Acceleration of Polarized Proton in High Energy Accelerators

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ABSTRACT

In low to medium energy accelerators, betatron tune jumps and vertical orbit harmonic correction methods have been used to overcome the intrinsic and imperfection resonances. At high energy accelerators, snakes are needed to preserve polarization. We analyze the effects of snake resonances, snake imperfections, and overlapping resonances on spin depolarization. We discuss also results of recent snake experiments at the IUCF Cooler Ring. The snake can overcome various kinds of spin depolarization resonances. These experiments pointed out further that partial snake can be used to cure the imperfection resonances in low to medium energy accelerators.

1. Introduction

The ability to accelerate of polarized protons to high energies is important for polarized proton collision experiments. During the acceleration process, polarized protons encounter thousands of spin depolarization resonances. These resonances arise mainly from the the horizontal fields in focusing quadrupoles. The horizontal fields rotate the spin away from the vertical axis. If the kicks are correlated each turn, then the resonance condition arises. There are two types of spin depolarization resonances[1]: intrinsic resonance at $K = kP \pm \nu_y$ and imperfection resonance at $K = k$, where k is an integer and P is the superperiodicity of the accelerator. K is the resonance tune obtained from Fourier analyzing quadrupole kicks around the accelerator. The resonance width or strength, ϵ, is defined as the corresponding Fourier amplitude.

In circular accelerators, the spin vector precesses around the vertical axis with a frequency of $G\gamma$ per turn[2], where G=(g-2)/2=1.792846 is the anomalous g-factor of the proton. The spin tune, ν_s, of the polarized proton is therefore $\nu_s = G\gamma$. When the spin tune equals the resonance tune, successive kicks add up coherently to give rise to depolarization. Fig. 1 shows resonance strength as a function of the energy for various accelerators. Observe that the intrinsic resonance strength is of the order of $|\epsilon| \leq 0.5$ for RHIC and $|\epsilon| \leq 5$ for the SSC.

We shall review the effect of snake resonances, and the effect of overlapping resonances and snake imperfections. The paper is organized as follows: Section 2 discusses the spin equation of motion. Section 3 deals with the spin motion in the presence of snakes. Section 4 reviews snake design. A conclusion is given in Section 5.

2. Spin Equation of Motion

The spin equation of motion for a moving particle in a static magnetic field is given by[3]

$$\frac{d\vec{S}}{dt} = \frac{e}{\gamma m}\vec{S} \times [(1+G\gamma)\vec{B}_\perp + (1+G)\vec{B}_\parallel]. \qquad (2.1)$$

where $\vec{B}_\perp$ and $\vec{B}_\parallel$ are the transverse and longitudinal components of the magnetic fields respectively. G is the anomalous gyromagnetic g-factor and γmc^2 is the energy of the moving particle. Let us use the coordinate system of the reference orbit, where $\hat{x}, \hat{s}, \hat{z}$ are unit vectors corresponding to radial outward, longitudinal, and transverse vertical respectively. Eq.(2.1) can then be transformed to,[1]

$$d\vec{S}/d\theta = \vec{S} \times \vec{F}, \qquad (2.2)$$

with $d\theta = ds/\rho$, where s is the longitudinal path length and ρ is the radius of curvature. The vector $\vec{F} = F_1\hat{x} + F_2\hat{s} + F_3\hat{z}$, can be expressed in term of particle coordinate as, $F_1 = -\rho z''(1+G\gamma)$, $F_2 = (1+G\gamma)z' - \rho(1+G)(\frac{z}{\rho})'$, $F_3 = -(1+G\gamma)$. Defining a 2-component spinor, Ψ, with $S_i \equiv <\Psi|\sigma_i|\Psi>$, Eq.(2.2) becomes,

$$d\Psi/d\theta = -\frac{i}{2}(G\gamma\sigma_3 - F_1\sigma_1 - F_2\sigma_2)\Psi = -\frac{i}{2}H\Psi\ . \qquad (2.3)$$

