On Sustaining Short, Intense Bunches in Linear and Circular Accelerators*

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ABSTRACT

The ability of existing analytical and numerical tools to predict beam performance at the short bunch lengths and high peak currents characteristic of contemporary accelerator designs is discussed. Recent advances in calculating the high frequency behavior of impedance and in describing bunched-beam collective dynamics are highlighted. A critical review is presented of outstanding problems that must be addressed before a thorough description of short, intense bunches is obtained.

INTRODUCTION

Performance optimization for linear and circular colliders, FEL drivers, damping rings, and synchrotron light sources often yields configurations with short bunches of high phase space density. For example, sub-centimeter interaction point β^*s are needed in high luminosity collider designs, and the variation of β over the scale β^* in turn demands sub-centimeter interaction lengths and, therefore, sub-centimeter bunch lengths. Chromatic effects can further constrain these highly charged, short bunches to low momentum spread. Similar demands are made on free electron laser drivers where high peak current at low momentum spread is necessary to achieve appreciable gain. Bunches must be longer than the slip distance $N\lambda$ (where N is the number of periods of the wiggler and λ is the wavelength of the radiation), and longer bunches offer better frequency definition (narrower bandwidth). However, for fixed peak current the advantages of reduced total bunch charge, rf phase length, and wakefields make short bunches attractive, and at IR wavelengths and below typical scenarios again involve centimeter and sub-centimeter bunches.

For a relativistic bunch of length $\ell = c\tau$, the width of the frequency spectrum $\Delta\omega$ of the wall currents due to either the gross charge distribution or perturbations will be of order $1/\tau$. A diffractive model of the coupling (machine impedance) of beam-induced fields to vacuum chamber discontinuities suggests a rolloff at frequencies above c/a, where a is the beam pipe radius. The sub-centimeter bunches discussed above are typically transported in multi-centimeter radius beam pipes, and, consequently, the beam coupling varies strongly over the frequency widths of possible collective modes and the rolloff region is sampled.

To fully understand this short bunch regime, which is more typical of electron than proton accelerators, two principal questions must be addressed. First, what is the frequency dependence of the machine impedance at frequencies well above the beam pipe cutoff (or alternatively, the time dependence of the wakefields at distances small compared to the beam pipe radius). Secondly, what is the correct description of collective phenomena for finite length bunches with strongly frequency dependent coupling. During the past few years there has been substantial progress in answering the first question, with various analytic approximations and numerical models yielding consistent conclusions on the scaling laws for high frequency machine impedances. The results have been more mixed with respect to the latter question, with reasonable success in explaining transverse instabilities in storage rings and curing emittance degradation in linacs, but only qualitative agreement with observed longitudinal, single-bunch instabilities in storage rings.

In September 1990 the Fourth Advanced ICFA Beam Dynamics Workshop focused on collective effects in short bunches, and the results presented at that meeting by a number of researchers strongly informs this present review. The proceedings of the workshop was published as a KEK Report[1] and, in addition to the individual papers, it provides overview notes and extensive references which are recommended.

IMPEDANCE BEYOND CUTOFF

In the last few years significant progress has been made in clarifying the asymptotic behavior of impedance in the ultrarelativistic limit $v = c$. First, a variety of approximate approaches[2] have consistently shown that the real (resistive) and imaginary (reactive) parts of the longitudinal impedance Z of an isolated cavity vary as $\omega^{-1/2}$ for high enough frequencies , $\omega \gg c/a$. For an infinitely periodic structure, on the other hand, the resistive impedance is found to rolloff asymptotically as $\omega^{-3/2}$ and the reactive, as ω^{-1}, consistent with causality. A relatively simple description of the transition between the two regimes is given by Gluckstern[3] in terms of the complex admittance per cell $NY_N(k) = N/Z_N(k)$ for $k = \omega/c$ and N cells:

$$NZ_0Y_N(k) \cong Z_0Y_1(k) + \alpha\sqrt{N-1}\ \arctan\left(\frac{\alpha}{2\sqrt{N}}\right) \quad (1)$$

