Bunch Length Measurement using Beam Spectrum *

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Abstract

A procedure for extracting the bunch length from the beam induced voltage spectrum is applied at CESR, for various bunch currents, to indicate whether the turbulent lengthening threshold is exceeded by short, high-current bunches. The approach is to monitor the signal spectrum from one of the detector buttons up to 8 GHz. The measuring system is calibrated by reference to a low-current bunch (1 mA), which is assumed to have a gaussian profile with the theoretically predicted length. In this experiment a single bunch was used with a natural length of 15 mm. No bunch lengthening was detected up to a current of 20 mA.

Introduction

Change of bunch length with current, indicating the turbulent lengthening threshold for short, high-current bunches was searched for in CESR. The approach, using readily available equipment, was to monitor the signal spectrum from one of the beam position detector buttons. To press the limit, the natural bunch length was reduced to $\sigma_t \cong 50 \ ps$ by using a lattice with $Q_h \cong 13.2(\alpha_p = 0.0069)$. To calibrate the measuring system we assume that at low current (1 mA) the bunch had a gaussian profile with the theoretically predicted σ_t .

Characteristics of Observed Spectrum

In the absence of coherent beam oscillations, the signal spectrum from a pickup electrode consists entirely of harmonics of the revolution frequency. The envelope of this spectrum is determined by the bunch length. The spectrum is distorted by the transfer function, $F(\omega)$, of the pickup, cable, and any added filters. As indicated below, this distortion is a major effect. However, as long as $F(\omega)$ remains constant, *i.e.*, does not itself depend in any way on bunch current, any <u>changes</u> in the observed spectrum can be attributed to changes in the bunch length. The pickup electrode receives a signal induced by the electric field of the passing bunch. If the electrode has negligible length,



Figure 1: Normalized Gaussian Power Spectrum

the signal from a gaussian bunch, with charge profile characterized by $\sigma_t = \sigma_s/c$, has a gaussian spectrum envelope with $\sigma_{\omega} = 1/\sigma_t$:

$$V(\boldsymbol{\omega}) = V_o e^{\frac{-\omega^2}{2\sigma_\omega^2}} = V_o e^{-2\pi^2 \sigma_t^2 f^2}$$
(1)

The corresponding power spectrum, expressed logarithmically, is

$$P(\omega) = 20 \log_{10} V(\omega) = 20 \log_{10} V_o - 171.4 \sigma_t^2 f^2 [dB]$$
 (2)

The normalized spectrum of equation 2 is plotted in Figure 1 for $\sigma_t = 50 \ ps$ and, for comparison, also for 55 ps, which might be considered a significant fractional bunch lengthening.

As mentioned above, this spectrum is modified by the transfer function, $F(\omega)$ of the monitoring system. The spectrum envelope observed is quite different from that of Figure 1, for the following reasons:

- a. If the length of the pickup button is not short compared to σ_s , the charge induced on it varies as the convolution of the button's geometry with the charge profile of the bunch.
- b. If the RC time constant of the button is short compared to σ_l , the button signal is given by

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the current, *i.e.*, by the time derivative of the induced charge. This introduces a factor ω into $F(\omega)$, de-emphasizing the low-frequency components. In practice $RC \simeq 30\Omega \cdot 2pF = 60 \ ps$, comparable to σ_t . Thus the actual shape of the low-frequency rolloff is more complicated.

- c. The button structure is not properly matched, and the feedthrough insulator has frequency-dependent losses. The high frequency response is extremely uneven: it dips sharply above about 3 GHz, but fortunately shows some 'windows' of usable transmission around 7-8 GHz.
- d. For convenience, the signal is brought all the way into the control room. A 26 m long cable has its own high frequency cutoff.
- e. Because the signal at the end of the cable can be as large as 100 V peak, it must be attenuated. This was done with a 4 GHz high-pass filter.