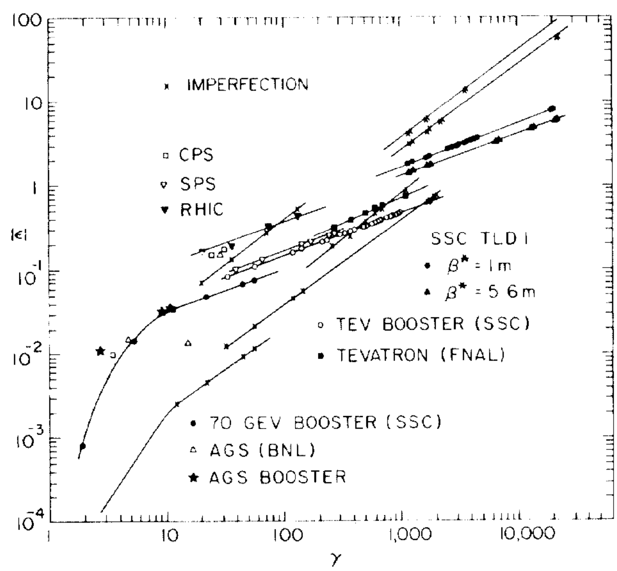

Fig.1 Compilation of intrinsic and imperfection resonance strengths for accelerators in the world. Note the scaling law of $\epsilon_{int} \sim \sqrt{\gamma}$ and $\epsilon_{imp} \sim \gamma$. The rms quadrupole misalignment assumed is ± 0.1 mm. The normalized emittance is $10\pi\mu$m-rad.

The off-diagonal matrix element of H, $\xi(\theta) \equiv F_1 - iF_2$, characterizes the spin depolarization kick by coupling the up and down components of the spinor wave function. Given the repetitive nature of circular accelerators, $\xi(\theta)$ can be Fourier analyzed as $\xi(\theta) = \sum \epsilon_j e^{iK_j\theta}$, where the Fourier amplitude, ϵ_j, is resonance strength and the resonance tune, K_j, is given by $K_j = k \cdot P \pm m\nu_y$ for intrinsic resonances and $K_j = k$ for imperfection resonances. Intrinsic resonances arise from the vertical betatron motion of particle, and imperfection resonances arise from

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the vertical closed orbit distortion. In a real accelerator, synchrotron and transverse betatron motions may also be coupled. The resonance condition becomes more generally $K_j = k \cdot P \pm m\nu_y \pm n\nu_x \pm l\nu_{syn}$, where k, l, m, and n are integers and ν_{syn} is the synchrotron tune.

For a single resonance, i.e. $\xi(\theta) = \epsilon \cdot e^{iK\theta}$, the spinor wave function for constant $G\gamma$ can be found easily[3], i.e. $\Psi(\theta_f) = t(\theta_f, \theta_i)\Psi(\theta_i)$ with

$t(\theta_f, \theta_i) = e^{-\frac{i}{2}K\theta_f\sigma_3} e^{\frac{i}{2}[\delta\sigma_3 + \epsilon_R\sigma_1 - \epsilon_I\sigma_2](\theta_f - \theta_i)} e^{\frac{i}{2}K\theta_i\sigma_3}$.

Here $t(\theta_f, \theta_i)$ is the spin transfer matrix, whose components are given by, $t_{11}(\theta_f, \theta_i) = ae^{i[c - K(\theta_f - \theta_i)/2]}$, and $t_{12}(\theta_f, \theta_i) = ibe^{-i[d + K(\theta_f + \theta_i)/2]}$, with $t_{21} = -t_{12}^*$, $t_{22} = t_{11}^*$ and $b = \frac{|\epsilon|}{\lambda}\sin[\lambda(\theta_f - \theta_i)/2] = (1 - a^2)^{1/2}$, $c = \arctan[\frac{\delta}{\lambda}\tan(\lambda(\theta_f - \theta_i)/2)]$, $d = \arg(\epsilon^*)$, $\lambda = (\delta^2 + |\epsilon|^2)^{1/2}$, and $\delta = K - G\gamma$. The off-diagonal matrix elements t_{12}, t_{21} are the depolarization driving terms, where the parameter b oscillates with an amplitude $|\epsilon|/\lambda$. When snakes are inserted into accelerator, the sine factor in the parameter b remains small until the next snake, which rotates the spin by 180^0 around a horizontal axis. The depolarization driving terms can thus be arranged to cancel each other.

