Calculations of the Conditions for Bunched-Beam e-p Instability in the Los Alamos Proton Storage Ring (PSR)*

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ABSTRACT

Recent observations are consistent with the possibility of an "e-p" instability in the PSR, with both bunched and unbunched beam. This instability requires stable trapping of electrons within the space charge potential of the protons and such trapping is not expected with bunched beam at PSR parameters. However, it is shown that electron trapping can occur if some of the beam leaks into the interbunch gap. Such leakage is observationally associated with the instability. Also it is shown that the leakage is consistent with the expected longitudinal dynamics within the PSR. Implications for improving PSR stability are discussed.

INTRODUCTION

Recent observations[1] support the hypothesis that the fast transverse instability observed in the PSR[2] is an electron-proton (e-p) instability, in which stray electrons are trapped within the space-charge of the circulating protons and unstable coupled transverse oscillations of the trapped electrons and protons develop. The instability requires a source of free electrons, stable trapping of electrons within the proton beam, and the exponential development of coherent coupled oscillations.

The oscillations can be described using a simplified linearized model[3],[4],[5] in which the proton beam and trapped electrons have uniform density within an elliptical cross section (a × b). Longitudinal variation is also ignored and the proton and electron densities are:

$$\rho_p = \frac{N}{(L \cdot \pi ab)}, \rho_e = \eta_e \rho_p \qquad (1)$$

where N is the total number of protons, L is the bunch length ($= 2\pi R$ for unbunched beam), and η_e is the neutralization factor. The equations of coupled vertical motions are:

$$\ddot{y}_p + (Q^2 + Q_p^2)\Omega^2 y_p = Q_p^2 \Omega^2 \bar{y}_e$$
$$\ddot{y}_e + Q_e^2 \Omega^2 y_e = Q_e^2 \Omega^2 \bar{y}_p$$

where

$$Q_e^2\Omega^2 = \frac{4Nr_e c^2(1-\eta_e)}{b(a+b)L}, \ Q_p^2\Omega^2 = \frac{4\eta_e N r_p c^2}{b(a+b)\gamma L} \qquad (2)$$

are the electromagnetic oscillation frequencies of the electrons and protons and Q is the PSR vertical tune. The equations describe dipole-mode oscillations coupled by the centers of charge $\bar{y}_p$, $\bar{y}_e$. Assuming harmonic motion obtains the dispersion relation:

$$(Q_e^2 - x^2)(Q^2 + Q_p^2 - (n-x)^2) = Q_e^2 Q_p^2 \qquad (3)$$

where n is the spatial harmonic of proton oscillation and $x = \omega/\Omega$ is the oscillation frequency in units of revolution frequency ($\Omega = v_p/R$). At PSR parameters, we obtain $Q_e \approx 40(\approx 100$ MHz). The relation can have complex solutions (instability) with $x \approx Q_e \approx n - Q$; thus the unstable oscillations occur at lower betatron sidebands near 100 MHz. Growth rates (from Im xΩ) of the sidebands can be quite fast; Im x$\Omega \approx 0.1 - 0.01\ \mu s^{-1}$ is readily obtained. The instability requires a minimal value of $Q_p(Q_p > 0.1$ or $\eta_e > 0.01)$, implying a relatively small neutralization is required. The PSR unstable oscillation frequencies and growth rates, and their dependences on beam size and density are in general agreement with the e-p model.

CONDITIONS FOR ELECTRON TRAPPING IN THE PSR

A key difference between the PSR and the simplified model is that the beam density in the PSR varies longitudinally by large factors, particularly with bunched beam. Stable trapping must be maintained with these variations. The trapping potential in the high intensity PSR beam is quite strong, and electrons should remain trapped within a continuous (debunched) beam. However, with bunched beam, a beam free interbunch non-trapping gap of 100 ns (25 m) passes through the electrons every PSR turn (360 ns). In that gap, even low-energy electrons will be detrapped, hitting the walls with high probability. (10–100 eV electrons travel 20–60 cm.)

The detrapping conditions can be quantified by representing the beam passage as a focussing transport section and the gap as a drift. The full transport is a product matrix:

$$M = \begin{bmatrix} 1 & L_1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \cos(k_e L_2) & \frac{1}{k_e}\sin(k_e L_2) \\ -k_e \sin(k_e L_2) & \cos(k_e L_2) \end{bmatrix} \qquad (4)$$

where $k_e = Q_e/R$ and L_2 and L_1 are the bunch and gap lengths. For stable trapping the magnitude of the trace of M must be ≤ 2. At PSR parameters the beam strongly overfocusses the electrons ($Q_e \gg 1$), and the total transport is almost always unstable. Equation 4 assumes a constant beam density; the density within the bunch can be modified to more realistic forms (*i.e.*, parabolic) and the

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transport recalculated (see reference [6]). At PSR parameters, the same pattern of general instability is obtained, provided that the gap (L_1) is beam-free. Therefore e-p instability should not occur in the PSR with bunched beam if a beam-free gap is maintained.

