

Tracking Studies for the Oxford Instruments Compact Electron Synchrotron

Chas. N. Archie

IBM Research, Thomas J. Watson Research Center, Yorktown Heights, N.Y. 10598

Jan Uythoven

Oxford Instruments Ltd., Synchrotron Division, Osney Mead, Oxford, England OX2 0DX

Abstract

Simulating electron trajectories (tracking) in a realistic compact ring introduces special problems because the magnetic bending radius can be as little as twenty times the horizontal displacement. Common simplifying approximations in tracking codes may not be appropriate. Results are presented from a thin lens tracking code which correctly handles the small bending radius virtual forces and real magnetic forces to octupole level. Nonsymplectic behavior is controlled by highly segmenting the Oxford superconducting dipoles. A qualitative comparison with experimental results is made.

Introduction

A superconducting electron storage ring intended primarily for X-ray lithography has recently been built by Oxford Instruments for IBM. This machine is currently being commissioned at the East Fishkill, New York IBM facility. The machine, called HELIOS, includes two 180° superconducting dipole magnets with a typical magnetic field of 4.5 T at an electron energy of 700 MeV. This gives a synchrotron light critical wavelength of 8.4 Å. The bending radius is 0.52 m and the total circumference is 9.6 m with about one third of this length within the dipole magnets.

Tracking an electron in this machine requires dealing with special computational problems not encountered in larger rings with conventional magnets. In particular, the usual procedure of switching to a coordinate system tied to the design energy closed orbit (a non-inertial reference frame) introduces *virtual* forces which are comparable to the higher order magnetic forces coming from a Taylor series expansion of the local magnetic field. Several of these virtual forces are examined in the case of HELIOS in the next section after which some tracking results are presented.

The Equations of Motion

In this paper there is room enough only for a brief outline of the derivation of the equations of motion to third order in the transverse coordinates and velocities.¹ Figure 1 shows the curvilinear coordinate system tied to the closed orbit of a design energy electron. Assuming that the design orbit is totally in the median plane, time derivatives of the unit vectors are

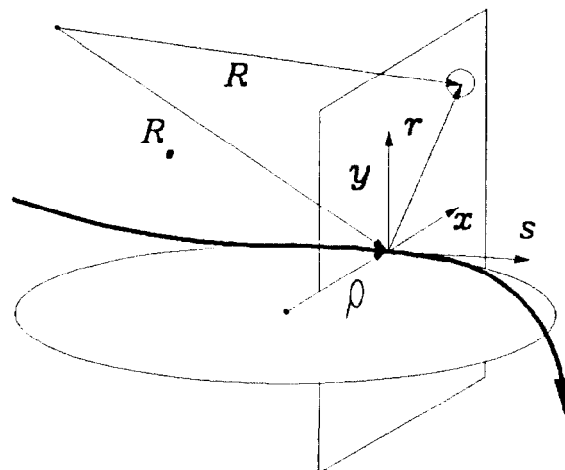


Figure 1. The Curvilinear Coordinate System

$$\dot{\vec{x}} = h\dot{s}\vec{s} \quad \dot{\vec{y}} = 0 \quad \dot{\vec{s}} = -h\dot{s}\vec{x} \quad (1)$$

where $h(s) = 1/\rho$. The position relative to a fixed arbitrary point, velocity, and acceleration are given by

$$\vec{R} = x\vec{x} + y\vec{y} + \vec{R}_0 \quad (2)$$

$$\vec{v} = \dot{x}\vec{x} + \dot{y}\vec{y} + \dot{s}(1 + hx)\vec{s} \quad (3)$$

$$\vec{a} = (\ddot{x} - h\dot{s}^2(1 + hx))\vec{x} + \ddot{y}\vec{y} + (\ddot{s}(1 + hx) + 2h\dot{x}\dot{s} + h\dot{x}\dot{s})\vec{s} \quad (4)$$

These relations should be substituted into the Lorentz equations for the particle motion

$$\dot{\vec{v}} = \frac{e}{m} (\vec{v} \times \vec{B}) \quad (5)$$

Time derivatives in the equations for the x and y directions can be rewritten in terms of s derivatives, denoted by ', with the help of the equation of motion for the s direction:

¹ C. N. Archie, D. J. Orrell and J. A. Uythoven, "Particle Tracking in Small Radius Electron Storage Rings," to be published.