3. Spin Motion in accelerator with Snakes

A snake is a local spin rotator[4], which rotates the particle spin by π radians about a horizontal axis without perturbing the particle orbits outside snake region. A partial snake differs only in the amount of spin rotation angle, e.g. a 10% snake rotates spin by 0.1π radians. Thus a snake is characterized by the amount of spin rotation angle, ϕ, and the **snake axis angle,** ϕ_s, with respect to $\hat{x}$ (radial outward direction).

The spinor wave function at a snake will be transformed locally according to

$$\Psi(\theta^+) = e^{i\frac{\phi}{2}\hat{n}_s \cdot \vec{\sigma}}\Psi(\theta^-) = S(\phi_s)\Psi(\theta^-),$$

where $\hat{n}_s = (\cos\phi_s, \sin\phi_s, 0)$ denotes the snake axis with respect to horizontal outward direction $\hat{x}$, and $\phi = \pi$ is spin rotation angle. $\theta^{\pm}$ depicts azimuthal orbit rotation angles just before and after snake.

Let us assume that there are N_s snakes with snake axes, $(\phi_1, \phi_2, \cdots, \phi_N)$ distributed in the accelerator. Let $\theta_{i,i+1}$ be the azimuthal orbit rotation angle between the i-th, and (i+1)-th snakes. The distribution of snakes should satisfy the following conditions

$$\sum_{k=odd}^{N_s} \theta_{k,k+1} = \sum_{k=even}^{N_s} \theta_{k,k+1} = \pi \; , \qquad (3.1a)$$

$$\nu_s = \frac{1}{\pi}\sum_{k=1}^{N_s}(-1)^k\phi_k = j + \frac{1}{2} \quad j = integer \; . \qquad (3.1b)$$

Eq.(3.1a) ensures that spin tune, ν_s, is independent of particle energy. Eq.(3.1b) can be used to set the spin tune to a most favorable number in avoiding snake resonances, which will be discussed in the following section. As an example, in an accelerator with two snakes, $N_s = 2$, the snakes should be separated by an orbital angle of π and the snake axes of these two snakes should be orthogonal to each other in order to maintain a spin tune of 1/2. For accelerators with a large number of snakes, there are many ways to organize snakes to obtain proper snake superperiodicity and proper spin tune.

Let us consider an accelerator with $N/2$ pairs of (ϕ_2, ϕ_1) snakes. The spin transfer matrix after passing through a pair of (ϕ_2, ϕ_1) snakes is given by,

$$\tau(\theta_0 + \frac{4\pi}{N}, \theta_0) = S(\phi_2)t(\theta_0 + \frac{4\pi}{N}, \theta_0 + \frac{2\pi}{N})S(\phi_1)t(\theta_0 + \frac{2\pi}{N}, \theta_0)$$

The components of spin transfer matrix are given by,

$$\tau_{11}(\theta_0 + \frac{4\pi}{N}, \theta_0) = -e^{-i(\phi_2 - \phi_1)}(1 - 2b^2e^{i\Phi}\cos\Phi), \quad (3.2a)$$

$$\tau_{12}(\theta_0 + \frac{4\pi}{N}, \theta_0) = -2iabe^{-i(c - 2K\pi/N_s + \phi_2)}\cos\Phi. \quad (3.2b)$$

with $\tau_{21} = -\tau_{12}^*$; $\tau_{22} = \tau_{11}^*$ and where $\Phi = K\theta_0 + 2K\pi/N_s + d - \phi_1$, and the parameters, a, b, c, and d are given by $b = \frac{|\epsilon|}{\lambda}\sin\frac{\pi\lambda}{N_s} = (1 - a^2)^{1/2}$; $\lambda = (\delta^2 + |\epsilon|^2)^{1/2}$; $\delta = K - G\gamma$; $c = \arctan[\frac{\delta}{\lambda}\tan\frac{\pi\lambda}{N_s}]$; and $d = \arg(\epsilon^*)$.

