow $\mu(M)$ to have a linear dependence on $M$:

$$\mu(M) = \mu + \alpha M, \tag{14}$$

such that Eq. (1) can be approximated by

$$\frac{\partial^2\xi(N,M)}{\partial N^2} + \mu^2 + 2\alpha M\mu \cong z(N,M), \tag{15}$$

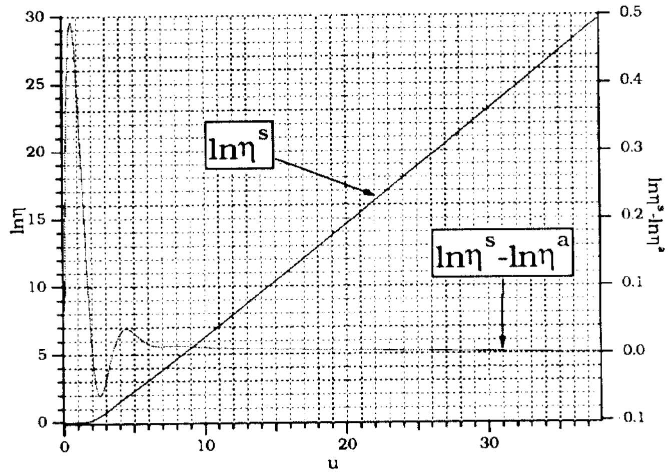


Figure 1: $\ln(\eta^s)$ and $\ln(\eta^s) - \ln(\eta^a)$.

we can obtain the solution for $\xi/\xi_o$ in the limit $u \gg 1$:

$$\frac{\xi(N,M)}{\xi_o} \cong \frac{1}{\sqrt{4\pi}} Re\left(\frac{e^{i\mu N + uf(v) + g(v)}}{\sqrt{ue^{-i\pi/6}}}\right). \tag{16}$$

The second universal parameter $v$ is defined as

$$v = \alpha\left(\frac{\mu}{r\omega\tau}\right)^{1/3} M^{1/3}\ N^{2/3} \tag{17}$$

and the functions $f(v)$ and $g(v)$, whose defining equations are given in Reference 5, are shown in Figs. 2 and 3. In

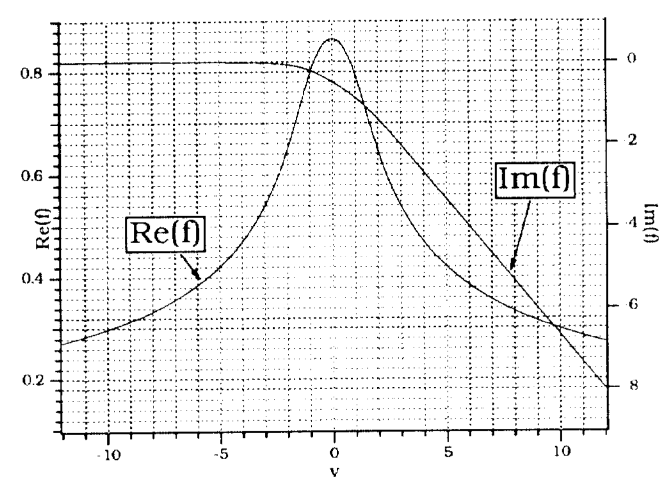


Figure 2: $Re\ f(v)$ and $Im\ f(v)$.

Fig. 4 we plot $\ln(\eta^s)$, obtained from numerical simulations, as a function of $v$ and compare it with the analytic value predicted by Eq. (16):

$$\ln(\eta^a) = u\ Re\ f(v) + Re\ g(v) - \frac{1}{2}\ln\ 4\pi u + \Delta(u), \tag{18}$$

where

$$\Delta(u) = \ln\ (\eta^s)\ |_{v=0} - \left(\frac{u\sqrt{3}}{2} - \frac{\ln\ 4\pi u}{2}\right) \tag{19}$$