$$x'' - \frac{h'x + 2hx'}{1 + hx} x' - h(1 + hx) = \frac{e}{p} \sqrt{x'^2 + y'^2 + (1 + hx)^2} \left[\frac{x'y'}{1 + hx} B_x - (1 + hx) \left(1 + \frac{x'^2}{(1 + hx)^2} \right) B_y + y' B_s \right] \quad (6)$$

and

$$y'' - \frac{h'y + 2hy'}{1 + hy} y' = \frac{e}{p} \sqrt{x'^2 + y'^2 + (1 + hy)^2} \left[(1 + hy) \left(1 + \frac{y'^2}{(1 + hy)^2} \right) B_x - \frac{x'y'}{1 + hy} B_y - x' B_s \right]. \quad (7)$$

The magnetic fields can be written down in more familiar accelerator physics terms. The expansions are limited to third order for B_x and B_y but only second order for B_s since B_s always enters in the equations of motion multiplied by a transverse velocity component.

$$\frac{e}{p_0} B_y = h + kx + \frac{1}{2} m_1 x^2 - \frac{1}{2} m_2 y^2 + \frac{1}{3} n_1 x^3 - n_2 xy^2 \quad (8)$$

$$\frac{e}{p_0} B_x = ky + m_1 xy + n_1 x^2 y - \frac{1}{3} n_2 y^3 \quad (9)$$

$$\frac{e}{p_0} B_s = h'y + (k' - hh')xy \quad (10)$$

Upon introducing these expressions and expanding the square roots we obtain

$$x'' = -(k + h^2)x - \left(\frac{1}{2} m_1 + 2hk + h^3 \right) x^2 + \frac{1}{2} m_2 y^2 + h'(xx' + yy') + \frac{1}{2} h(x'^2 - y'^2) + \left(\frac{1}{3} n_1 + m_1 h + h^2 k \right) x^3 + (n_2 - hm_2) xy^2 - hh'x^2 x' - \left(\frac{3}{2} k + 2h^2 \right) xx'^2 + kx'y'y' - \frac{1}{2} kxy'^2 + k'xyy' \quad (11)$$

and

$$y'' = ky + (m_1 + 2hk)xy + h'xy' - h'x'y + hx'y' + (n_1 + 2hm_1 + h^2 k)x^2 y - \frac{1}{3} n_2 y^3 - k'xx'y - hh'xx'y - (k + 2h^2)xx'y' + \frac{1}{2} kx'^2 y + \frac{3}{2} kyy'^2. \quad (12)$$

The higher order terms on the right hand sides of Equations (11) and (12) partition into those which stem solely from the higher order magnetic fields present and *virtual* forces which result from the transformation to a noninertial reference frame, the curvilinear coordinate system. Clearly all forces on the real particles are magnetic forces but this artificial grouping into *virtual* forces and magnetic forces is instructive for showing the importance of keeping certain terms in the truncations of the equations of motion in the curvilinear coordinate system. We focus on the relative strengths of the sextupole order forces as these largely determine the dynamic aperture.

With these terms dependent on various powers of x , x' , y and y' , it is not obvious how they compare. One way to do this relies on the higher order force terms being generally small compared to the dipole and quadrupole field terms as is certainly true for small excursions about the design orbit. Hence the pseudo-harmonic solution to the strong focussing terms alone describes the motion fairly well over a few revolutions in the lattice. It is therefore meaningful to use time averages of the pseudo-harmonic solution to make relative comparisons of the higher order terms. This

assumption is only used to gain an appreciation of the relative magnitudes of the forces in the problem.

For simplicity the discussion is restricted to comparing force terms for a purely horizontal deviation of the particle from the closed orbit. The solution to the linear part of the equation of motion is given by

$$x = \sqrt{\epsilon_x \beta_x} \cos \mu_x \quad (13)$$

where β_x is the usual accelerator physics horizontal betatron function, ϵ_x is the conserved horizontal emittance and μ_x is the betatron phase. Let the brackets $\langle \rangle$ denote time averaging at a given lattice position. Then it can be shown

$$\langle x'^2 \rangle = \frac{1}{4} (4 + \beta_x'^2) / \beta_x^2 \langle x^2 \rangle \quad (14)$$

and

$$\langle xx' \rangle = \frac{1}{2} \beta_x' / \beta_x \langle x^2 \rangle. \quad (15)$$

Returning now to Equation (11), the purely horizontal sextupole order force terms on the right hand side can now be compared. Figure 2 shows the lattice parameters β_x , β_y and the dispersion η for a possible operating tune point of the HELIOS lattice. Using the above time averages and the calculated dipole fields,