The spin motion in accelerator can then be obtained iteratively by using the spin tracking equation through pairs of snakes:

$$T(\theta_{n+1}) = \tau(\theta_{n+1}, \theta_n)T(\theta_n) \; , \qquad (3.3)$$

where $\theta_{n+1} = \theta_n + 4\pi/N_s$. Eq.(3.3) can be solved using a power series expansion in the strength parameter b^2; i.e.

$$T_{11} = T_{11}^{(0)} + T_{11}^{(1)} + T_{11}^{(2)} + \cdots, \qquad (3.3a)$$

$$T_{12} = T_{12}^{(1)} + T_{12}^{(2)} + T_{12}^{(3)} + \cdots, \qquad (3.3b)$$

where $T_{11}^{(i)} = O(b^{2i})$ and $T_{12}^{(i)} = O(ab^{2i-1})$. A set of hierarchy equations to solve Eq.(3.3) iteratively can be obtained. By solving the spin tracking equation, one can find the spin tune and snake resonance condition. The final polarization is obtained from the expectation value of σ_3 in spinor wave function ,i.e.

$$< S >= |T_{11}|^2 - |T_{12}|^2 = 1 - 2|T_{12}|^2$$

where the unitarity condition, $|T_{11}|^2 + |T_{12}|^2 = 1$, has been used.

3.1 Snake resonances

Without lose of generality, we shall first discuss an accelerator with two snakes (ϕ_2, ϕ_1), located at an orbital angle of π from each other. The **one turn map(OTM))** is given by

$$\tau_{11}(\theta_0 + 2\pi, \theta_0) = -e^{-i\pi\nu_s}(1 - 2b^2e^{i\Phi}\cos\Phi) \; , \quad (3.4a)$$

$$\tau_{12}(\theta_0 + 2\pi, \theta_0) = -2iabe^{-i(c - K\pi + \phi_2)}\cos\Phi \; , \quad (3.4b)$$

where $\pi\nu_s = \phi_2 - \phi_1$ and $\Phi = K\theta_0 + K\pi + d - \phi_1$ is the characteristic phase of the orbital motion. The perturbed spin tune, Q_s, given by the trace of OTM, is $\cos\pi Q_s = b^2\sin(2\Phi)$ with $\nu_s = 1/2$. The parameter b is 1 when $|\epsilon| = N_s/2$. Thus during acceleration through a resonance with strength $|\epsilon| \approx N_s/2$, the perturbed spin tune, Q_s, will range over a whole integer unit and will cross the intrinsic resonance comdition many times. Therefore polarization can not be preserved. When the maximum of $|Q_s - \frac{1}{2}|$ equals the deviation of the vertical betatron tune from half integer, the resonance condition recurs. The maximum tolerable resonance strength is thus given by

$$< \epsilon_c >= \frac{\arcsin(|\cos \pi K|^{1/2})}{\pi} N_s \,. \qquad (3.5)$$

Eq.(3.5) indicates that the tolerable critical resonance strength will be larger when the betatron tune is nearer to an integer. Numerical simulations agree well with the prediction of Eq.(3.5). The spin motion in the accelerator can then be obtained iteratively from the spin tracking equation, Eq.(3.3). To first order, we obtain easily

$$T_{12}^{(1)}(\theta_{n+1}) = iab(-1)^n e^{-i(c-K\pi+\phi_2)}\{e^{i(\Phi+nK\pi)} \zeta_{n+1}(K+\nu_s) + e^{-i(\Phi+nK\pi)}\zeta_{n+1}(K-\nu_s)\} \qquad (3.6)$$