where

$$\alpha = \frac{(1+j)a\sqrt{\pi k}}{\sqrt{L}} \quad (2)$$

and where $Z_0Y_1(k)$ is the averaged admittance for a single cell

$$Z_0Y_1(k) = \frac{(1+j)a\sqrt{\pi k}}{\sqrt{g}} \quad (3)$$

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The cell gap is g and the intercell spacing is L. For $N \to \infty$

$$N Z_0 Y_N(k) \to \frac{(1+j)\pi a\sqrt{\pi k}}{\sqrt{g}} + \frac{j\pi k a^2}{L} \quad (4)$$

and $\omega^{-3/2}$ dependence is obtained for the resistive part of the impedance if the second term is large compared to the first; *i.e.*, $ka^2 \gg L^2/g$. The derivation demands that $ka^2 \gg L$. It would appear that for large spacing L the impedance returns to a sum of single cavities values. If, however N is held fixed and the limit $k \to \infty$ is taken

$$N Z_0 Y_N(k) \to \frac{(1+j)\pi a\sqrt{\pi k}}{\sqrt{g}} \left[1 + \frac{\sqrt{g(N-1)}}{2\sqrt{L}}\right] \quad (5)$$

Note that for $N \gg L/g$ the impedance per cavity is reduced by a factor $1/\sqrt{N}$.

The analogous longitudinal couplings for dipole and higher modes, which are excited by offset beams, have been found to exhibit (up to constants) the same behavior. Neither wall resistivity nor beam pipe curvature has been included in the analyses to date, and it must be pointed out that all of the results have involved some level of approximation. Iterative methods or smoothing may not be convergent, and truncation of matrices and finite mesh size may introduce spurious behavior, but the fact remains that a broad range of approximate methods agree on the basic asymptotic frequency behavior of impedance. A rigorous result for some closed geometry with beam pipe, unfortunately, has yet to be achieved.

Experimentally, the clearest evidence of $\omega^{-1/2}$ behavior comes from reinterpretation of an ISR experiment by Hoffmann, Risselada, and Zotter.[4] It is argued that parasitic losses are due to single protons individually interacting with the machine impedance. The interaction frequency is determined by the width of the field lines at the wall, $\Delta\omega \sim c\gamma/a$. For high energies (31.4 GeV) frequencies over 60 GHz are sampled. Measurements of energy loss at three different energies (3.6 GeV, 15.4 GeV and 31.4 GeV) are consistent with the $\omega^{-1/2}$ behavior of isolated cells.

LOSS FACTORS AND WAKE POTENTIALS

The longitudinal and transverse wake potentials (W_ℓ and W_t, respectively) are the effective Green functions for beam self-interaction in a quasistatic limit. They are Fourier conjugate to the impedance functions. The associated loss factors, k_ℓ and k_t, are averages of the respective wake potentials over a given particle distribution, and are, in particular, functions of the rms bunch length σ. For $\omega^{-1/2}$ behavior, we have

$$k_\ell \propto \sigma^{-1/2} \quad (6)$$
$$k_t \propto \sigma^{1/2} \quad (7)$$
$$W_\ell \propto \tau^{-1/2} \quad (8)$$
$$W_t \propto \tau^{1/2} \quad (9)$$
$$k_\ell \propto \sigma^{0} \quad (10)$$
$$k_t \propto \sigma^{1} \quad (11)$$
$$W_\ell \propto \tau^{0} \quad (12)$$
$$W_t \propto \tau^{1} \quad (13)$$

where τ is the distance behind the exciting charge, and τ and σ are assumed small.

