Computer simulation of the coherent beam-beam effect in the LHC

W. Herr, CERN-SL CH-1211 Geneva 23

Abstract

For the Large Hadron Collider in the LEP tunnel (LHC) a non-zero crossing angle is foreseen to avoid multiple collisions outside the interaction point. The two beams can however suffer from long range beam-beam interactions during the time they are in one common vacuum chamber. These long range interactions can excite coherent beam-beam modes and due to the finite separation they are in general non-linear leading to possible coherent resonances of higher order. A computer simulation has been developped and was used to investigate this coherent beam-beam effect. Possible constraints on the LHC parameters are investigated and discussed.

1 Introduction

The LHC is designed to reach very high luminosities above $10^{34} cm^{-2} s^{-1}$ [1] and to achieve this a very large number of closely spaced bunches (4725) is foreseen. Normally, the two beams are separated into two beam pipes except in the interaction regions where they share a vacuum chamber. To avoid that all those bunches collide, a crossing angle of 200 μrad if foreseen. However, long range beam-beam effects cannot be avoided between the incoming and outgoing bunches and it was shown [2] that these long range forces constrain the choice of parameters for the crossing angle and the β^* functions at the interaction point. In this paper the coherent effect of long range interactions will be examined. The excitation of coherent bunch oscillations induced by the beam-beam effect is well known and studied extensively [3, 4] for the case of head-on collisions where the coherent motion is excited by a small transverse displacement of the colliding bunches. For the LHC these coherent oscillations can be excited by long range collisions since they are dipolar kicks and one can expect the excitation of coherent rigid dipole oscillations, i.e. small deviations from the nominal orbit. Their stability is investigated with a simulation program.

2 LHC parameters

Parasitic long range collisions in interaction region

The bunch spacing of 15 ns corresponds to a bunch distance of 4.5m and for the LHC geometry one calculates a total number of 19 parasitic collisions on each side of the collision point. The actual kicks received from opposing bunches are relatively small but since their effect is cumulated over many collisions, the total kick can become very significant [2]. Because a given bunch "collides" with many other bunches in every interaction region, a coupling of all bunches and a large number of coherent modes can be expected. To ensure a stable system, the motion of all these modes with different frequencies must be stable independently.

Beam separation in interaction region

The normalised emittance of the LHC beams is $\epsilon = 3.75\ \mu$m ($\epsilon = \sigma^2 \gamma/\beta$) and with a β^* value of 0.5 m this results in a beam size of about 15 μm. The separation between the two orbits d(s) increases linearly with the distance from the interaction point and the normalised separation $d_{sep} = d(s)/\sigma(s)$ is constant between the interaction region and the first quadrupole and can be written as $d_{sep} = \alpha\beta^*/\sigma^*$ with α the full crossing angle and β^* and σ^*, the betatron function and beam size at the interaction point (see e.g. [2]). For the nominal LHC parameters this gives a separation of about 6.5 σ for all long range collisions.

3 Simulation model

Phase space variables and particle transport

The variables used for the simulation are the transverse coordinate and angle of a bunch with respect to the design orbit (x,x'). The bunches are considered as rigid objects in this simulation since only the coherent dipole oscillation is considered. A complete simulation with more than 4000 bunches is unrealistic since the computer time required for the simulation would be too large. The maximum number of bunches in the simulation was 256 per beam. For the particle transport a linear, uncoupled machine is assumed. All the interaction regions are considered identical in the simulation, i.e. same length and same β^*. Details about the simulation can be found in [6].

Beam-beam interaction

For a given bunch the distance from its design orbit can be written as x_1, the distance of the opposing bunch to its design orbit as x_2 and the distance of the two design orbits is d(s). The distance between the two bunches is then given as $\delta x = x_2 - x_1 + d$. The beam-beam interaction between the two bunches is approximated by rigid dipole kicks of the form:

$$f(r) = \frac{8\pi\xi\sigma^2 x_{x,z}}{r\beta}(1 - e^{\frac{r^2}{2\sigma^2}})$$

where: $r = \sqrt{x^2 + z^2}$, σ is the transverse beam size and ξ is the linear beam-beam tune shift.