where $\zeta_n(q)$ is the enhancement function and is given by,

$$\zeta_n(q) = \frac{\sin nq\pi}{\sin q\pi}. \qquad (3.7)$$

At $q = integer$, we find that $\zeta_n(q) \to n$. This means that the off-diagonal kicks add up coherently on each turn through snake pairs. This condition is indeed a nominal resonance condition, since the spin tune equals a spin resonance position of $\pm K$. However since the betatron tunes of accelerator are not half integers, the condition $K \pm \nu_s = integer$ will never occur. Avoiding snake resonances, polarization will fall within the envelope of

$$\ll S \gg = 1 - 2|T_{12}|^2 = 1 - 8a^2b^2.$$

A few important observations can be drawn here:

1. $T_{12}^{(1)}(\theta_{n=even}) \equiv 0$ at an imperfection resonance, $K = integer$. This means that imperfection kicks cancel each other every two turns around the accelerator. Thus snakes cure most effectively imperfection resonances. At $2nK = integer, K \neq 1/2$, a similar cancellation of $T_{12}^{(1)}(\theta_m)$ occurs at $m = 2n$ *turns* around accelerator.

2. The envelope function $\ll S \gg$ has many nodal points, where the depolarization driving term vanishes, i.e. $b = 0$ or 1. These nodal points corresponds to the spin matching condition[3,5], where $G\gamma = K \pm \sqrt{(integer \cdot N_s)^2 - |\epsilon|^2}$. Thus these nodal locations are separated approximately by N_s units of $G\gamma$. These nodal points play an essential role in spin restoration during the passage through a depolarization resonance.

Based on the linear response theory of Eq.(3.6), we expect that snakes will not work at a betatron tune equal to a half integer. Fig. 2 shows the polarization vs. the fractional part of the vertical betatron tune. A surprisingly many depolarization resonances appear at a betatron tune of rational numbers, e.g. 1/6, 5/6, 1/10, 3/10, etc.. To understand these resonances, we have to study the spin tracking equation beyond linear order in b. These higher order snake resonances can also be studied by solving spin hierarchy equations[3]. In general, the snake resonance condition is given by, $m\nu_s + nK =$ integer, with $m, n =$ odd integers.

Since the betatron tunes of colliders, such as RHIC, SPS, Tevatron, and SSC, have to avoid low order betatron resonances for a long term orbital stability, snake resonances do not impose further constraints to the operational condition of the colliders. The resulting tolerable resonance strength[3] agrees well with the critical resonance strength of Eq.(3.5). One can generalize the discussion to multi-snake accelerators, in which the snake resonance condition is modified by the snake superperiodicity P_s. At higher snake superperiodicities, there are fewer snake resonances, yet the resonance width is increased. The basic physics remains however unchanged.

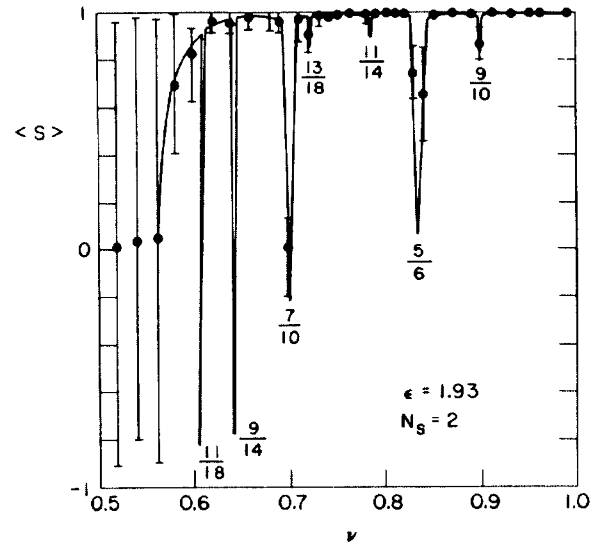

Fig.2 Beam polarization after passage through a single spin resonance is shown as a function of the fractional part of spin resonance tune. Higher order snake resonances are seen clearly.