As is clear from equations (6-13), extrapolations of measurements performed with relatively long bunches and of numerical models at the limits of computer capacity depend on which asymptotic regime is applicable. Consider for example, the choice of bunch length in a linear collider. If $\omega^{-1/2}$ behavior is realized, then exceedingly short bunches would appear unattractive since energy losses and bunch-induced energy spreads (of order $2k_\ell Q$) would be exacerbated while transverse wakes would only be modestly reduced. It is also noted that $\tau^{-1/2}$ small time behavior for W_ℓ yields somewhat more curvature energy spread than does a constant longitudinal wake. If $\omega^{-3/2}$ behavior is obtained, the transverse wakes would be strongly reduced with short bunch length with little impact on energy loss and energy spread. Although pulsed, room temperature linacs such as SLAC appear to be safely in the latter regime, the wide spacing L of standing wave superconducting cavities in superconducting linacs may not satisfy the condition $a^2/\sigma \sim ka^2 \gg L^2/g$ for equation (4) to yield $\omega^{-3/2}$ rolloff.

BEAM DYNAMICS OF LINACS AND STORAGE RINGS

For the relatively short time a bunch is in a relativistic linac, the longitudinal motion is essentially frozen. Longitudinal electric fields can change the energy of beam particles, but since there is no slip associated with energy offset, current modulations are not induced. Thus, the principal longitudinal concerns are energy spreads induced by the gross charge distribution. Transversely, there can be amplification from the head to tail of the bunch since transverse wakes can induce betatron oscillations which can in turn excite further wakes. This is described by[5]

$$\begin{aligned}\frac{d}{ds}\left(\gamma(s)\frac{d}{ds}x(z,s)\right) + \left(\frac{2\pi}{\lambda(s)}\right)^2\gamma(s)x(z,s) \\ = r_0\int_z^\infty dz'\rho(z')W_t(z'-z)x(z',s)\end{aligned} \quad (14)$$

The emittance degradation is dominated by the the transient amplification of the largest perturbations, element alignment and jitter, which are of relatively low frequency compared to typical bunch lengths. In this regime, where the perturbation $x_0(z,s)$ is independent of z along the bunch, an energy variation from head to tail can cancel the wake force through chromatic variation of the focusing strength. This, of course, is the principle of BNS damping or autophasing which has been successfully applied to SLAC. Note that this effect scales with $\Delta\gamma$, not $\frac{\Delta\gamma}{\gamma}$,[6] for fixed betatron wavelength, and is therefore more effective at high energies.

In any case, the frozen longitudinal motion and short linac propagation time allows for effective numerical simulation with the beam bunch divided into slices which can be successively updated for ultra relativistic wakes. Together with the better estimates as described above of the short time or high frequency behavior of beam coupling, the necessary fundamental numerical modeling tools are well in hand. The situation for storage ring bunches remains more clouded in spite of considerable work. The fundamental difference is clearly the importance of longitudinal motion in the dynamics of storage ring bunches. Synchrotron motion is an important ingredient, but the principal physics is in the fact that perturbations in energy can lead to current variation which can in turn excite wakefields. An additional feature is the need to address long term stability rather than transient growth.

MODE COUPLING ANALYSIS

Internal bunch instabilities, both transverse and longitudinal, have provided a fundamental limitation in the design of short-pulse-length synchrotron light sources, high-phase-space-density damping rings, and single pass FEL drivers. Although several formalisms have been developed to describe this class of beam instability, they share a common structure. The starting point is typically the linearized Vlasov equation,

$$\frac{\partial f}{\partial t} + \dot{x}\frac{\partial f}{\partial x} + \dot{p}_{ext}\frac{\partial f}{\partial p} + \frac{\partial f_0}{\partial p}G[f] = 0 \qquad (15)$$

where x and p are appropriate generalized coordinates, $\dot{p}_{ext}$ is the external focusing, and $G[f]$ is some linear functional acting on the distribution f. For a coasting beam f_0 is independent of x, and Fourier analysis yields a simple algebraic equation. For a bunched beam, however, f_0 is no longer independent of x, and Fourier analysis yields a convolution integral.

