

# DESIGN OF 4-8 GHz STOCHASTIC COOLING EQUALIZERS FOR THE FERMILAB ACCUMULATOR

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*Abstract*--The 4-8 GHz core stochastic cooling system in the Fermilab Accumulator uses a coaxial cable transmission line to transmit signals from the pickup electrodes to the kicker electrodes. Because of the long length of the cable and the high frequency range of the cooling system, the system suffers a 40 dB gain slope and a 360° phase variation across the 4-8 GHz Band. To remedy this situation, a balanced stripline equalizer was designed featuring offset coupled lines and Schiffman phase shifters. The design was optimized by defining an effective bandwidth density and programming the OUTVAR and OUTEQN blocks of the microwave CAD program TOUCHSTONE to maximize bandwidth density.

## I. INTRODUCTION

Stochastic cooling is a means of reducing the beam emittance using a microwave feedback system incorporated into a circular accelerator [1]. Particles that are not on the correct orbit are detected by a pickup array. This signal is then feed to a kicker array which corrects the particle position. The cooling rate is proportional to the bandwidth of the system. For the 4-8 GHz Betatron systems in the Accumulator, the distance between the pickup and kicker is approximately one third of the circumference of the accelerator. The signal is transmitted from the pickup to the kicker by means of a 1/2" diameter coaxial trunk line that spans the chord between pickup and kicker. The length of the coaxial line is approximately 350 ft. The insertion loss of the coaxial cable is 20 dB at 4 GHz and increases to 40 dB at 8 GHz. The signal level is boosted along the trunk line by placing amplifiers at the beginning and the end of the cable.

## II. COOLING THEORY

As shown in Fig. 1, a network analyzer is inserted into the trunk line to measure the gain and phase of the cooling system at a number of Schottky bands throughout the bandwidth of the cooling system. The response is the product of the electronic gain of the cooling system and the beam transfer function from kicker to pickup. The response of the core horizontal betatron system before the equalizers were built is shown in Fig. 2. (The vertical betatron system response is similar to the horizontal response.) Because the stochastic cooling system is a feedback loop, the desired phase for all frequencies is 180°. Phase excursions greater than ±90° from 180° will cause emittance growth (heating). As shown in Fig. 2, there is a large region of the frequency band of the system that will cause the beam to heat. Also, the gain of the system

falls off significantly at 8 GHz which causes this portion of the band to be useless for cooling.

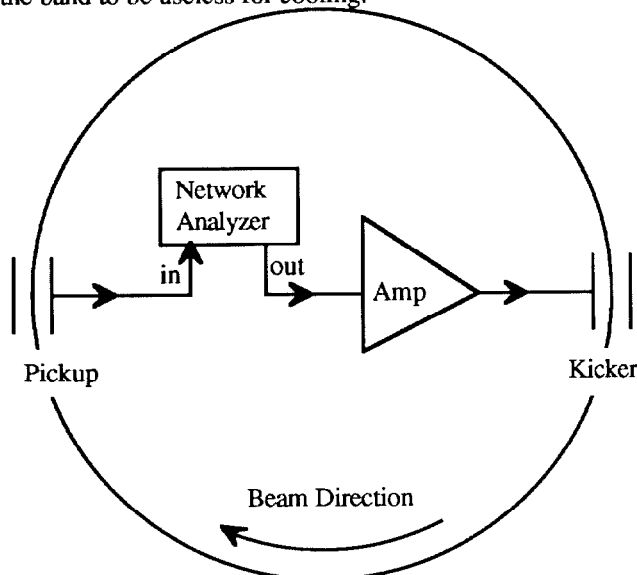


Fig. 1. Schematic of network analyzer measurement on a stochastic cooling system.

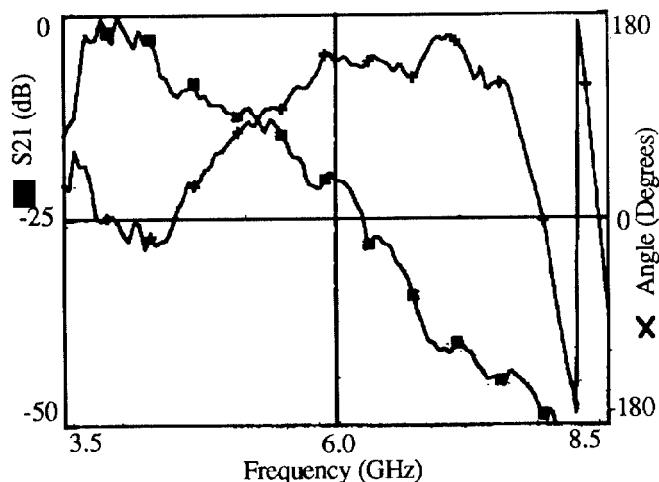


Fig. 2. Gain and phase response of the core horizontal stochastic cooling system without an equalizer.