3.2 Overlapping Resonances

The spin resonance tune and resonance strength are intrinsic properties of the lattice design as well as the beam emittance. Important intrinsic resonances are normally well separated and can be treated as isolated resonances. However intrinsic and imperfection resonances may overlap. Section 3.1 showed that imperfection kicks cancel each other every two turns around accelerator. When an intrinsic resonance is present, the self cancellation mechanism of depolarization kicks disappears. Spin becomes susceptible to depolarization kicks.

Fig. 3 shows tolerable resonance strengths of overlapping imperfection and intrinsic resonances. When the strength of an intrinsic resonance, $|\epsilon_{int}|$, is very small, the tolerable imperfection resonance strength, $|\epsilon_{imp}|$, becomes very large due to the self cancellation mechanism of Section 3.1. However, when $|\epsilon_{int}|$ is slightly increased, tolerable $|\epsilon_{imp}|$ decreases drastically until about $|\epsilon_{imp}|/\frac{N_s}{2} \leq 0.3$, where the imperfection resonance strength plays a minor role in depolarization process. Tolerable intrinsic resonance strength can then be increased until about $|\epsilon_{int}|/\frac{N_s}{2} \leq 0.4$. The tolerable imperfection strength will be $|\epsilon_{imp}|/\frac{N_s}{2} \leq 0.3$. For the SSC and RHIC, we expect

$|\epsilon_{imp}|/\frac{N_s}{2} \leq 0.05$ with 0.3 mm rms closed orbit distortion[3]. Beyond this intrinsic resonance strength, the perturbed spin tune plays a decisive role in determining the tolerable intrinsic and imperfection resonance strengths until the critical resonance strength, $< \epsilon_c >$, is reached. The relationship between tolerable intrinsic and imperfection resonances is also valid for an accelerator with multi-snakes. A plateau for a limiting imperfection resonance is clearly seen on Fig. 3, which indicates the sensitivity of spin to imperfection errors when an intrinsic resonance is present nearby. The sensitivity is clearly due to the disappearence of the self cancellation mechanism. To achieve a higher tolerance to imperfection resonances, we can set a limit on the tolerable intrinsic resonance strength as $|\epsilon_{int}|/\frac{N_s}{2} \leq 0.4$, or $N_s \geq 5|\epsilon_{int}|$.

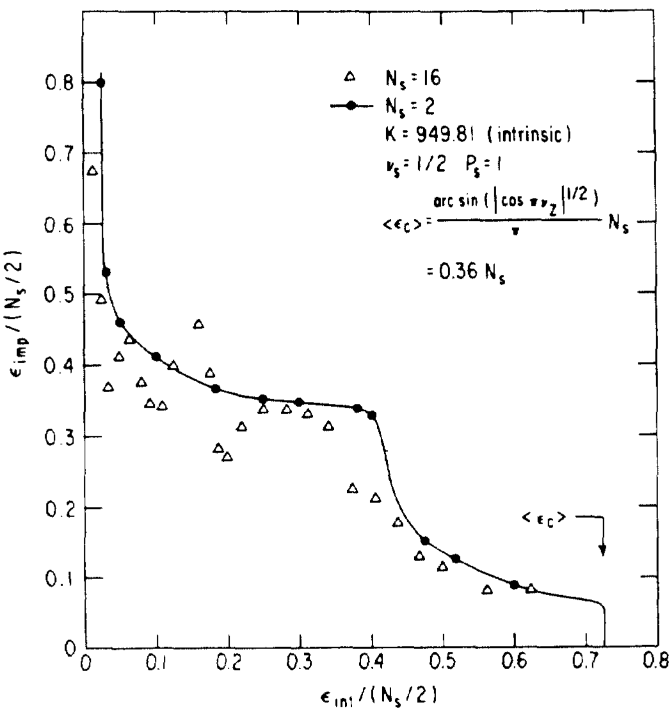

Fig.3 Correlation between tolerable intrinsic and imperfection resonance strengths for $N_s = 2$ and $N_s = 16$. See Section 3.2 for further discussion.