Typically, a set of basis states (possibly degenerate) is chosen with the higher states corresponding roughly to shorter wavelength internal ripples. For each mode there is an associated eigenfrequency $m\omega_s$, a multiple of the synchrotron frequency. The impedance and beam current generate an additional interaction between the states which is expressed as a perturbing matrix generated by expectation values of impedance in the space of the eigenstates. In general reactive impedance can couple a basis element to itself and generates diagonal frequency shifts as its leading term. Resistive impedance provides the primary coupling between neighboring states. In this manner, the integral equation implicit in the Vlasov equation for a finite length beam is converted into an infinite dimensional matrix equation.

Determination of the threshold current for longitudinal and transverse instability ostensibly requires solution of an infinite dimensional matrix problem. In practice, the matrix is truncated and numerically diagonalized. Instability can evolve in a number distinct ways. First, modes (which at zero current are spaced by the synchrotron frequency) can be shifted as a function of current by the diagonal elements of the perturbation matrix. Modes of the right class can couple when their frequencies match, and can yield instability if there exists a nonzero off-diagonal resistive coupling. For transverse instabilities in storage rings this picture appears to give a reasonable description of experiment, with the lowest m=0 mode shifted until it collides with the m = -1 mode. The spectrum of the m=0 mode primarily samples the better known, lower portions of the impedance spectrum. Spectral shifts and stability enhancement from chromaticity are also observed and are in reasonable agreement with theory.[7] Predictions based on higher modes are more problematic.

For longitudinal dynamics the lowest $m = \pm 1$ mode ($m = 0$ corresponds to the unperturbed distribution) significantly samples impedance in the rolloff region for short bunches where the reactive impedance is changing sign and there is a strong resistive component. Whether bunch lengthening or shortening is predicted is particularly sensitive to assumptions about the impedance spectrum. Calculation of mode shifts is less certain and, in fact, the onset of instability is not typically associated with mode shifts.[8] A second method for generating instability in a matrix theory is for the off diagonal resistive elements to become comparable in size to the unperturbed mode spacing. For coupling of two neighboring modes, shifts in frequency would be expected, but this may not be the case when many modes are involved. The success of the infinite matrix approach for the transverse mode coupling instability depends very strongly on the *very finite* dimensionality ($m = 0$ to $m = -1$) of the underlying physics. Experience with the longitudinal instability suggests that its solution may not be so well behaved, and results have not been significantly more predictive than simple "massaged" scaling laws.

OPERATOR EQUATIONS AND INFINITE MATRICES

Successful conversion of an integro-differential equation and its associated eigenvalue problem into a truncated infinite matrix equation depends on both the boundedness of the operators and the basis set chosen for the expansion. Typically, one is provided with necessary conditions for convergence of the approximation scheme, but a particular operator may, in fact, submit to the approximation even if it does not satisfy such a condition. Such *luck* requires experimental confirmation. It will be argued heuristically in this and the following section that the matrix techniques used to describe longitudinal bunch instabilities do not satisfy some of the simplest necessary conditions for convergence. This is particularly the case for impedances which asymptotically scale as $\omega^{-1/2}$.

Intuition from finite dimensional matrices to infinite dimensional operators is clearest for a subset of bounded operators called *completely continuous*. Such operators map bounded sequences of vectors to convergent sequences of vectors;[9] in other words, the matrix elements M_{mn} fall off rapidly with m and n. For example, the identity operator is bounded, but not completely continuous. Simple differentiation corresponds to multiplication by harmonic number (or matrix index) n in a Fourier series decomposition, and is neither bounded nor completely continuous. For completely continuous operators, one can be rather cavalier in the choice of basis set.

For operators whose elements either grow, tend to constants, or decrease too slowly (*e.g.*, inverse square root) with index, infinite matrix decompositions are not generally well behaved except for carefully selected basis sets.[10] These basis sets should share, for example, boundary values or boundary behavior with the underlying problem. Other choices of basis can lead to misleading results.