Including the effect of the total transfer function $F(\omega)$ on the measured signal, the power spectrum is:

$$P(\omega) = 20 \log_{10} F(\omega) + 20 \log_{10} V_o - 171.4 \sigma_I^2 f^2 [dB]$$
 (3)

where σ_I is the bunch length at current I. (The bunch shape is assumed to remain gaussian.)



Figure 2: Digitized power spectrum at 12.6 mA in a window of 10 MHz around 6.55 GHz

Measurement Procedure

The measurement consists of recording parts of the spectrum with a low-current bunch, and then repeating the readings at the same frequencies with bunches of progressively larger current. It is clear from Figure 1 that bunch lengthening is most readily detected by covering the spectrum to the highest accessible frequency. The procedure, which eliminates the dependence of the signal on V_o , is to



Figure 3: Values of σ_I at 12.7 ma obtained at the different frequencies of the power spectrum

monitor the ratio between the spectrum envelope at some selected frequency, f_n , and at a reference frequency, f_1 . If P_n and P_1 are the power spectrum measured at frequencies f_n and f_1 , then:

$$(P_n - P_1)_I = 20 \log_{10} F_{n+} - 171.4 (f_n^2 - f_1^2) \sigma_I^2 [dB]$$
 (4)

where

$$F_{n1} = \frac{F(\omega_n)}{F(\omega_1)} \tag{5}$$

At low current, we assume that the bunch has the theoretical bunch length,

$$\sigma_o = \frac{\alpha_p \cdot \sigma_E}{\Omega_s \cdot E_o} \tag{6}$$

where $\alpha_p, \sigma_E, \Omega_s$ and E_o are the momentum compaction, energy spread, synchrotron frequency and electron energy, respectively. The low-current measurement then yields

$$20\log_{10}F_{n\,1}=(P_n-P_1)_o+171.4(f_n^2-f_1^2)\sigma_o^2$$
 (7)

and

$$\sigma_I = \left(\frac{(P_n - P_1)_o - (P_n - P_1)_I}{171.4 \left(f_n^2 - f_1^2\right)} + \sigma_o^2\right)^{1/2} \qquad (8)$$

where the index c_o is used to denote low current. The error in σ_I due to a measurement error in the spectrum is:

$$\Delta \sigma_I = \frac{\Delta \left[(P_n - P_1)_o - (P_n - P_1)_I \right]}{342.8 \left(f_n^2 - f_1^2 \right) \sigma_o} \tag{9}$$

Note that, the larger the distance between f_n and the reference frequency f_1 , the smaller the error in σ_I .

At the time of the experiment, CESR was operating with a single RF cavity. The data were taken with a lattice of high horizontal tune, $Q_h = 13.2, \alpha_p = 0.0069, \sigma_o =$ 50psec, with a single bunch of positrons. The output of the spectrum analyzer was connected to a digitizer and the



Figure 4: σ_t for different bunch current

data were recorded by a computer. A sample spectrum is shown in Figure 2.

The spectrum was measured at a few different frequencies f_n for each current. For each measurement the power spectrum peaks (see Figure 2) were averaged in a 10 MHz ($\simeq 25$ revolution harmonics) window centered on f_n . The reference frequency f_1 was taken at 8 GHz. (For more details about the measurement, see reference [1]). Figure 3 shows values of σ_I at 12.7 mA calculated from equation 8 at the different f_n with the error calculated from equation 9 for $\Delta P = 1 \ dB$. The straight line is a least-square fit to the data. Figure 4 shows the bunch length measured in CESR at three currents using the above procedure.

Conclusions

No significant bunch lengthening in CESR was observed up to a bunch current of 20 mA, as seen in Figure 4. This observation agrees with a previous result, obtained in 1983 by detecting the synchrotron light, before the installation of horizontal separators in CESR [2]. Both experiments assume that the bunch shape is gaussian and are thus unable to detect shape changes involving a particular mode [3].

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