Beam Detector Impedance Calculation Using Circuit Models

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Abstract

Expressions for the transfer impedance and longitudinal impedance of arbitrarily terminated stripline detectors are developed using a circuit model. Attenuation effects and distributed element effects are considered without requiring a field solution. A rectangular button is treated as a center-tapped transmission line.

Introduction

The Supercollider facility requires position measurement at some 2400 locations, in six different accelerators, beginning at the 35 keV ion source and as beam progresses to the 20 Tev Collider rings. Given that SSC is a green site, that measurement requirements, impedance budgets, signal power, and mechanical envelopes are being determined and vary with each machine, a study of detector responses appeared useful. In the circular machines the bunch interval is 5 m and FWHM bunch width is 14 cm. The short wavelength content of the bunch current is weak. To couple sufficiently at 60 MHz, a reasonable electrode area is required. A properly designed stripline detector can be long enough to produce strong signals while exhibiting a tolerable beam impedance. This discussion concerns the frequency domain analysis of strip transmission lines, arbitrarily terminated, and in one case, center terminated. This familiar beam detector geometry [1,2] is shown in Figure 1.

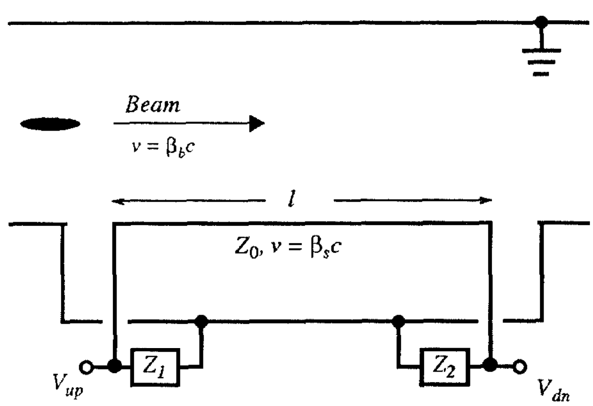

Figure 1. Longitudinal view of stripline beam detector

A stripline may be considered as a section of transmission line of length l which is exposed to the beam's electromagnetic fields. Transverse physical dimensions determine the impedance Z_0. The strip is terminated in an impedance Z_1 at its upstream end, and an impedance Z_2 downstream. The analysis does not restrict Z_1 or Z_2 to be purely real. Signals between the strip and ground conductor travel at the velocity $\beta_s c$. The beam velocity through the detector is $\beta_b c$, where c is the velocity of light in vacuum. The fraction of the total beam current induced on the electrode is g. This analysis considers detectors having only one electrode. Transverse impedances are not calculated.

TEM Circuit Model

Relativistic charged particle beams impose quasi-TEM fields in the metallic structures that enclose them. These fields couple to the stripline, and by a hand-waving argument and application of Lorentz reciprocity [3], the coupling occurs at the ends of the strip. This simplification, and taking the beam as a current source as shown in Figure 2, allows construction of an equivalent circuit. The sources are in phasor form, with the downstream source delayed by the beam flight time $l/\beta_b c$. Since the model contains lumped and distributed elements, it is useful to replace the line by a lumped element T-network [4] of admittances Y_A and Y_B, which is equivalent when $Y_A = Y_0(\coth\gamma_s l + \operatorname{csch}\gamma_s l)^{-1}$ and $Y_B = -Y_0 \sinh\gamma_s l$. The propagation constant for signals on the electrode is given by $\gamma_s = \alpha + i[\omega/(\beta_s c)]$, in which α is the loss term. Converting Z_1 and Z_2 to admittances, the circuit may then be solved using node equations.

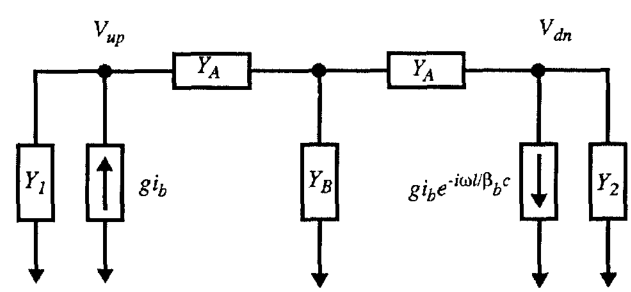

Figure 2. Equivalent circuit for stripline beam detector.

