COUPLED BUNCH DIPOLE MODE MEASUREMENTS OF ACCELERATING BEAM IN THE FERMILAB BOOSTER

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Abstract--The dipole oscillation of a coupled bunch mode is measured using a fast single shot digitizing oscilloscope. The phase centroid of the bunches is determined from the digitized waveform. The phase error of the bunches as a function of bunch number is then Fourier analyzed which provides the coupled bunch mode spectrum. The resulting spectrum is used to determine the evolution of coupled bunch mode instabilities in the Fermilab Booster. This method can determine the coupled bunch mode spectrum in a single turn in contrast to frequency domain methods which take at least a full synchrotron period to complete the measurement.

I. INTRODUCTION

The Fermilab Booster accelerates protons from 200 MeV to 8 GeV in 33.3 milliseconds. Coupled bunch oscillations occur when the intensity of the Booster reaches 1×10^{12} protons. While these oscillations are not a problem under the present operating conditions, the intensity in the Booster after the Fermilab Linac upgrade is completed is expected to be 5×10^{12} protons. With these intensities, the coherent motion of the bunches within the RF buckets causes a dilution of the longitudinal emittance. The dilution occurs primarily because of the unpredictable position of the bunches relative to the RF voltage when the beam is transfered from the Booster to the Main Ring. Presently, there is an effort to build longitudinal dampers that reduce the coupled bunch mode oscillations[1].

The Booster RF frequency swings from 30 MHz to 53 Mhz. The RF frequency slew rate has a maximum of 3 MHz/mS at the beginning of the acceleration cycle and is about 200 kHz/mS midway through the cycle. The synchrotron frequency ranges from 30 kHz at injection to 2 kHz at extraction. Bunch oscillations can be detected by examining the phase modulation (PM) sidebands around revolution lines of the longitudinal frequency spectrum [2]. The spacing between the PM sidebands is equal to the synchrotron frequency. Because of the large RF frequency slew rate, the 2 kHz resolution needed to resolve the PM sidebands is impossible to obtain with a conventional spectrum analyzer. With this lack of resolution in spectrum analyzer data, it is not possible to distinguish between unequal bunch population phenomena and coupled bunch modes.

This dilemma can be resolved by mixing the longitudinal signal with a harmonic of the revolution frequency which is derived from the RF. The mixed signal is then fed into a spectrum analyzer were the resolution can be increased so that the PM sidebands can be detected. The disadvantage to this approach is that a mixing circuit has to be built for each coupled bunch mode to detect all the coupled bunch modes with a single shot of beam. Also, because the signal is Fourier analyzed, the time at when a coupled bunch mode appeared in the acceleration cycle is uncertain to within the inverse of the resolution bandwidth of the spectrum analyzer.

II. THEORY OF MEASUREMENT

According to the theory of Sacherer [3], the distribution function for a single bunch in the presence of collective forces can be written as:

$$g(r,\theta) = g_0(r) + \sum_{m=1}^{\infty} g_m(r) \cos(m\theta) \qquad (1)$$

where g is the distribution including the instability and go is the unperturbed distribution. The variables r and θ are the radius and angle in the longitudinal phase space that have been normalized to make the unperturbed trajectories circular. The sum is taken over the unstable modes: m=1 for dipole, m=2 for quadrapole, etc. The distribution functions of the other bunches are identical to Eqn. 1 except that unstable modes a multiplied by a phase that advances by $2\pi n/K$ per bunch where K is the number of bunches. The complete distribution for the K bunches contains a double sum over m (mode type) and n (azimuthal mode number) with independent coefficients for each (m,n) pair:

$$g(r,\theta,k) = g_0(r) + \sum_{n=0}^{K-1} \sum_{m=1}^{\infty} B_n g_m(r)\, e^{j2\pi nk/K} \cos(m\theta) \qquad (2)$$

where k is the bunch index. If the left hand side of Eqn. 2 is measured, the dipole instability term can be isolated by multiplying both sides of the equation by $\phi = r\cos(\theta)$ and integrating over r and θ. The resulting equation:

$$<\phi_k> = \pi \int r^2 g_1(r)\, dr \sum_{n=0}^{K-1} B_n e^{j2\pi nk/K} \qquad (3)$$

tells us that the average phase error is proportional to the strength of the dipole perturbation. While the isolation of the dipole term is exact within Sacher's formulation, the experimental technique is not sufficiently precise to guarantee that there is no admixture of higher modes. We have neglected changes the bucket area that occur during acceleration; these changes are relatively small during the portion of the cycle when the instability is observed.