3.3 Snake Imperfections

When the spin rotation angle, ϕ, of Eq.(3.1) deviates from π by $\Delta\phi = \pi - \phi$, the spin transfer matrix of snake becomes,

$$S(\phi_s) = e^{-i\frac{\Delta\phi}{4}\hat{n}_s\cdot\vec{\sigma}} e^{i\frac{\phi}{2}\hat{n}_s\cdot\vec{\sigma}} e^{-i\frac{\Delta\phi}{4}\hat{n}_s\cdot\vec{\sigma}}. \tag{3.8}$$

Thus the spin rotation angle error is equivalent to an imperfection spin resonance, with $\epsilon_{imp}^{eq} = \frac{\Delta\phi}{\pi}$. The corresponding spin tune then becomes energy dependent. To leading order, we obtain

$$\cos \pi\nu_s = \frac{N_s}{2} \cos(2G\gamma\pi/N_s) \sin^2(\frac{\Delta\phi}{2}), \tag{3.9}$$

where the linear dependence of Eq.(3.9) on N_s is due to the assumption that each snake has an identical systematic error in the spin rotation angle. In reality, the snake rotation angle may deviate from π randomly, so the resulting spin tune modulation will not increase linearly with the number of snakes. Tracking calculation shows that the characteristic behavior is similar to that of Fig. 3.

Besides the error in ϕ, the snake axis angle, ϕ_s may also deviate from the ideal situation. The resulting spin tune is again energy independent (Eq.(3.1b)). The snake resonance condition determines the tolerable snake axis angle[3].

3.4 Snake Experiments

Recently, Krisch et al.[6] have carried out successfully a series of experiments in the IUCF Cooler Ring to test snake concept. Using a single solenoid snake, polarized protons have been accelerated through imperfection and intrinsic resonances without losing polarization. The experiments also discovered synchrotron spin resonance in a proton storage ring. Synchrotron spin resonances have played important roles in electron storage rings, but have never before been found in the proton storage ring. Snakes cure synchrotron spin resonance as well. Along with experimental tests with full snakes, the partial snake concept of Roser[7] has also been studied extensively. Indeed partial snakes can be used in low to medium energy machines for correcting imperfection resonances. An interesting question involves the evolution of spin tune when snake is adiabatically turned on. Numerical tracking calculations have been performed to study the problem. A new series of experiments using a solenoid rf kicker[6] have been approved at the Cooler Ring to study more complex problems, such as overlapping resonances, spin tune, etc.

4. Snake Design

Since the invention of the snake idea by Derbenev and Kondratenko, the design of snake and/or spin rotator has become an interesting task. There are many varieties of snake designs[6,8]. For low to medium energy accelerators, Helical type snakes[8] seems to offer advantages in obtaining smaller transverse orbit displacements. At higher energies, snake design is flexibile.

The essential feature of the Steffen snake is the symmetric arrangement of vertical bending magnets and anti–symmetric horizontal bending magnets. These features can be preserved in the following modified snake configuration

$$S_m = (-H,\ -V,\ mH,\ 2V,\ -mH,\ -V,\ H),$$

where $m = 2$ corresponds to the Steffen's snake. The number m is determined by geometry, i.e.

$$(m-1)\left(d + \tfrac{1}{2}(m-1)\ell_x + \ell_y + \ell_g\right) = \ell_x + \ell_y + 2\ \ell_g\ .$$

The spin rotation angle, ϕ, and snake axis angle, ϕ_s, are given by

$$\cos\frac{\phi}{2} = \cos^2\psi_y + \cos m\psi_x \sin^2\psi_y \tag{4.1}$$

$$\cos\phi_s = \frac{-\sin\frac{m\psi_x}{2}\cos\psi_y}{\sqrt{\cos^2\frac{m\psi_x}{2} + \sin^2\frac{m\psi_x}{2}\cos^2\psi_y}} \tag{4.2}$$

Note that $m\psi_x$ and ψ_y are the relevant variables in determining ϕ and ϕ_s. For a partial snake, we have $\phi < \pi$.