A different pattern is obtained if the gap does not remain beam-free, but significant amounts of beam leak into the gap. For that case we can approximate the beam density as a sum of a continuous sinusoidal distribution plus a constant background ϵ, similar to profiles observed in the PSR at onset of instability (!):

$$\rho(z) = \frac{N}{2\pi R}[(1 + \cos(z/R)) + \epsilon] \tag{5}$$

At PSR parameters, stability conditions (Abs (Tr M) $\leq$ 2) are obtained for almost all conditions (see Figure 1), provided $\epsilon > 0$. The overall stability situation is facilitated by the relatively large distance scale in the PSR bunch; the transitions from high to low density beam are "adiabatic".

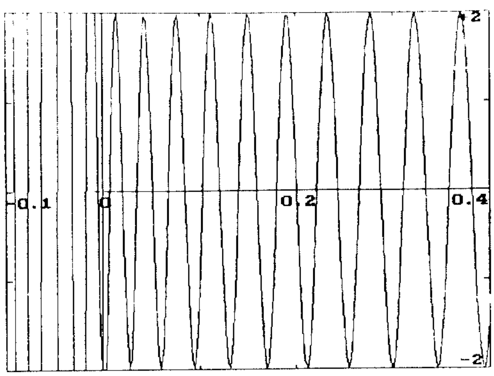

Figure 1. Tr[M] (ϵ) for $N = 2.25 \times 10^{13}$. Stability (electron trapping) is obtained for almost all $\epsilon > 0$.

The calculations demonstrate that electron trapping, and hence e-p instability, can occur if the interbunch gap has filled in with low-density beam. Observationally, instability can occur if the gap has indeed filled in, to some extent, and does not occur if the gap is maintained.

CONDITIONS FOR BUNCH LEAKAGE

Observations in the PSR show leakage of beam from the confining rf bucket when instability occurs. The critical question is whether such leakage may occur within the expected longitudinal dynamics, and in this section results of simulation explorations of this dynamics are reported.

The dominant longitudinal forces are expected to be due to the rf buncher and longitudinal space charge. The PSR has a low-frequency first-harmonic (2.8MHz, h=1) rf system with relatively low voltage ($V_{rf} \approx 10kV$). As a high-intensity machine at relatively low energy (800 MeV), it also has strong space charge forces. In a simple 1-D model the longitudinal space charge debunching force is:

$$F_z = -\frac{eg}{\gamma^2 4\pi\epsilon_0}\frac{d\lambda}{dz} \tag{6}$$

where λ is the beam line density, $\gamma = E/m_p c^2$, and $g = 1 + 2ln(r/b)$ with b and r the beam and pipe radii. With these forces the equations of longitudinal motion are:

$$\frac{d\phi}{dn} = -\frac{2\pi h\eta}{\beta^2}\frac{\Delta E}{E} \tag{7}$$

$$\frac{d}{dn}\left(\frac{\Delta E}{E}\right) = \frac{eV_{rf}}{E}(\sin(h\phi) - \sin(h\phi_s)) - \frac{egR}{2\gamma^2\epsilon_0 E}\frac{d\lambda(z)}{dz} \tag{8}$$

The phase ϕ and relative energy ($\Delta E/E$) have been chosen as dependent variables and turn number (n) is the independent variable. $\eta = (1/\gamma^2 - 1/\gamma_T^2)$ is the frequency slip factor. In the PSR the rf harmonic $h = 1$, and $\Phi_s \approx 0$ (no acceleration).

In addition, beam particles have energy losses of $\sim$ 500 eV per turn, with energy spread, from passing through the stripping foil. Energy losses from impedance couplings may also occur. Transverse variations and transverse-longitudinal couplings may also be important. However, these effects were ignored in initial simulations.

Injection into the PSR is not phase-space matched. The revolution period is 360 ns. The injected (200 MHz) beam is chopped into micropulses within that period centered about 0 with a width of $\sim$ 250 ns, so that no beam is injected near the unstable phase; the interbunch gap is initially beam-free. The beam is injected with small energy spread, but over the injection time the beam rotates to fill most of the rf bucket, with substantial variations in bunch shape and densities. Injection continues for $\sim$ 360 to 720 μs (2000 turns).

In the simulations, the entire beam ($> 10^{13}$ protons) is represented by $\sim$ 6000 macroparticles. The time step used is one turn; an rf kick plus single-turn transport represents the single-particle dynamics. The space charge is proportional to $d\lambda/dz$. λ, the density, is found by splitting the circumference into 64 or 128 bins and finding the macroparticle density within the bins. The derivative $d\lambda/dz$ is found from the difference $(\lambda_{i+1} - \lambda_{i-1})$ of the density of adjacent bins. The method has inaccuracies from the coarseness of the binning and from the macroparticle statistics and the simplified 1-D force representations. The injection procedure is simulated by adding more macroparticles over the injection time, with new particles injected randomly in phase within the injection width and randomly within a small energy spread. A typical run would include 1200 turns of injection followed by 1000 turns of storage. Beam leakage is observed by particle motion outside the confining bucket and into the interbunch gap. The simulations were performed on an IBM PC, which provides instantaneous turn-around and immediate color graphics display of the motion.