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Given the average values $<\phi_k>$ for the K bunches in the machine, one can use discrete Fourier analysis to determine the relative strengths B_n of the various azimuthal modes:

$$B_n = \frac{\sum_{k=0}^{K-1} <\phi_k> e^{-j2\pi nk/K}}{K\pi \int r^2 g_1(r)\, dr} \qquad (4)$$

III. MEASUREMENT ALGORITHM

Step 1 The instantaneous beam current as a function of time is obtained for a single turn in the acceleration cycle. This is done by placing a detector (a resistive wall monitor) at a fixed location in the ring and digitizing the wall current as a function of time for one turn with an high speed oscilloscope. The oscilloscope used in this paper was the Tektronix DSA 602 which has a single shot digitizing rate of 2 gigasamples/second. Since the average length of a single turn in the Fermilab Booster is about 2 υS, a record length of 4096 points is needed (the DSA 602 has a maximum record length of 32,768.).

Step 2 The approximate location of the bunch centers is determined. This can be done by searching for the first bunch and then finding the rest of the centers aided with the knowledge of the RF frequency for the turn when the data was taken.

Step 3 For each bunch, the centroid is computed according to Eqn. 3. This is done by multiplying the bunch profile by time and integrating. The centroid is normalized by dividing by the total charge in the bunch. In this way variations due to unequal bunch populations are eliminated.

Step 4 The phase advance of the bunches due to the RF frequency and slew rate is found and removed from the data. The centroid spacing between adjacent bunches will be a combination of the coupled bunch mode oscillations and the phase due to the RF frequency. The phase differences due to the RF can be found by making a least squares fit to the the phase centroids according to the following form:

$$<\phi_k>_{fit} = \phi_0 + \phi_1 k + \phi_2 k^2 \qquad (5)$$

The coefficient ϕ_0 is due to when the measurement was taken with respect to the RF. The coefficient ϕ_1 is due to the linear increase of phase due to a fixed RF frequency. The coefficient ϕ_2 is due to the slewing of the RF frequency with respect to time. Higher order corrections are small enough to be neglected. Once the coefficients ϕ_0, ϕ_1, ϕ_2 are obtained, they are subtracted from the actual phase centroid data to obtain the phase error due to the coupled bunch motion. The coupled bunch phase error for a typical turn in the Booster is shown in Fig. 1.

Step 5 Fourier analysis according to Eqn. 4 is performed to provide the coupled bunch mode coefficients B_n. From Eqn. 3 it can be shown that the number of phase oscillations for mode n is the same as the number of oscillations for mode K-n. The coupled bunch mode content of the wavefrom shown in Fig. 1 is displayed in Fig. 2.

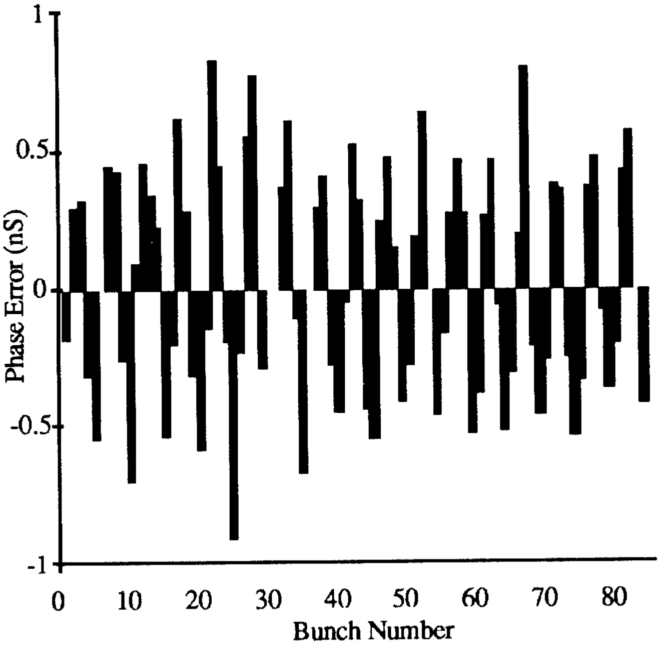

Fig. 1 Phase error of each bunch for 32 mS into the acceleration cycle. Intensity = $1\text{x}10^{12}$ protons.

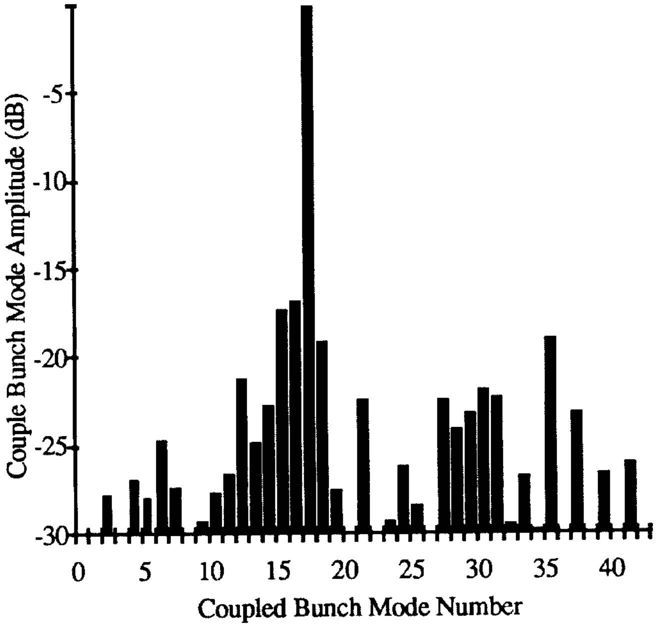

Fig. 2 The coupled bunch mode content of Fig. 1.