The cooling rate for a betatron cooling system is [1]:

$$\frac{1}{\tau} = \frac{W}{M N_p} \left( 2 \operatorname{Re}[gM] - |gM|^2 \right) \quad (1)$$

where  $W$  is the bandwidth of the system,  $M$  is the mixing factor of the beam,  $N_p$  is the number of particles, and  $g$  is proportional to the electronic gain of the system and the number of particles. The product  $gM$  is proportional to the

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response measured by the network analyzer. The mixing factor is inversely proportional to frequency and can be written as:

$$M = \frac{M_c f_c}{f} \quad (2)$$

where  $f_c$  is the center frequency of the system and  $M_c$  is the mixing factor at the center frequency. Since the gain and the mixing factor are a function of frequency, the cooling rate of Eqn. 1 should be a sum over all the Schottky bands. Since the spacing between the Schottky bands is much smaller than the bandwidth of the system, this sum can be replaced by an integral:

$$W_{\text{eff}} = \int_{\text{bandwidth}} \left( 2\text{Re}[gM] - |gM|^2 \right) \frac{f df}{f_c} \quad (3)$$

where the cooling rate be rewritten as:

$$\frac{1}{\tau} = \frac{W_{\text{eff}}}{M_c N_c} \quad (4)$$

The kernel of the integral of Eqn. 3 can be called the bandwidth density of the cooling system. The goal of the equalizer design is to maximize this bandwidth density.

### III. EQUALIZER DESIGN

To maximize the bandwidth density, the equalizer should have high insertion loss at low frequencies and low loss at high frequencies. This response can be obtained by using the capacitive properties of electrically short coupled transmission lines. To obtain the proper gain slope a number of these are placed in series. To avoid resonances between other devices in the cooling system, the reflected power at low frequencies is terminated in a balanced arrangement of  $90^\circ$  hybrids. The mechanical tolerances of the  $90^\circ$  hybrid can be relaxed if the circuit is built in stripline.

The phase characteristic of the equalizer is shaped by Schiffman phase shifters [2]. These devices are all-pass tightly coupled transmission lines. The phase shape is determined by the length of the coupled lines. The magnitude of the phase excursion can be increased by adding a number of phase shifters in series. A schematic of the circuit is shown in Fig. 3.

### IV. OPTIMIZATION OF EQUALIZER

The circuit was designed using a microwave CAD program called TOUCHSTONE™. To maximize the bandwidth density, the frequency domain response of the cooling system shown in Fig. 2 is multiplied by the equalizer response. The bandwidth density can be defined by using the OUTVAR and OUTEQN block. In the OUTVAR block,  $gM$  is defined as the product of the network analyzer response shown in Fig. 2 times the S21 response of the equalizer. The OUTEQN block then computes the square root of the kernel of the integral in Eqn. 3 at each frequency. Because the optimizer uses a least squares formulation and the square root of the bandwidth density is the optimized variable, the error function will be equal to the

effective bandwidth ( $W_{\text{eff}}$ ). A random maximizer is used to maximize the error function.

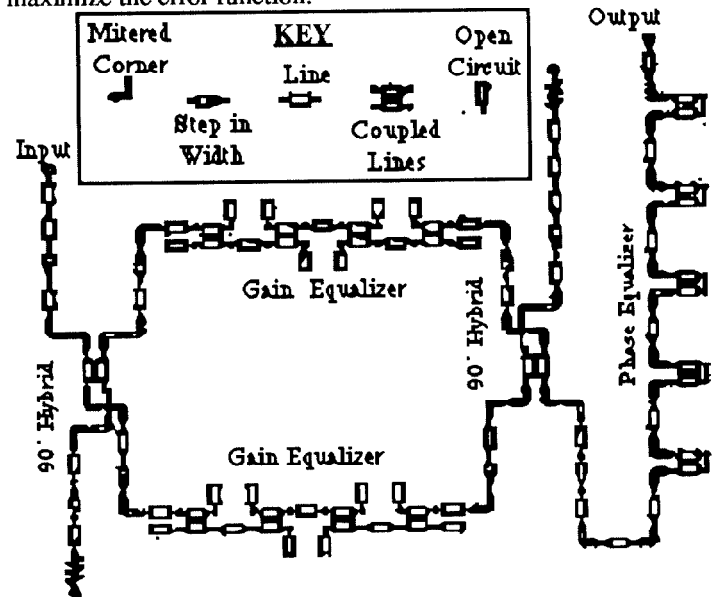


Fig. 3. Schematic of equalizer.

