Effect of Non-planar Undulators on Beam Dynamics in ELETTRA

Lidia Tosi and Ryutaro Nagaoka
Sincrotrone Trieste, Padriciano 99, 34012 Trieste, Italy

Abstract

There is a strong interest in the installation of devices that will generate circularly polarized radiation in ELETTRA. However, the impact of such undulators on beam dynamics has been found to be more serious with respect to conventional planar devices, since further strong non-linearities are introduced in both horizontal and vertical planes. An investigation of the consequences of these effects on beam dynamics has been carried out for different types of devices. Various pratical means to compensate their effects, including local optics modifications, are attempted.

I. INTRODUCTION

ELETTRA[1], under construction at Trieste (Italy), is a 1.5-2.0 GeV third generation light source which at full operation will accomodate up to 11 insertion devices. Recently there has been also an increasing interest in the installation of a circularly polarized light source. However, previous studies[2] on the dynamic apertures in presence of helical insertion devices have shown an unacceptable reduction in the maximum stable amplitudes due to the additional non-linearities which these structures introduce in both the horizontal and the vertical planes with respect to the usual plane devices. In fact, the transverse and longitudinal components of the magnetic field in a general helical structure may be written as [3]:

$$B_x = B_o\frac{k_x}{k_y}\,sh(k_x x)sh(k_y y)cos(kz) + B_o'\,ch(k_x' x)ch(k_y' y)sin(kz)$$

$$B_y = B_o\,ch(k_x x)ch(k_y y)cos(kz) + B_o'\frac{k_y'}{k_x'}\,sh(k_x' x)sh(k_y' y)sin(kz) \quad (1)$$

$$B_z = -B_o\frac{k}{k_y}\,ch(k_x x)sh(k_y y)sin(kz) + B_o'\frac{k}{k_x'}\,sh(k_x' x)ch(k_y' y)cos(kz)$$

where k_x, k_y, k_x', k_y' must satisfy the two divergence conditions $k_x^2+k_y^2=k^2$ and $k_x'^2+k_y'^2=k^2$ with $k=2\pi/\lambda_o$, λ_o being the insertion device period length. Thus, the field may be looked upon as the superposition of the field **B** generated by a conventional horizontal plane device, whose non-linearities in the transverse planes scale as $(1/\rho k)^2(k_x)^m(k_y)^n$ [4], and of a field **B'** generated by a vertical plane device, which will introduce additional linear and non-linear forces scaling as $(1/\rho' k)^2(k_x')^n(k_y')^m$ [12,3], with ρ and ρ' the bending radii in the fields B_o and B_o' and with m, n = 0, 2,... Considering the exchanging roles that k_y and k_x' play in the two fields and the fact that usually for a plane horizontal device by construction $k_y \cong k$, all the non-linearities which were present in the vertical plane for the latter are now also present in the horizontal plane, whose maximum stable amplitude for ELETTRA will mostly suffer because of the larger beta value of 8.2 m at the device location against 2.6 m for the vertical one.

A further cause for the strong reduction in the horizontal plane may be searched also in the fact that, whereas for the plane device described by the **B** field a particle injected with zero vertical amplitude will always remain in the horizontal plane, the **B'** field generates a finite vertical amplitude. The lifting of the particle off the horizontal plane will eventually in addition activate non-linearities coming from the strong sextupoles and from the **B** field.

Thus, it is important to be able to minimize the difficulties that a circularly light source may cause to machine operation, by a scanning through the design parameters of different proposals[5], which include a crossed undulator[6], a crossed scheme[7], an asymmetric wiggler[8] and an elliptical wiggler[9], whose most significant dynamic apertures are presented in the following section. An optimal dynamic aperture, which garantees a sufficient safety margin for the dynamic aperture requirements for Touschek and beam-gas scattering and for the injection process[10], has been found for the elliptical multipole wiggler.

In the last section, an enlargening of the maximum horizontal stable amplitude is attempted by lowering the beta value at the device location. Such a modification is found to be quite helpful, especially when no sextupoles are included. However, the inclusion of the latter presents a whole series of problems typically encountered when designing a lattice.