Using $V_{up} = i_b Z_{up}$ and $V_{dn} = i_b Z_{dn}$, the complex transfer impedances expressed by Eqns. 1 and 3 give the relationships between the beam current and the terminal voltages across the loads, Z_1 and Z_2, respectively.

$$Z_{up}(\omega) = g\frac{Z_0}{2}(1+\Gamma_1)\frac{1-(1+\Gamma_2)e^{-\gamma_s l}e^{-\gamma_b l}+\Gamma_2 e^{-2\gamma_s l}}{1-\Gamma_1\Gamma_2 e^{-2\gamma_s l}} \qquad (1)$$

By symmetry, $\gamma_b \equiv 0 + i[\omega/(\beta_b c)]$ may be defined as the beam propagation constant. The reflection coefficients

$$\Gamma_1 = \frac{Z_1 - Z_0}{Z_1 + Z_0} \quad \text{and} \quad \Gamma_2 = \frac{Z_2 - Z_0}{Z_2 + Z_0} \qquad (2)$$

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$$Z_{dn}(\omega) = g\frac{Z_0}{2}(1+\Gamma_2)\frac{(1+\Gamma_1 e^{-2\gamma_s l})e^{-\gamma_b l} - (1+\Gamma_1)e^{-\gamma_s l}}{1-\Gamma_1\Gamma_2 e^{-2\gamma_s l}} \quad (3)$$

are defined in the standard way. Note that it is possible to include loss effects on the electrode simply by using the hyperbolic functions in Y_A and Y_B.

Longitudinal Impedance

An important parameter for stripline sensors is the impedance the strip presents to the beam. In as much as the detector removes real power from the passing beam, that impedance cannot be zero. The impedance $Z_\parallel$ is calculated from the transfer impedances by evaluating the ratio of terminal voltages at the upstream and downstream ports, divided by beam current. The transit time phase lag of the bunch at the downstream port must be considered.

$$Z_\parallel = \frac{i_b Z_{up}}{i_b} - \frac{i_b Z_{dn}}{i_b e^{-\gamma_b l}} = Z_{up} - Z_{dn}e^{\gamma_b l} \quad (4)$$

Evaluating Eqn. 4 for a lossless ($\alpha = 0$) electrode, and allowing $\beta_b = \beta_s$ gives

$$Z_\parallel(\omega) = g^2\frac{Z_0}{2}\frac{(\Gamma_1\Gamma_2 + 2\Gamma_1 + 1)(1 - e^{-i2\theta_b})}{1-\Gamma_1\Gamma_2 e^{-i2\theta_b}} \quad (5)$$

$$\text{where } \theta_b = \frac{\omega l}{\beta_b c} \text{ and later } \theta_s = \frac{\omega l}{\beta_s c}. \quad (6)$$

The factor g^2 appears because only the fraction g of the image current sees the potential $i_b Z_\parallel$ at the discontinuities [5]. When $\Gamma_1 = 0$, Eqn. 5 reduces to the well known expression derived by Shafer [6].

Center-Tapped Stripline

A strip identical to the previous case might be loaded, not at each end, but at its center with a resistance Z_L. The calculation proceeds considering two equal transmission lines, each of length $l/2$, and writing the admittance matrix. The transfer impedance magnitude is given by Eqn. 7.

$$|Z_T(\omega)| = \frac{g2Z_L \sin\frac{\theta_b}{2}}{\sqrt{\cos^2\frac{\theta_s}{2} + \left(\frac{2Z_L}{Z_0}\right)^2 \sin^2\frac{\theta_s}{2}}} \quad (7)$$

The longitudinal impedance, when $\theta_s = \theta_b$ and $Z_L = Z_0$, is given by Eqn. 8.

$$Z_\parallel(\omega) = 4g^2 Z_0 \frac{2\sin^2\theta_b + i(1+\cos\theta_b)\sin\theta_b}{4\sin^2\theta_b + (1+\cos\theta_b)^2} \quad (8)$$

The quantity $i_b|Z_T(\omega)|$ is plotted against the response of a matched stripline detector using $Z_0 = 50\,\Omega$, $g = .125$, $\beta_s = \beta_b = 1$, $i_b = .192$ A, $l = 15$ cm. See Figure 3. The button's response peaks at twice the frequency of the stripline, but encloses more $\omega H(\omega)$ area. This "rectangular button" detector exhibits weak resonances whenever $\theta_s \neq \theta_b$, and stronger resonances whenever $Z_L \neq Z_0$, as would a stripline with the wrong upstream termination. Considering the resonances, the low signal obtained from a small button, and possibly more difficult mechanical implementation of a large button, use of button detectors is not foreseen at SSC.