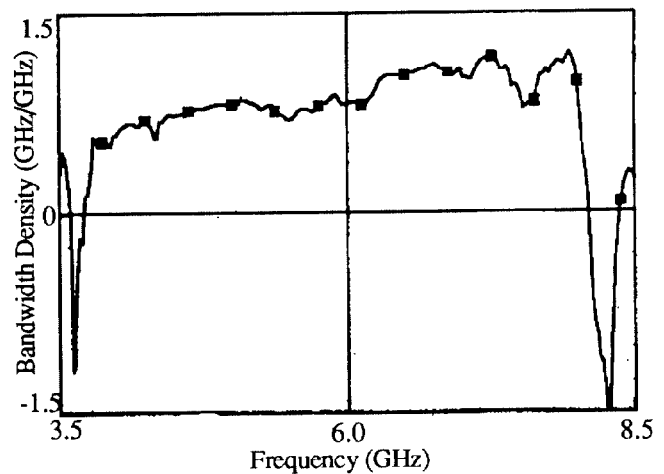


Fig. 4. Bandwidth density of cooling system with an equalizer.

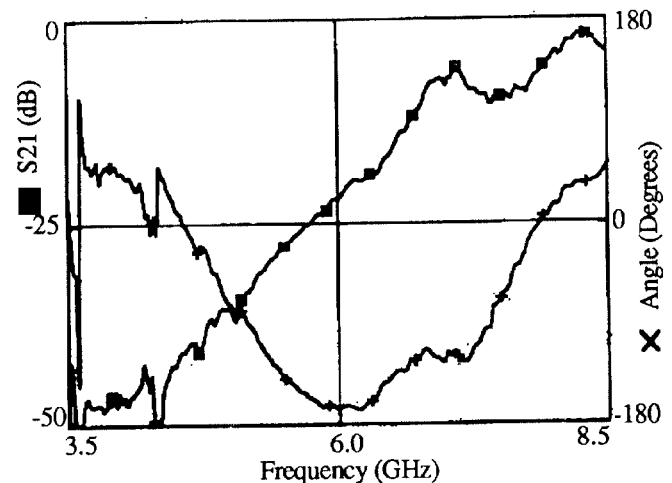


Fig. 5. Measured transfer response of the equalizer.

The following circuit parameters were varied during the optimization:

- 1.) The electrical lengths of series coupled lines.
- 2.) The impedances of the series coupled lines
- 3.) The spacing between the series coupled lines.
- 4.) The electrical lengths of the phase shifters.
- 5.) An arbitrary gain, delay, and phase offset.

An optimized bandwidth density is shown in Fig. 4. The measured response of the equalizer is shown in Fig. 5. The product of the beam response and the equalizer response is shown in Fig. 6. The effective bandwidth for this combination is 3.720 GHz.

The equalizers were installed into the core betatron cooling systems. The gain and phase response of the cooling systems is very similar to the response shown in Fig. 6 with the exception of a phase offset. The phase differences between the measurement and the response showed in Fig. 6 reduced actual cooling bandwidth to 3.3 GHz. This situation can be corrected by flipping the polarity of a 90° hybrid in the trunk line of the cooling systems.

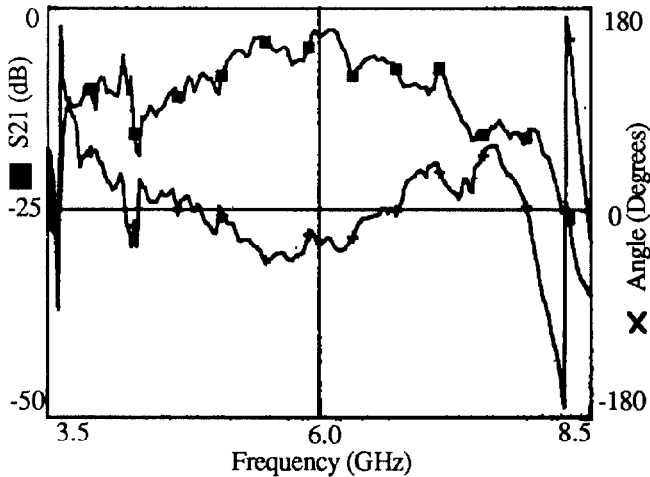


Fig. 6. Gain and phase response of the core horizontal stochastic cooling system with the equalizer.

## V. CONCLUSIONS

Using the formulation of an effective bandwidth density, an equalizer can be designed for stochastic cooling systems that does not rely on the arbitrary judgements of the designer as to the gain and phase flatness across the band.

## VI. ACKNOWLEDGEMENTS

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