When ψ_x, ψ_y are small, one obtain then $\phi \approx 2m\psi_x\psi_y$ and $\phi_s \approx \frac{\pi}{2} + \frac{m\psi_x}{2}$. Fig. 4 shows ψ_x, ψ_y relationship of Eq. (4.1) and ϕ_s vs. ψ_y for $m = 2$. When $\phi_s = 0$, or π, the snake axis is along the radial $\hat{x}$ axis. The compact snake configuration saves about 15% of total length. Besides, total $\int Bd\ell$ and the horizontal orbit displacement, D_x, are also reduced.

By adjusting the parameter m, we can obtain a proper distance $2d$ between two halves of a snake. The orbit displacements at middle of two halves of a snake can be corrected by a orbit shifter of $(-V', V')$ at both ends of a snake. However when the distance $2d$ becomes large, we will have m$\simeq$1. Obviously, the total length and the radial orbital displacement of a snake increases as well. Such a split snake configuration is not practical for an insertion detector area. It can be used in accelerators with two adjacent straight sections separated by a quadrupole. Such a snake configuration eases design criteria of low energy ($\leq$ 30 GeV) accelerators.

To fit a collider interaction region (IR) into the space between the split snake, the snake configuration can be modified[8] to obtain split snake with $m = 2$. The advantage of the split snake configuration is that the spin in the mid section of a snake will be in the horizontal plane. Such a snake therefore serves a dual purpose of being a snake and a spin rotator for helicity state experiments. For a spin up particle passing through a half snake, the spin orientation becomes $S_x = -\sin m\psi_x \sin\psi_y$; $S_y = \sin^2\frac{m\psi_x}{2}\sin 2\psi_y$; $S_z = 0$. Such a scheme can save the need of four spin rotators in a polarized proton experiment.

5. Conclusion

The current understanding of polarized proton acceleration in the high energy accelerator has been reviewed. With proper closed orbit correction, the overlapping resonance between the intrinsic and imperfection resonances can be controlled. The number of snakes needed is found to be proportional to the intrinsic resonance strength, i.e. $N_s \approx 5|\epsilon_{int}|$. The snake imperfection is more important with a large number of snakes. We also discuss the snake design issues. A split snake configuration can serve as a snake and as a spin rotator for the helicity state experiments.

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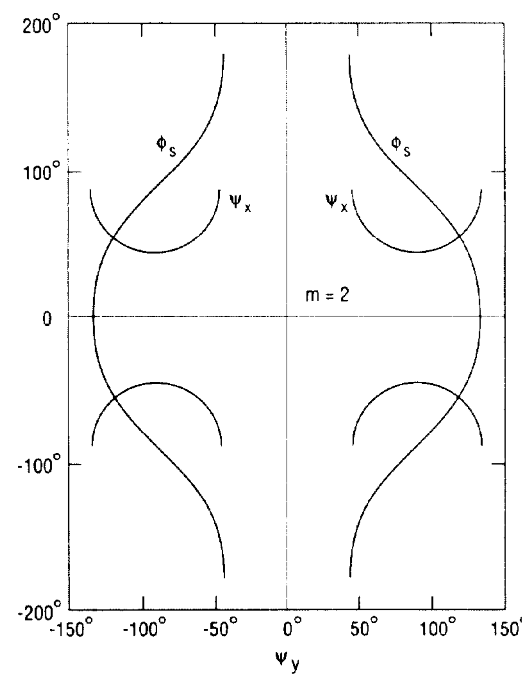

Fig.4 Relation between ψ_x and ψ_y is shown for the Steffen snake configuration. The corresponding snake axis angle is also given.