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# BEAM EMITTANCE MEASUREMENT TECHNIQUE FOR PLS-IM-T LINAC*

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*Abstract*

The PLS 60 MeV injector (PLS-IM-T) for the 2-GeV linac is expected to be commissioned by the end of 1991. A simple technique has been developed for the beam emittance measurement, which employs the bending magnet and the quardrupole magnets in the beam transport system. By varying the quadrupole strength, one can simultaneously determine the horisontal and vertical emittances, and the momentum spread of the beam.

## I. Introduction

The PLS-IM-T, an injector of the PLS 2.0 Gev linac, will be commissioned by the end of 1991. In the PLS-IM-T, there will be no scintillation target in the beam line which can be used for emittance measurement. Thus, the measurement of the beam emittance will be performed at the beam energy analyzing station, differently from the ordinary method . The layout is illustrated in Fig. 1.

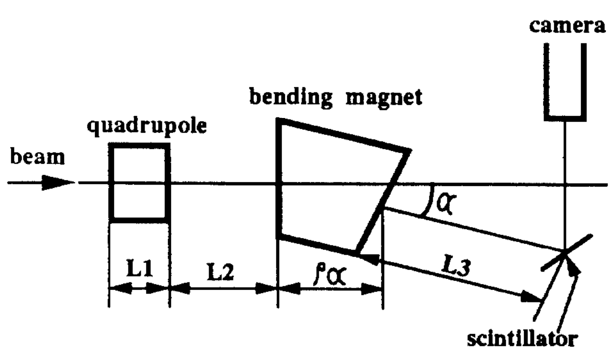


Fig.1 Layout of the beam analyzing station

*This project is supported by POSCO and MOST, KOREA.
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## II. Theory

It is taken that $z$ is the direction of the beam transport, $x$ and $y$ are the horisontal and vertical coordinates respectively. It is also assumed that the quadruple is focusing in the $x$ direction and defocusing in the $y$ direction, and the bending magnet will bend the beam in the $x$ direction. The beam matrix at the entrance of the quadruple is defined as $\sigma_o$, which is discribed by the super-ellipsoid equation as follows:

$$X^T \sigma_o^{-1} X = 1, \tag{1}$$

where

$$X = \begin{pmatrix} x \\ x' \\ y \\ y' \\ \delta \\ \frac{\Delta p}{p} \end{pmatrix}.$$

$x' = dx/dz$, $y' = dy/dz$, $\delta$ is the half width of the beam, and $\sigma_o$ is symmetric, *i.e.*

$$\sigma_{oji}, \quad i, j = 1, 2, \ldots, 6.$$

Then, the beam matrix will satisfy the following fomula at the scintillator in the beam analyzing station.

$$\sigma = R\sigma_o R^T, \tag{2}$$

where $R$ is the total transfer matrix from the quadruple to the scintillator.

Expression for $R$ is:

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$$
R = \begin{pmatrix} R_{11} & R_{12} & R_{13} & R_{14} & R_{15} & R_{16} \\ R_{21} & R_{22} & R_{23} & R_{24} & R_{25} & R_{26} \\ R_{31} & R_{32} & R_{33} & R_{34} & R_{35} & R_{36} \\ R_{41} & R_{42} & R_{43} & R_{44} & R_{45} & R_{46} \\ R_{51} & R_{52} & R_{53} & R_{54} & R_{55} & R_{56} \\ R_{61} & R_{62} & R_{63} & R_{64} & R_{65} & R_{66} \end{pmatrix} \tag{3}
$$

$$
= \begin{pmatrix} 1 & L_3 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & L_3 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & \frac{L_3}{4w^2} \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} \cos\alpha & \rho\sin\alpha & 0 & 0 & 0 & \rho(1-\cos\alpha) \\ -\rho\sin\alpha & \cos\alpha & 0 & 0 & 0 & \sin\alpha \\ 0 & 0 & 1 & \rho\alpha & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ -\sin\alpha & -\rho(1-\cos\alpha) & 0 & 0 & 1 & \rho(\sin\alpha-\alpha) \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \cdot
$$

$$
\begin{pmatrix} 1 & L_2 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & L_2 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & \frac{L_2}{4w^2} \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} \cos(kL_1) & \frac{1}{k}\sin(kL_1) & 0 & 0 & 0 & 0 \\ -k\sin(kL_1) & \cos(kL_1) & 0 & 0 & 1 & \rho(\sin\alpha-\alpha) \\ 0 & 0 & \cosh(kL_1) & \frac{1}{k}\sinh(kL_1) & 0 & 0 \\ 0 & 0 & k\sinh(kL_1) & \cosh(kL_1) & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & \frac{L_1}{4w^2} \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}.
$$

At the scintillator, one can get a graph as shown in Fig. 2, from which it is easy to get the HWHM of the beam, $\Delta x$ and $\Delta y$. As well known, the HWHM is related to the beam matrix by $\Delta x = \sqrt{\sigma_{11}}$ and $\Delta y = \sqrt{\sigma_{33}}$ . If the quadruple strength is changed four times, one can get four graphs and therefore four different $\Delta x$ and $\Delta y$.