Consider, for example, the Legendre equation

$$\frac{d}{dx}(1-x^2)\frac{df}{dx} = -\omega^2 f \tag{16}$$

which describes space charge waves on a cold, parabolic bunch in the approximation of the electric field being proportional to the derivative of the longitudinal charge distribution. As a simple test of matrix truncation, this system was analyzed using a simple Fourier series in sines and cosines. The infinite matrix generated was truncated and numerically solved. Diagonalization produced good values ($\sim n(n+1)$) for the lowest even eigenvalues at a reasonably small matrix dimension, but the lowest odd eigenvalues had failed to converge even for a 180 by 180 matrix. The symmetry and nodes of the eigenfunctions, however, were suggestive of Legendre polynomials. Apparently the zero boundary condition of the sine functions has introduced convergence problems in the expansions of the Legendre polynomials, which are nonzero at the bunch ends. The Legendre operator has one other feature of note – it has positive expectation values, and it is this fact which allows the infinite matrix problem to work with carefully chosen basis sets. The Legendre model above represents a finite bunch with a sharp, but physically interesting interaction, and may exhibit more singular behavior than smoother distributions and interactions.

The question, then, with respect to the bunched beam problem is whether the underlying infinite matrix is either 1) completely continuous and susceptible to an arbitrary basis expansion or 2) well-behaved, but requiring a carefully matched basis set, or 3) pathological.

SINGULARITY OF LONGITUDINAL EQUATIONS

For frequency shifts Ω large compared to the synchrotron frequency, the matrix equation for longitudinal bunch motion takes a particularly simple form, which is sufficient to illustrate the issue. Following Wang,[11] we expect an infinite dimensional equation for high harmonic number of the form

$$\rho_m = \sum_{n=-\infty}^{\infty} T_{mn}\rho_n \tag{17}$$

where

$$T_{mn} = \frac{IZ(n)}{n}\lambda_{m-n} H\left(\frac{\Omega}{\sqrt{mn}}\right) \tag{18}$$

$$H\left(\frac{\Omega}{\sqrt{mn}}\right) \propto \int_{-\infty}^{\infty} d\omega \frac{g'(\omega)}{\frac{\Omega}{\sqrt{mn}} - \omega} \tag{19}$$

and I is the average current, $Z(n)$ is the impedance at harmonic n, and λ_n is the Fourier coefficient of the unperturbed charge distribution. Note that for $\Omega/\sqrt{mn}$ large compared to the width of the revolution frequency distribution g, $H \propto mn$. This implies, for example, that although the diagonal elements T_{nn} will scale as $Z(n)/n$ for small frequency shifts, for sufficiently large shifts the scaling can be as strong as $nZ(n)$. Consider now, the two asymptotic forms of impedance described earlier. In the infinite periodic limit, the leading behavior will be a reactive $1/n$ for $Z(n)$. In the limit $\Omega \to \infty$ T_{nn} remains bounded, but does not rolloff. This situation is marginal for convergence of arbitrary series expansions. For the isolated cavity, both the reactive and resistive impedance falls as $1/\sqrt{n}$, and for large shifts T_{nn} tends to a $\sqrt{n}$ behavior. Since this $\omega^{-1/2}$ scaling, as discussed earlier, is most likely for storage rings, this estimate suggests that special care must be taken in choosing basis sets. The large resistive term, in fact, may prevent the process from converging at all at large values of current. Typically, convergence becomes problematic when the self-adjoint (in this case, the reactive part) is not dominant.

It should be noted that for finite γ, the impedance is sharply rolled-off at frequencies greater that $\gamma c/a$, and the matrix is indeed finite but exceedingly large in dimension. Also, for fixed Ω, T_{mn} does finally rolloff at sufficiently large index, say n_Ω. However, n_Ω increases with increasing Ω. The primary lesson to be drawn from from the discussion is that matrix truncation can be misleading, and that physical solutions, at a minimum, may require matrices of a dimension considerably larger than first expected.