II. DYNAMIC APERTURES

In order to define the design parameters of the helical device whose dynamic aperture would garantee some safety margin for the aperture requirements and to understand the influence of the parameters on the dynamics, investigations on the dynamic behaviour of several possibilities have been carried out. In all the computations, the original tunes have been re-installed by a global compensation, leaving a residual beta beat whose maximum was found to be less than 3% for all cases. After doing the chromaticity correction, four particles with different initial conditions were tracked over 250 turns with the computer code RACETRACK [11,12].

Since the non-linearities introduced by **B'** scale as $1/\rho'^2$, investigations on the influence of the value for B_o' were done for a crossed undulator[6] whose parameters at 2 GeV were taken to be B_o = 0.35 T, λ_o = 0.06 m and N_p = 75. Lowering B_o' from 0.35 T to 0.15 T brought an improvement up to 21 mm in the horizontal and 16 mm in the vertical maximum stable amplitudes against the original values of about 10 mm.

A second series of investigations were made for the crossed scheme[7], in which the light is generated by placing in the same straight section a horizontal plane device U1 with a field **B** followed by a vertical plane device U2 generating a field **B'**.

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The two devices were chosen to have at 2 GeV the same maximum on axis fields of 0.39 T, $\lambda_o = 0.066$ m and $N_p = 24$. The transverse propagation constants of U1 were maintained constant with $k_y = k$, introducing non-linearities in the vertical plane. The ones for U2 were instead varied in such a way as to realize linearly a horizontally defocussing device with $k_x' = i\ 123\ m^{-1}$, a horizontal drift with $k_x' = 0$, an equally focussing device in both planes with $k_x' = k_y'$ and a vertical drift with $k_x' = k$. As shown in the resulting dynamic apertures in figure 1, there is a strong reduction in both planes for the case in which U2 is horizontally defocussing, since the ratio of the transverse and longitudinal propagation constants is large. All the other cases present different reductions due to the intrinsic non-linearities of the two devices and due to the coupling effects of U2. They may be explained by considering the following equations of motion for U2:

$$x'' = -\frac{1}{2\rho'^2 k^2}\left(k_x'^2 x + \frac{k_x'^4}{6}x^3 + \frac{k_x'^2 k_y'^2}{2}xy^2\right) - \frac{\sin(ks)}{\rho'}\left(k_y'^2 xy + \frac{k_y'^4}{6}xy^3 + \frac{k_x'^2 k_y'^2}{6}x^3 y\right) + y'\frac{\cos(ks)}{\rho'}k\left(x + \frac{k_x'^2}{6}x^3 + \frac{k_y'^2}{2}xy^2\right) \quad (2)$$

$$y'' = -\frac{1}{2\rho'^2 k^2}\left(k_y'^2 y + \frac{k_y'^4}{6}y^3 + \frac{k_x'^2 k_y'^2}{2}x^2 y\right) - \frac{\sin(ks)}{\rho'}\left(\frac{k_y'^2}{2}y^2 + \frac{k_x'^2}{2}x^2 + \frac{k_y'^4}{24}y^4 + \frac{k_x'^4}{24}x^4 + \frac{k_x'^2 k_y'^2}{4}x^2 y^2\right) - x'\frac{\cos(ks)}{\rho'}k\left(x + \frac{k_x'^2}{6}x^3 + \frac{k_y'^2}{2}xy^2\right)$$

For $k_x' = 0$, U2 introduces practically the same non-linear terms in the vertical plane as U1. The two combined together may explain the large reduction in the maximum vertical amplitude and because of the lifting of particles off the horizontal plane by U2, they contribute with the sextupoles to the reduction of the horizontal one. For the case $k_x' = k$, the above terms do not exist in U2, leading to an enlargement of the maximum vertical amplitude. However, analogous terms exist in the horizontal equations, whose maximum amplitude will suffer mostly. The best compromise seems to be the case in which $k_x' = k_y'$, where even though the equations of motion are the most complex, the strengths of the non-linearities are smaller with respect to the previous cases. Of course the whole mechanism of the dynamics is much more complex, depending on the system and on the combined effects of the sextupoles and the device.