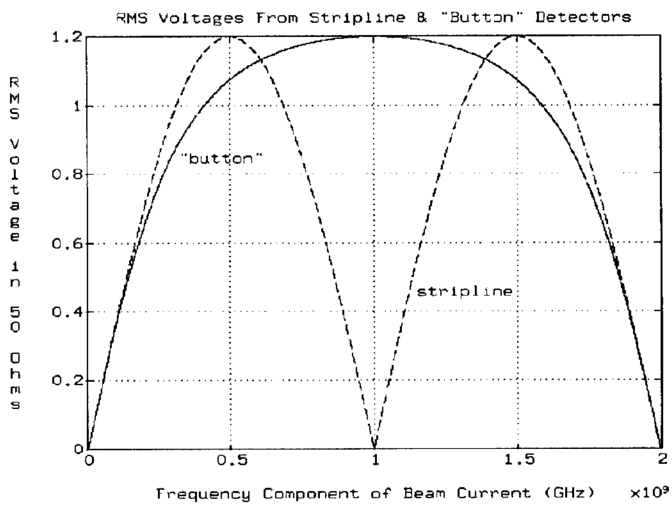

Figure 3. Frequency Response of Button and Stripline

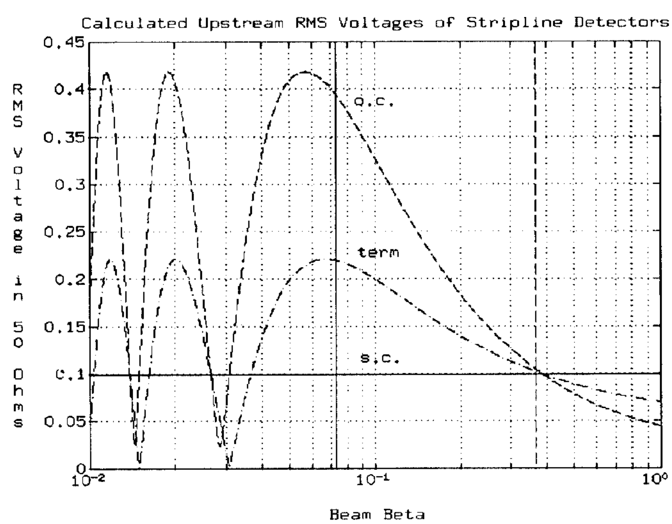

Figure 4. Stripline Response vs. Beam Velocity (β_b)

Open, Shorted, and Terminated Stripline

It is useful to consider the special real values for Γ_2, i.e. values of the downstream resistance Z_2. The cases of interest are the terminated, $\Gamma_2 = 0$, short-circuited, $\Gamma_2 = -1$, and open-circuited, $\Gamma_2 = +1$ loads. The equations describing the frequency responses of