The transverse kick δx' is computed as $\delta x' = f(x_2 - x_1 + d(s)) - f(d(s))$. The term f(d(s)) has to be subtracted since it is an orbit kick caused by the beam-beam interaction and has to be compensated by other means. In this study we are only interested in oscillations of bunches around the orbit.

The separation for the nominal LHC parameters is about 6.5 σ and a linearised beam-beam force as it is usually used to study coherent beam-beam effects from head-on collisions is not sufficient since it would suppress all resonances of orders higher than two and result in too optimistic limits for the stability (for details see [6]). In order to make the model realistic for a smaller number of bunches, several parasitic collisions have been accumulated into a single kick for the test bunch. The phase advance between the long range collisions on one side of the interaction region is small and such a cumulation is justified but the total effect is overestimated since the bunches normally collide with different phases.

Simulation strategy

The program has been written for two dimensions, but the stability was only studied in the horizontal plane where the

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two beams cross at an angle. The initial orbit displacements of all bunches were chosen as random numbers in the range of $\pm 0.1\sigma$. The results have been shown to be not very sensitive to the initial displacement. For the results presented in the next section only the long range forces are considered.

The program follows all bunches of both beams and keeps track of the phase space variables. An unstable situation is assumed whenever a bunch exceeds a certain limit on the displacement, in most cases a limit of twice the initial deviation from the design orbit is enough and the detection of instability is relatively insensitive to the actual choice of this value. For this detection, the number of turns tracked in each case is in the order of 250 - 1000, but in some cases up to 16000 turns have been tracked to study the possible dependence of the number of turns on the onset of unstable motion. Should the motion remain stable, i.e. bound, then the linear beam-beam tune shift ξ is increased in small steps to find the maximum value where the motion remains stable. To find stability diagrams in the (Q,ξ)-plane, the initial Q-values of the machine are scanned.

4 Results

Equally spaced interaction regions

Although equally spaced interaction regions with a limited number of bunches are not a good model for the LHC, such a scheme is treated for completeness and to compare with the results obtained for clustered interaction regions and many bunches.

Spectrum and stability A simulation has been performed to investigate the stability of a scheme with four bunches and equally spaced interaction regions, in this particular example with 4 collision points, i.e. every second collision is omitted. Fig.1 shows a Fourier spectrum of a bunch where the beam-beam tune shift used was ξ = -0.0034, i.e. the nominal LHC tune shift, the separation was 6.5 σ and 19 long range collisions were cumulated on each side in every interaction region. The horizontal tune was Q_h = 71.28. Three peaks in

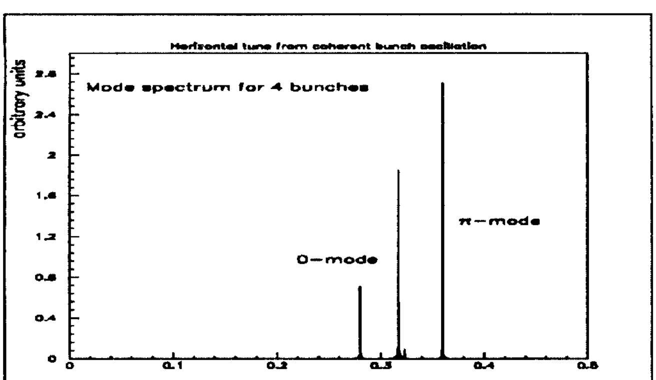

Figure 1: Mode spectrum for four bunches and equally spaced interaction regions

the spectrum can be seen: the peak at the lowest frequency corresponds to the so-called 0-mode without a frequency shift where the bunches move together and which is always stable. The mode where the bunches move maximally out of phase is the so-called π-mode which experiences the maximum frequency shift. This mode can become unstable. Between the 0-mode and the π-mode are the multi-bunch modes and in this very simple case of equally spaced and equal bunches these modes are degenerated into a single mode. The symmetry of the system is the reason for this degeneracy and breaking the symmetry by using two non equidistant interaction points or different phase advances between the interaction points the other modes become visible [5]. The instability is usually presented in the form of a (Q,ξ)-diagram where for every value of the tune Q the tune shift ξ is plotted where instability occurs, i.e. at values of ξ for which the π-mode (or any other mode) is shifted onto a resonance which can be driven by the force. The stability diagram in the (Q,ξ)-plane is shown in Fig.2. Two re-