OTHER APPROACHES

The lack of startling quantitative success for truncated matrix methods may be due to numerical difficulties. On the other hand, it may be the case that some important physics is missing from the model, and there has been some activity with this perspective. Clearly, the first concern would be that the impedance function itself is not well estimated. It seems to be the case, however, that fitting to magnitude and shape leads to internal inconsistency, for example, between threshold current and parasitic losses.[12] Oide and Yokoya[13] argue quite legitimately that the impact of potential well distortion of the underperturbed distribution must be included, and have produced a Vlasov matrix analysis (using a new set of basis states) which indicates that threshold behavior can be dramatically altered by inclusion of potential well distortion in a self-consistent manner. Agreement appears good between this theory and simulation for some specific impedance shapes. Of particular note is that a strong capacitive component enhances thresholds by shortening the bunch and preventing deterioration of the incoherent synchrotron frequency. A thermodynamic approach to turbulent bunch lengthening has been proposed by Meller.[14] The free energy of numerically-obtained, time-dependent distributions is compared to the conventional stationary Maxwell distribution for impedances with a resistive component. It is found that above a threshold current a time-dependent solution has a lower free energy and is therefore the preferred state for the beam. This transition is interpreted as the threshold for turbulent bunch lengthening. Comparison to SPEAR observations and simulation are promising. Hirata[15] also finds time dependent solutions, but for localized structures, and demonstrates that they can be the preferred state of the beam. More effort, both analytic and numerical, should be focused on such nonperturbative methods. Finally, many computer simulations[8] have been implemented and the general features of bunch lengthening with momentum growth have been observed. At least qualitative agreement with experiment is found. From the arguments in previous sections, some care may be necessary to model the isolated cavity with $\omega^{-1/2}$ asymptotic behavior.

SYNCHROTRON RADIATION IMPEDANCE

For small storage rings there may be another important source of coherent interaction of the beam with its environment – the synchrotron radiation process. Synchrotron radiation in bends and wigglers is suppressed at frequencies below a cutoff value many times the TM mode cutoff. For a chamber consisting of two infinite parallel plates, for example, separated by $2h$, the synchrotron radiation power takes on free space values only for frequencies ω satisfying[16]

$$\omega \gtrsim \frac{c}{\rho}\left(\frac{\rho}{h}\right)^{3/2} \tag{20}$$

where ρ is the bending radius. If this power loss is interpreted as an effective resistance, in the spirit of machine impedances, the peak value of

$$\frac{Z}{n} \simeq 300\frac{h}{R}\text{ohms} \tag{21}$$

where R is the average radius of the machine.

Taken at face value, this impedance could be the limiting component for small machines. Consider a compact synchrotron light source with $\rho = 1$ meter, $R = 2$ meters, and vacuum chamber half height of 1 centimeter. Then a maximum value of Z/n of 1.5 ohms is obtained at a frequency $\omega = (2\pi)50$ GHz. For short bunches where the impedance from vacuum chamber discontinuities is rolling off, the synchrotron impedance could be dominant. However, the synchrotron-radiation-induced coherent interaction may be of a different character. Wavelengths are now of the order of the transverse beam dimensions, the interaction occurs over an extended region (bending magnets and wigglers), and synchrobetatron coupling is inherent in the process. Much work has been done on the electromagnetics of synchrotron radiation,[17],[18] primarily in analysis of the resonance structure of a toroidal beam pipe. The beam dynamics has received only rudimentary attention. With the commissioning of small, clean compact synchrotron light sources, some experiments may be possible.

CONCLUSIONS

There has been much analytic progress in understanding the high frequency behavior of accelerator impedance. Analysis of beam dynamics has been successful for low-order transverse mode coupling, but there is no cogent analysis of longitudinal turbulent bunch lengthening. For single-cavity impedances, the interaction may be too singular to be well treated by a small-dimensioned matrix approach, and more global treatments may hold the best promise for prediction. Computer simulations have been exceedingly successful in modeling linac beam dynamics, where both the finite duration of beam propagation and frozen longitudinal motion with causal wakes ease calculations. For storage rings, it remains that the compute-efficient codes necessary for simulating many turns require compromises in electromagnetic modeling which limit quantitative agreement with experiment. Finally, the effects of coherent synchrotron radiation on beam stability remains an open question that could soon be addressed experimentally in compact synchrotron light sources.

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