Step 6 The coupled bunch mode spectrum is obtained as a function of time in the cycle. This is done by repeating Steps 1-5 for different turns throughout the acceleration cycle. However, to record the entire acceleration cycle with a digital oscilloscope would require a record length of 66 million points. Since such a record length is not possible, a new batch of beam can be injected for each measurement. This will yield satisfactory results if the operating conditions of the booster remains stable during the measurements. Experimentally this has been found to be the case. Figures 3 and 4 show the coupled bunch mode spectrum as a function of time for two different beam intensities.

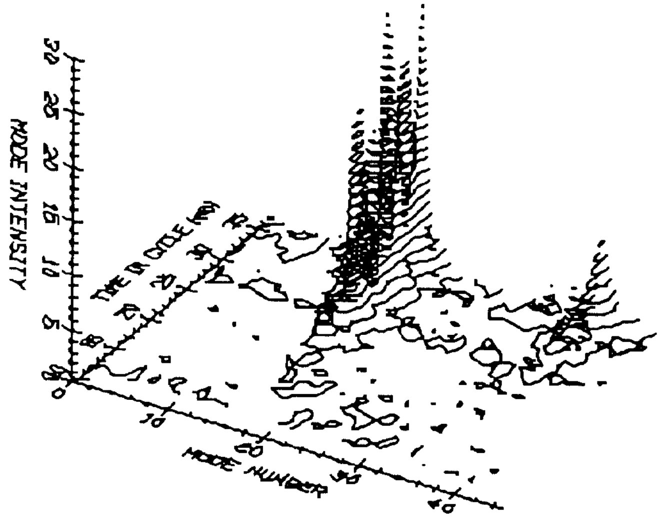

Fig. 3 Coupled bunch mode spectrum as a function of time for a beam intensity of $1.2x10^{12}$ protons. The mode intensity is units of nSx84

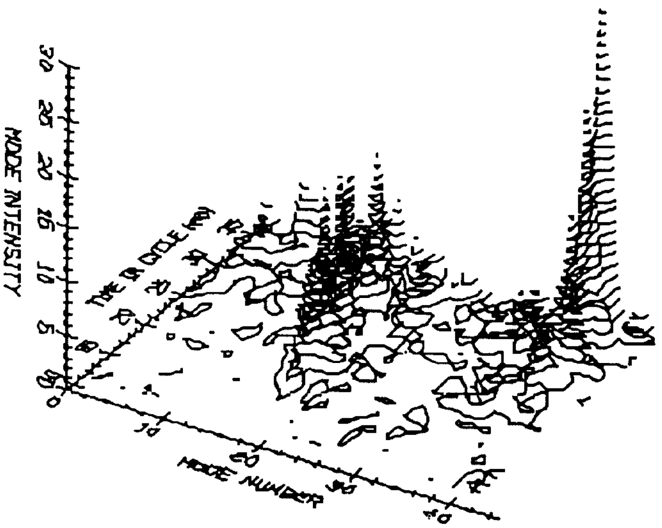

Fig. 4 Coupled bunch mode spectrum as a function of time for a beam intensity of $2.5x10^{12}$ protons. The mode intensity is units of nSx84

Once the mountain range display shown in Figs. 3 and 4 is obtained, the evolution of a particular mode as a function of time may be analyzed. Figure 5 shows the evolution of mode 17 for a beam intensity of $1.2x10^{12}$ protons. The data can be fitted to an e-folding rate of 5.5 mS. However, this number may not reflect the maximum growth rate due to nonlinearities during the growth process.

IV. SUMMARY

Using a fast single shot digitizing oscilloscope, one can determine the coupled bunch mode profile in a single turn. This has an advantage over frequency domain techniques which require at least a synchrotron period to determine the profile. However, one disadvantage to the time domain method is the higher order multipole contributions (quadrupole, sextapole,...) can not be completely resolved because of the limited digitizing rate of the oscilloscope.

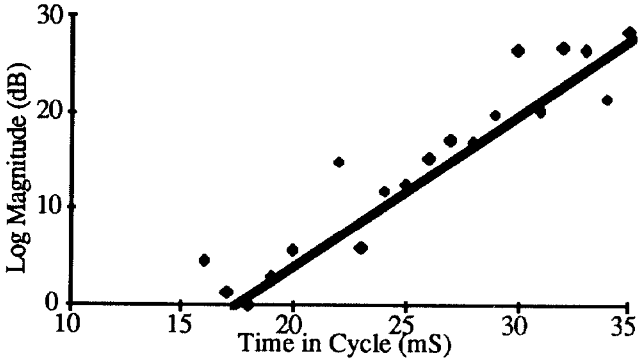

Fig. 5 Intensity of mode 17 versus time in the acceleration cycle for 1.2x1012 protons.

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