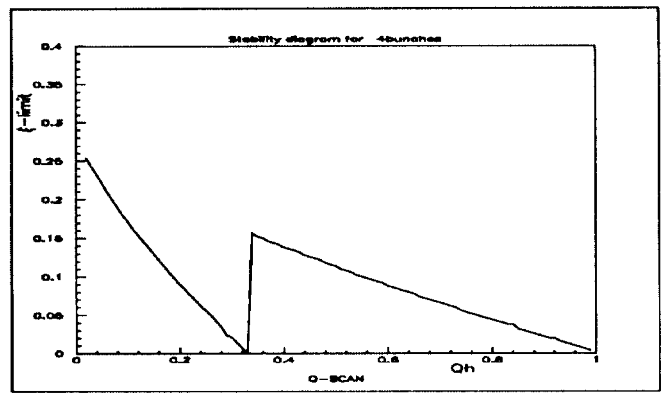

Figure 2: Stability diagram for four bunches and equally spaced interaction regions

gions are visible where the bunch motion becomes unstable: the regions for tune values just below the integer and around the third order resonance. The resonances at 1/2 and 2/3 are suppressed by the symmetry of the system. The stability plot for a single interaction region would show these resonances. For a linearised beam-beam force the third order resonance disappears. Non-linear resonances of orders higher than three have not been seen in the simulations.

Frequency shift An important quantity is the frequency shift of the π-mode since it determines the frequency span of the coherent beam-beam modes. From simulations with simple configurations it can be shown that this shift is only dependent on the total number of long range collisions, the linear beam-beam tune shift ξ, the number of interaction points and the normalised separation. This frequency shift is independent of the total number of bunches as long as the above parameters are kept constant, i.e. for smaller number of bunches the long range kicks are accumulated into a single kick. This allows a simple extrapolation of the frequency span to the nominal LHC parameters.

Clustered interaction regions

In the SSC and the LHC the interaction points are clustered (SSC) or not equally distributed around the ring (LHC), i.e. with different phase advance. In addition, the long range collisions are clustered around the interaction regions since normally the two beams are separated elsewhere.

Frequency shift Assuming three identical interaction regions with a phase advance of Q/2, Q/4 and Q/4 between the collisions, a separation of 6.5 σ and 19 long range collisons on both sides of the collision point, a frequency shift of the π-mode of about 11 ξ is found, i.e. with the current tune values of

Q_x=71.28 and Q_z=70.31 this results in a shift to 71.317 of the horizontal π-mode. The coherent tune shift from the head-on collision at the interaction point however has a opposite sign and partially compensates the long range tune shift. The total shift is therefore $\approx 7.5\ \xi$. It can therefore be extrapolated that a linear tune shift of more than 0.0075 is necessary to get unstable motion. For a linearised beam-beam force a linear beam-beam tune shift as large as ξ=-0.02 would be needed since only first and second order resonances would be excited.

Clustered long range interactions with many bunches To investigate the effect of many long range interactions clustered around the collision points, a larger number of bunches has been simulated (up to 256 per beam) and the long range collisons have been localised around the interaction points. Because a given bunch collides with many other bunches, all bunches couple together and a very large number of multi-bunch beam-beam modes can be observed. In Fig.3