From (2), it can be derived that

$$
\begin{aligned}
\sigma_{11} &= R_{11}^2\sigma_{o11} + 2R_{11}R_{12}\sigma_{o12} + R_{12}^2\sigma_{o22} \\
&\quad + R_{16}^2\sigma_{o66}, && (4) \\
\sigma_{33} &= R_{33}^2\sigma_{o33} + 2R_{33}R_{34}\sigma_{o34} + R_{34}^2\sigma_{o44}, && (5) \\
\sigma_{66} &= \sigma_{o66}. && (6)
\end{aligned}
$$

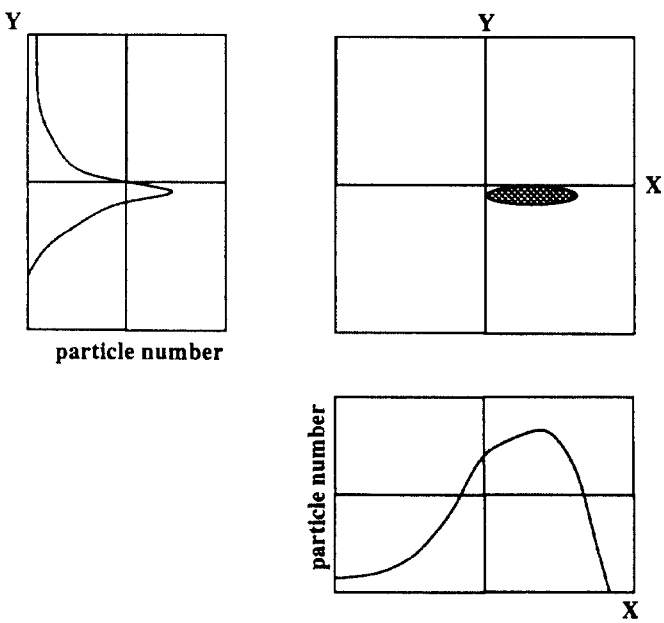


Fig.2 beam profile at the scintillator

From (3), one can get

$$
\begin{aligned}
R_{11} &= \cos(kL_1)(\cos\alpha - \frac{L_3}{\rho}\sin\alpha) - k\sin(kL_1) \times \\
&\quad \left[(L_2+L_3)\cos\alpha - \frac{L_2L_3}{\rho}\sin\alpha + \rho\sin\alpha\right], && (7) \\
R_{12} &= \frac{1}{k}\cos(kL_1)(\cos\alpha - \frac{L_3}{\rho}\sin\alpha) + \cos(kL_1) \times \\
&\quad \left[(L_2+L_3)\cos\alpha - \frac{L_2L_3}{\rho}\sin\alpha + \rho\sin\alpha\right], && (8) \\
R_{16} &= \rho(1-\cos\alpha) + L_3\sin\alpha, && (9) \\
R_{33} &= \cosh(kL_1) + k\sinh(kL_1)(L_2+L_3+\rho\alpha), && (10) \\
R_{34} &= \frac{1}{k}\sinh(kL_1) + \cosh(kL_1)(L_2+L_3+\rho\alpha). && (11)
\end{aligned}
$$

# III. Beam emittance measuring method for PLS-IM-T

Values of Eqs. (7) through (11) will be changed as the quadruple strength varies. In the case of PLS-IM-T, $L_1 = 0.2\,m$, $L_2 = 1.14\,m$, $L_3 = 1.36\,m$, $\rho = 0.52\,m$ $\alpha = 0.418\,rad$, and $k = 24.245\sqrt{G/2w}$; $w = 60$ MeV, where $G$ is the field gradient of the quadruple.

Substituting four $\Delta x$'s into (4) and solving the equation set, one can get $\sigma_{o11}$, $\sigma_{o12}$, $\sigma_{o22}$, $\sigma_{o66}$, and thus the horizontal emittance

$$
e_x = \sqrt{\sigma_{o11}\sigma_{o22} - \sigma_{o12}^2} \quad (\pi m.rad), \tag{12}
$$

and the beam momentum spread

$$\Delta p/p = \pm\sqrt{\sigma_{066}}. \qquad (13)$$

Using the same procedure for Eq. (5), it is possible to get the vertical emittance from the three $\Delta y$ values.

Ordinarily, measuring the beam momentum spread by means of measuring $\Delta x$ of the beam at the analyzing station always includes the contribution of either the beam dimension or divergence at the focusing point. However, it is noticeable that the Eq. (13) has an expression of a pure beam momentum spread.

## References

[1] K. L. Brown, et al., Transport. SLAC-91, Rev. 2, UC-28(1/A).