Results for a typical case are shown in Figures 2A-2D. The tracking clearly shows beam leakage into the gap. Initial injection places beam in a square wave pulse with small $\Delta E/E$ (Figure 2A). After $\frac{1}{4}$ synchrotron oscillation (600 turns), the rf bunch rotation has introduced a large

$\Delta E/E$ and a large beam density concentration near the center, with large space charge forces (Figure 2B). The space charge force pushes the beam at the edges of the bunch outside the bucket (Figure 2C), some beam spreads into the gap (2D).

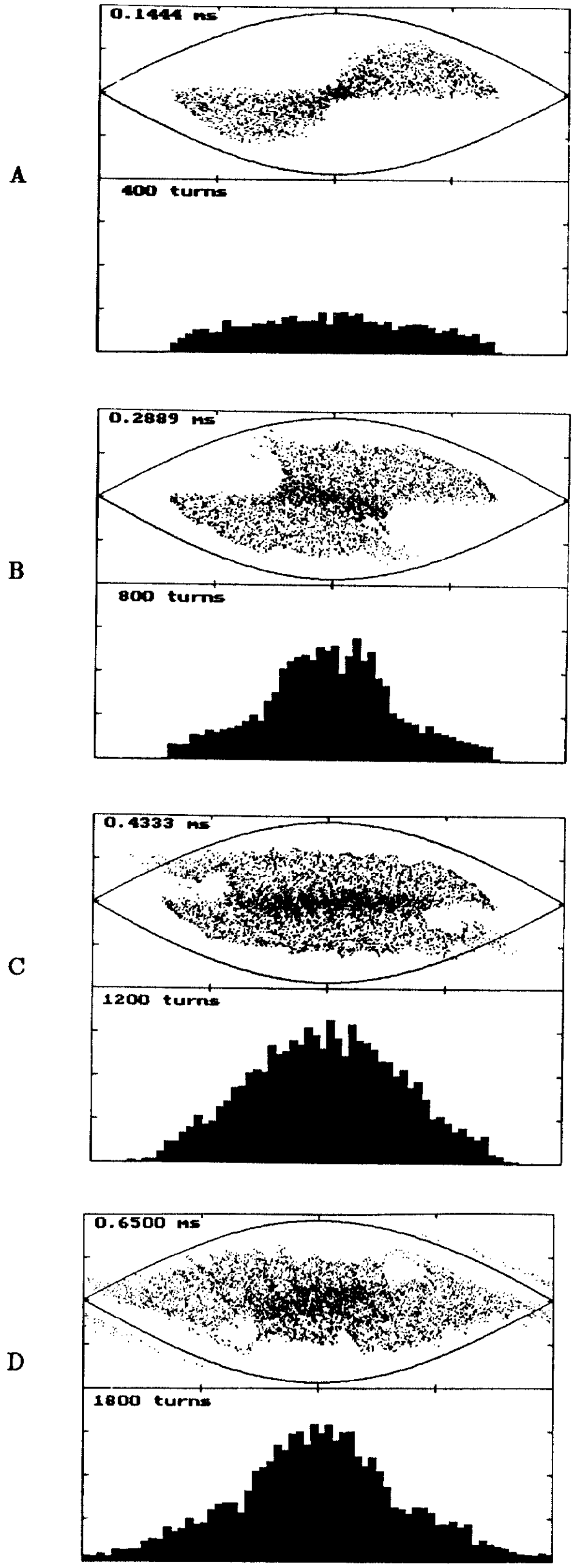

Figure 2A-D Simulation of bunch leakage in the PSR. ($N = 4 \times 10^{13}$, $g = 4$, $V_{rf} = 10kV$)

Injected intensity can be varied to find a leakage threshold. For $N > 3.0 \times 10^{13}$ large leakage occurs, while for $N \leq 1.5 \times 10^{13}$ no leakage occurs and intermediate values show small bunch leakage. Reducing rf voltage reduces the leakage threshold; at $V_{rf} = 6kV$ the threshold is reduced to $1.0 - 1.5 \times 10^{13}$.

In bunched beam simulations, leakage does not occur until after $\sim \frac{1}{2}$ synchrotron oscillation (1200 turns at $V_{rf} = 10kV$), which allows time for the space charge force to develop. With unbunched beam, simple drift fills the gap and this requires much less time. The same bunched-beam time delay is seen in PSR instability observations.

The simulation conditions for bunch leakage are in good agreement with observed PSR conditions for instability. The combination of rf bunching (weak), longitudinal space charge (large), and injection mismatch (large) is sufficient to explain the existence of bunch leakage at high intensities in the PSR.

The calculations show that e-p instability should not occur unless beam leaks into the interbunch gap, and that such leakage can occur within the PSR longitudinal motion. Manipulation of PSR parameters ($V_{rf}(t)$, Φ_s, injection width) to minimize leakage has improved stability and permitted higher intensities in PSR operations, and further optimizations (*i.e.*, with larger V_{rf} or a multiharmonic "barrier-bucket" system) are possible.

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