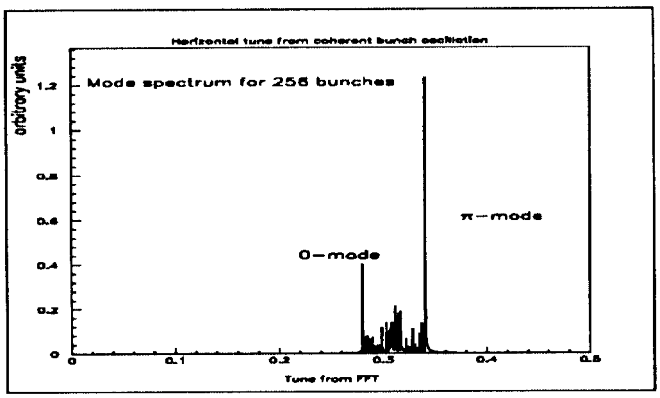

Figure 3: Mode spectrum for clustered long range collisions

the spectrum is shown for 256 bunches and one interaction point with eight clustered long range collisons. A large number of beam-beam modes can be seen between the 0-mode and the π-mode. For the limit of a very large number of bunches the spectrum will become a continuum of modes and the entire tune region up to the π-mode is potentially unstable if a low order resonance is in this region. The stability diagram for the above conditions is shown in Fig.4. The resonances of

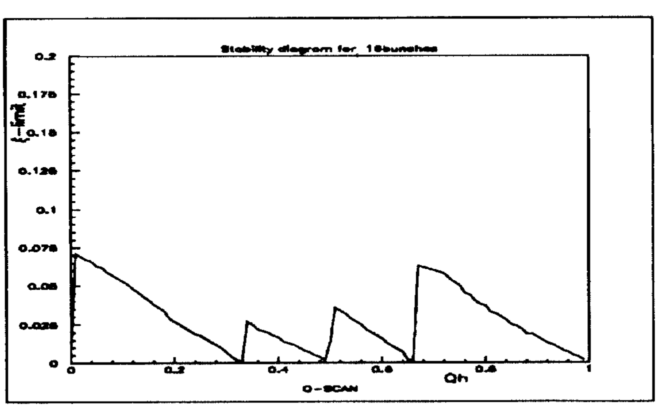

Figure 4: Stability diagram for clustered long range collisions

orders one, two and three are clearly visible. No resonances are suppressed since the symmetry is broken. The widths of the individual resonances are decreasing very fast with the number of parasitic collisions: the long range collisions all occur at different phases between the bunches and a decoherence effect is observed. For larger number of collisions (≥ 6) the linear beam-beam tune shift in the input has to be adjusted with a precision of $\approx 10^{-5}$ to get unstable motion. The limit of instability in Fig.4 had to be carefully searched since with a too coarse scan of ξ the instability could be missed and only the first and second order resonance would be seen. A scan with the required granularity in ξ would not be feasible since the computer time required would be enormous. A method was therefore implemented in the simulation program to find the stability limit for the required resonance by adjusting the π-mode on to the required frequency. Higher order resonances were also studied using this method, but have not been identified, at least not with the number of turns studied (up to 16000 turns). With the nominal LHC parameters, i.e. almost 40 long range kicks per interaction region, a strong decoherence can be expected and the excitation of resonances with orders higher than two will be weak or non-existing.

Dependence on separation The separation determines not only the total frequency shift of the π-mode and therefore the frequency span of the beam-beam modes, but also the non-linearity of the beam-beam potential. For the working point of Q_h = 71.28 a separation of more than 5 σ is necessary in order to remain stable [6]. For the nominal separation of 6.5 σ the motion is stable for $\xi \leq 0.0075$.

5 Conclusion

The coherent dipole oscillations induced by long range beam-beam interactions for the LHC have been investigated and the results can be summarised as follows:

- Coherent dipole oscillations can be excited by long range collisions.
- The separation of several sigma causes the excitation of non-linear coherent resonances of low (3rd) order.
- Clustered interaction regions increase the number of beam-beam modes which can potentially become unstable The frequency space between the 0-mode and the π-mode should be free from low order resonances.
- For the LHC parameters a linear tune shift of ξ = 0.0075 per interaction is necessary for unstable motion.
- However, strong decoherence from long range collisions and other damping mechanisms will make it unlikely that the coherent beam-beam effect is a limitation for the LHC with the current working point.

References

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