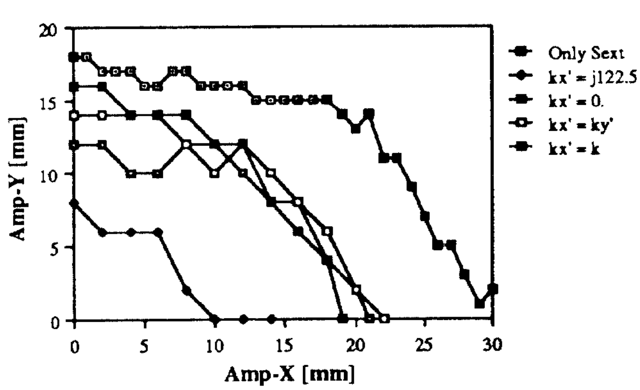

Figure 1. Dynamic aperture for the crossed scheme varying the design parameters of U2

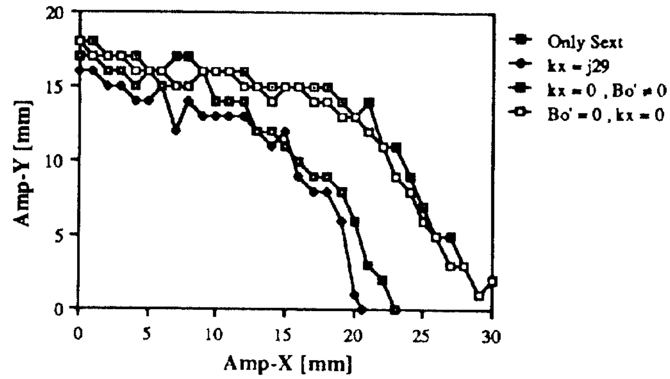

Figure 2. Dynamic aperture for the elliptical multipole wiggler

Investigations have been also carried out for an asymmetric wiggler[8], which is a horizontal plane device whose vertical and longitudinal field components may be written as a sum of harmonics. The dynamic aperture for the device chosen to have at 1.5 GeV an equivalent linear effect corresponding to a maximum on axis field of 0.35 T, $\lambda_o = 0.24$ m and $N_p = 20$ showed no significant reduction with respect to the one with sextupoles alone. In fact, this device presents a small value of k, important for small non-linearities, it produces a small linear distorsion and most important of all it is a plane horizontal device with no **B'** field.

From the above results, it has become quite evident that the required device must be searched for in the class of helical structures which present low values for all the propagation constants, a small value for B_o' and an adequate value for B_o. While the first two conditions contribute in containing the non-linear and coupling effects, the third must assure a small linear distorsion around the ring in order to not excite additional resonances due to the optical symmetry break. The device was chosen to be an elliptical multipole wiggler[9,13] with $B_o = 0.33$ T, $B_o' = 0.054$ T, $\lambda_o = 0.2$ m and $N_p = 20$ at 1.5 GeV. Since this device allows the switching of the polarization of the light by inverting only the **B'** field, the matching to the original tunes has been done only for **B**. Various sets of transverse propagation constants have been scanned in order to find an optimal dynamic aperture and the two most significant corresponding to $k_x = i\ 29\ m^{-1}$, $k_x' = 34.8\ m^{-1}$ and $k_x = 0$, $k_x' = k$ are reproduced in figure 2. In order to give an idea to what extent the presence of **B'** may limit the horizontal aperture, the dynamic aperture produced by the device when the above field is zero in the case $k_x = 0$ is shown. Since the non-coupling non-linearities introduced by **B'** are effectively small, it is reasonable to deduce that the small coupling ones combined with the sextupoles and the **B** field is the main cause of the reduction. Since the elliptical device with $k_x = 0$ seems to garantee some safety margin for the fulfilment of all the necessary aperture requirements, this device may be the most suitable as a circularly polarized light source for ELETTRA.