Table 1. Frequency Response Characteristics of Stripline Beam Detectors

	Matched Line $\Gamma_2 = 0$	Open Circuit Line $\Gamma_2 = +1$	Short Circuit Line $\Gamma_2 = -1$
$\lvert Z_{up}\rvert$ $\beta_s \neq \beta_b$	$g2(Z_1 \parallel Z_0)\left\lvert\sin\frac{1}{2}(\theta_s+\theta_b)\right\rvert$	$gZ_1\left[\frac{\cos^2\theta_s - 2\cos\theta_s\cos\theta_b + 1}{\xi^2\sin^2\theta_s + \cos^2\theta_s}\right]^{1/2}$	$gZ_0\frac{\lvert\sin\theta_s\rvert}{\sqrt{\xi^{-2}\sin^2\theta_s + \cos^2\theta_s}}$
$\lvert Z_{up}\rvert$ $\beta_s = \beta_b$	$g2(Z_1 \parallel Z_0)\lvert\sin\theta_b\rvert$	$gZ_1\frac{\lvert\sin\theta_b\rvert}{\sqrt{\xi^2\sin^2\theta_b + \cos^2\theta_b}}$	$gZ_0\frac{\lvert\sin\theta_s\rvert}{\sqrt{\xi^{-2}\sin^2\theta_s + \cos^2\theta_s}}$
$\lvert Z_{dn}\rvert$ $\beta_s = \beta_b$	$gZ_0\Gamma_1\lvert\sin\theta_b\rvert$	$g(Z_1 - Z_0)\frac{\lvert\sin\theta_b\rvert}{\sqrt{\xi^2\sin^2\theta_b + \cos^2\theta_b}}$	0
$\angle Z_{up}$ $\beta_s = \beta_b$	$\frac{\pi}{2} - \theta_b$	$\tan^{-1}(\xi^{-1}\cot\theta_b)$	$\tan^{-1}(\xi\cot\theta_b)$
$\Delta\omega_{3dB}$ $\beta_s = \beta_b$	ω_0	$\frac{4}{\pi}\omega_0\tan^{-1}\xi$	$\frac{4}{\pi}\omega_0\tan^{-1}\xi^{-1}$
$\left\lvert\frac{V_{up}^{\Rightarrow}}{V_{up}^{\Leftarrow}}\right\rvert$	$\left\lvert\frac{\sin\frac{1}{2}(\theta_s+\theta_b)}{\sin\frac{1}{2}(\theta_s-\theta_b)}\right\rvert$	1	1

$\theta_s \equiv \frac{\omega l}{\beta_s c}$ $\quad\theta_b \equiv \frac{\omega l}{\beta_b c}$ $\quad\xi \equiv \frac{Z_1}{Z_0}$ $\quad\omega_0 \equiv \frac{\beta c\pi}{2l}$ $\quad\Gamma_1 = \frac{Z_1 - Z_0}{Z_1 + Z_0}$ $\quad\Gamma_2 = \frac{Z_2 - Z_0}{Z_2 + Z_0}$

these cases are summarized in Table 1. The peak response of all detectors occurs at $f_0 = \omega_0/(2\pi) = (\beta_b c)/(4l)$ if $\beta_s = \beta_b$. According to the model, the detector response recurs indefinitely in ω. In practice, a bandwidth limitation of the transition pieces between the strip and terminations, as well as propagation of the TE_{11} mode, reduces the detector bandwidth to a few cycles of the sine function.

When $\Gamma_1 = 0$, the three cases converge, the response being independent of Γ_2. The directivity of each detector is the ratio of the upstream voltage $V_{up}^{\Rightarrow}$ obtained when beam travels from port 1 to 2, to the voltage obtained at the upstream port with beam traveling from 2 to 1, $V_{up}^{\Leftarrow}$. Only the downstream terminated detector exhibits directivity, which depends upon the beam and signal propagation constants, and hence is frequency sensitive. When $0 < |\Gamma_2| < 1$, a detector having partial directivity is obtained.

Figure 4 shows the equations in row 1 of Table 1 applied to the SSC Linac. The frequency response as a function of beam velocity, using the values $\beta_s = .386$ (alumina loading), $Z_0 = Z_1 = 50\,\Omega$, $g = .125$, $i_b = .025$ A, $l = 2$ cm, and $f = 428$ MHz, is plotted. Fig. 4 demonstrates the large signal produced by an open - circuited pickup when excited by a non - relativistic beam.

References

[1] Q. A. Kerns, D. B. Large, "Analysis Of A Traveling-Wave Beam Electrode", LRL, UCRL-11551, July 7, 1964.

[2] R. E. Shafer, R. C. Webber, T. H. Nicol, "Fermilab Energy Doubler Beam Position Detector", IEEE Transactions on Nuclear Science, Vol. NS-28, No.3, June 1981.

[3] G. R. Lambertson, "Physics of Particle Accelerators", editors M. Month and M. Dienes, AIP 153, Vol. 1, 1987.

[4] R. E. Collin, "Foundations For Microwave Engineering", McGraw - Hill, pg.198. See problem 4.9.

[5] K. Y. Ng, "Stripline Monitors as Transmission Lines", Fermi National Accelerator Laboratory, Note FN - 534, Feb. 1990.

[6] Ref. [2], Eqn. 5.