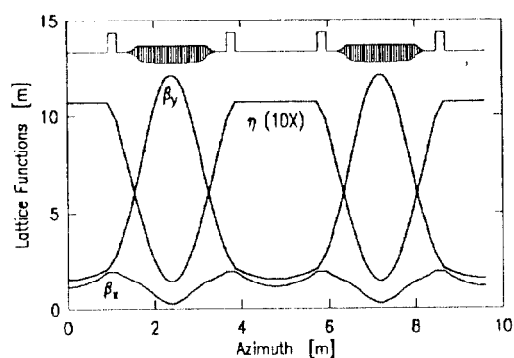


Figure 2. HELIOS Lattice Parameters:

the various coefficients of second order force terms are shown in Figure 3. Several features are worth noting. The magnetic field contributes kick-counterkick pairs at each end of the dipole magnet and hence there is partial cancellation. By contrast, the three *virtual* force terms, some of which extend throughout the body of the magnet, do not similarly pair up.

Tracking Application to HELIOS

The previous section demonstrates that significant higher order forces extend throughout the HELIOS dipole. This, together with the relatively large length of the dipoles, implies that a thin lens kick code can only be used provided the dipoles are highly segmented. The results reported here come from a thin lens kick code in which gradient dipoles are treated exactly by a matrix formalism and all higher order force are represented by kicks between the magnet segments. Because HELIOS is a compact ring with few actual components in the ring, the dipoles can be

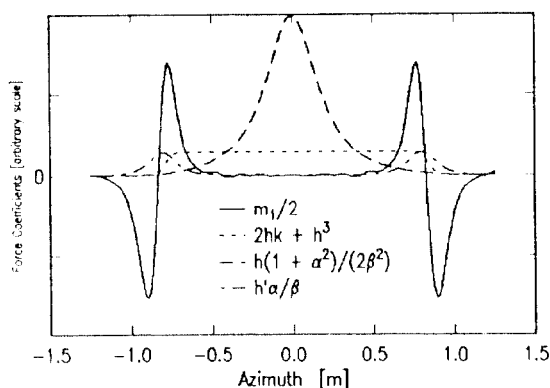


Figure 3. Time Averaged Sextupole Order Force Comparison: Note $s = 0$ at the dipole center and $\alpha = -\frac{1}{2}\beta'$.

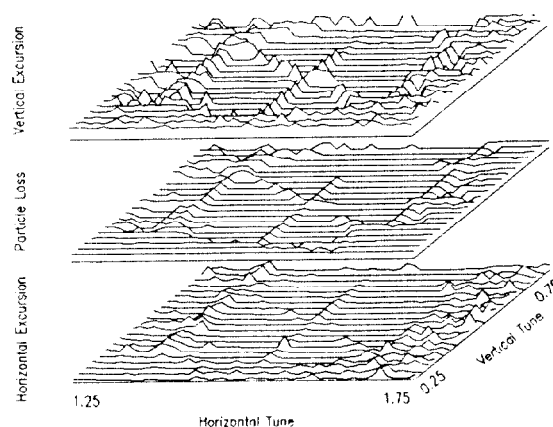


Figure 4. Stored Beam Tune Scan

highly segmented without serious computational difficulty.

In Figure 4 a scan over tune space for HELIOS is presented. Adjustable sextupoles in the dipoles and one straight are set to zero the chromaticities at each tune point. The initial excursion of the particle at the middle of the ring straight section is calculated at each tune point to be ten times the natural beam size ($10\sigma_x$) with 10% coupling between horizontal and vertical motion. The particle is tracked for a maximum of 200 revolutions and the maximum vertical and horizontal excursions are noted. Doubling the number of maximum revolutions has little effect on the topology of the figure. If the excursion exceeds 50 mm in either of the two transverse directions, the particle is considered lost. The middle graph is a quantity which is proportional to 200 minus the number of successful revolutions of the particle. In some sense this graph is a composite of the other two graphs. If the particle is lost then this quantity is zero.

Of course, these studies do not take into account collective effects which may have significant effects on beam stability at any tune point. Nevertheless, preliminary commissioning experience on HELIOS provides some qualitative confirmation of these tracking results in that sizable currents have been injected and stored in many of the quiet regions of Figure 4. It should be emphasized that the comparison of tracking results and commissioning experience reported is preliminary and that considerably more experience is necessary in order to go beyond qualitative remarks.

The authors wish to thank particularly Mark Barton for his suggestions and constructive criticism and the people at Daresbury Laboratory and Oxford Instruments who designed HELIOS.