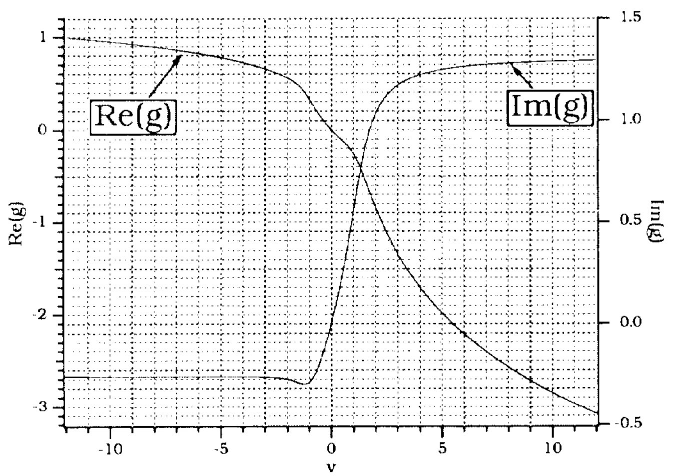


Figure 3: $Reg(v)$ and $Img(v)$.

corrects for the small inaccuracy of Eq. (11) for $v = 0$ shown in Fig. 1. Once again, the agreement between the simulations and the predictions is excellent, particularly over the range $-2 < v < 10$.

Figure 4 contains the necessary information to determine the gradient of $u$ necessary to suppress beam breakup. Specifically, one calculates $u$ from Eq. (12), and uses the appropriate curve to determine the value of $v$ for which $\ln(\eta) \cong 1$. The desired $\alpha$ is then obtained from Eq. (17).

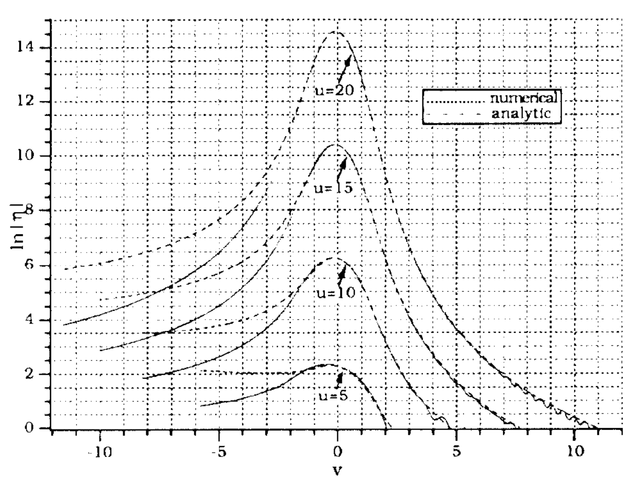


Figure 4: $\ln(\eta^s)$ and $\ln(\eta^a)$.

The relation between the accurate results in Fig. 4 and the approximation in Eq. (10) can be obtained by observing that suppression occurs at relatively large values of $v$. Considering only the term $u\ Re\ f(v)$ in the exponential in Eq. (16) and using the large $v$ limit, $f(v) \cong (8/9v)^{1/2}$, we find

$$u\ Re\ f(v) \cong (8u^2/9v)^{1/2} = (2M\ r\ \omega\tau/\mu\alpha)^{1/2}. \quad (20)$$

If we assume that suppression takes place when $u\ Re\ f(v)$ is of order 1, we predict

$$\delta\mu^2(M) = (\mu + \alpha M)^2 - \mu^2 \cong 2\alpha\ M\ \mu \sim M^2 r\ \omega\tau, \quad (21)$$

in close agreement with the simplified autophasing version in Eq. (10).

Finally, we reiterate that our analysis is for a coasting beam bunch of uniform density, with a linear variation of focussing force with position within the bunch. These restrictions can be removed by using numerical simulations. It is also necessary to include the longitudinal wakefield in determining the focussing force within the bunch. Another concern is that our formulation assumes that each macroparticle transverses the cavity before the next one enters. This can be justified by averaging the displacement of the macroparticles over each cavity.[8]

## V. REFERENCES


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