III. OPTICS MODIFICATIONS

Since ELETTRA presents a large horizontal beta value of 8.2 m against 2.6 m of the vertical, in this section an attempt of containing the non-linear effects in the horizontal plane by lowering the horizontal beta is presented. However, since this operation accomplished at only one straight section renders the sextupole optimization difficult, the optics have been modified in such a way as to pass from a 12-fold symmetry to a 6-fold one. Each superperiod is composed of two of the original ones, in which there is a high beta straight section followed by a low beta one, as shown up to the symmetry point in figure 3. In order to localize as much as possible the modifications, the optical functions before the quadrupole triplet in the low beta section have been fixed to the original values. By the introduction of an additional quadrupole family, the betas were simultaneously lowered and the alfas set to zero at the symmetry point. The quadrupole triplet in the high beta section was then used to globally re-adjust the fractional parts of the tunes to appropriate values. While the horizontal beta is reduced to 1.60 m in the low beta section leading to an increase in the tune of unity, the vertical one remained almost the same. Associated with this change, a sextupole optimization was carried out and the best configuration was found to be just the original harmonic sextupoles powered differently in the two sections. However, a reduction of about 30% in the horizontal dynamic aperture with respect to the original lattice occurs, due to the increase in the number of harmonics influencing the motion noticed in the analysis in single resonance approximation[14].

In order to see the effectiveness of lowering the horizontal beta value, the crossed undulator of the previous section with B_0' = 0.35 T was introduced without sextupoles in the ring. The maximum horizontal stable amplitude was found to be 60 mm against 20 mm in the original lattice. Subsequent trackings with sextupoles showed a large sensitivity to the location of the working point, due to the combined effect of the device and of the deterioration of the sextupole distribution compared to the former. The best dynamic aperture, shown in figure 4, was found by shifting slightly the vertical tune.

On this occasion, also a new proposed tracking routine for plane horizontal devices[15] has been extended to non-planar devices.

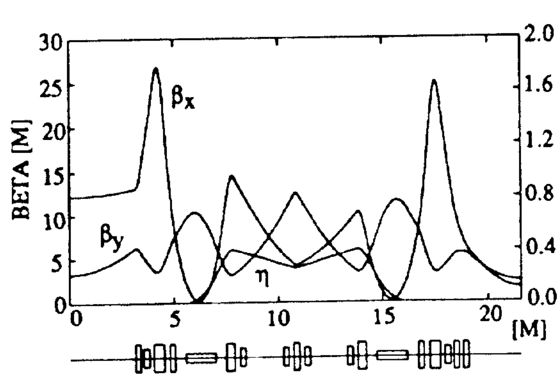

Figure 3. Modified optics

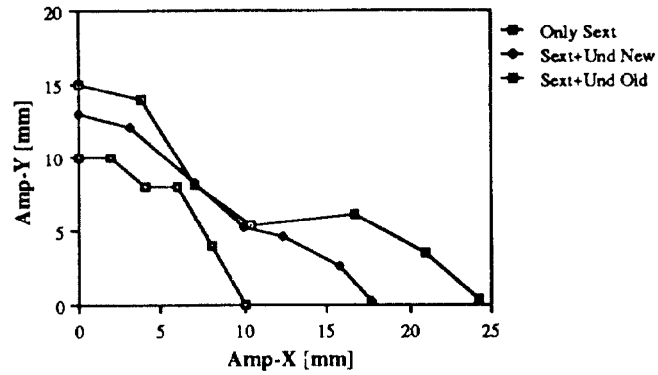

Figure 4. Comparison of the dynamic apertures for the crossed undulator in the original lattice and in the modified one.

IV. CONCLUSIONS

The presence of a helical insertion device in ELETTRA may deteriorate critically the dynamic aperture, due to the introduction of additional non-linearities in both planes. Thus, the request from the users of having a circularly polarized light source has required the necessity of finding a device which fulfills all the aperture requirements. After investigating the beam performance for various possibilities, an elliptical multipole wiggler has been found to be the most suitable. Furthermore, the lowering of the horizontal beta to suppress the non-linearities in this plane has shown to be useful for the crossed undulator, even when the presence of sextupoles may hinder the effectiveness because of the optical symmetry break.

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V. REFERENCES

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