Tuning and Coupling Mismatch Tolerance in Cavities Driven by a Quadrature Hybrid[1]

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Abstract

If cavities having identical complex reflection coefficients are driven by a quadrature hybrid, all of the reflected power appears at the terminated port of the hybrid. Mismatch of tuning angle or coupling coefficient results in voltage standing wave ratio greater than unity in the hybrid input port. In this paper, allowed mismatch is related to the allowed standing wave ratio.

I. INTRODUCTION

An attractively simple and economical drive concept for accelerator cavities employs quadrature hybrids to drive cavities in pairs (Fig. 1). This protects the klystron from reflected power from the cavities during ordinary (non-fault) operation, so long as their complex reflection coefficients and beam loading are equal, without recourse to high power circulators. assuming a perfectly balanced split and correctly matched phase lengths in the drive legs to the two cavities, the voltage standing wave ratio (VSWR) seen by the klystron will be entirely due to differences in the tuning, coupling, and beam loading of the two cavities. Here, we set out to find the allowable mismatches in tuning and coupling for a given maximum VSWR, assuming no beam present.

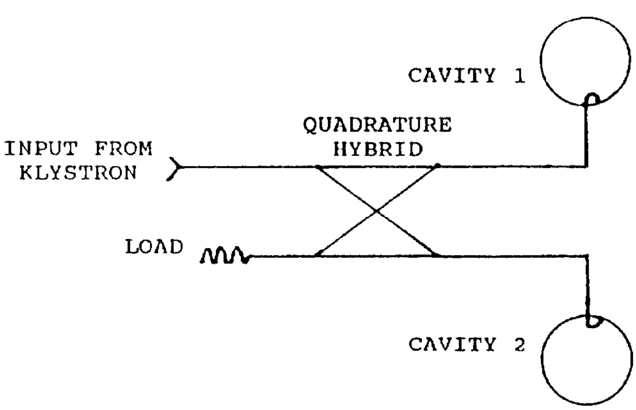

Figure 1. Cavity drive circuit schematic.

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II. BACKGROUND: Q CIRCLE, REFLECTED VOLTAGE AND TUNING ANGLE

As drive frequency is swept upward through resonance, the complex reflection coefficient of a cavity moves along a circle tangent to the unit circle at the left side of the Smith chart [1]. This circle is called the Q circle, presumably because it contains information from which the loaded, unloaded, and external Q's of the cavity can be calculated. Figure 2 shows a schematic representation of a Q circle, defining various angles, phasors, etc. used herein.

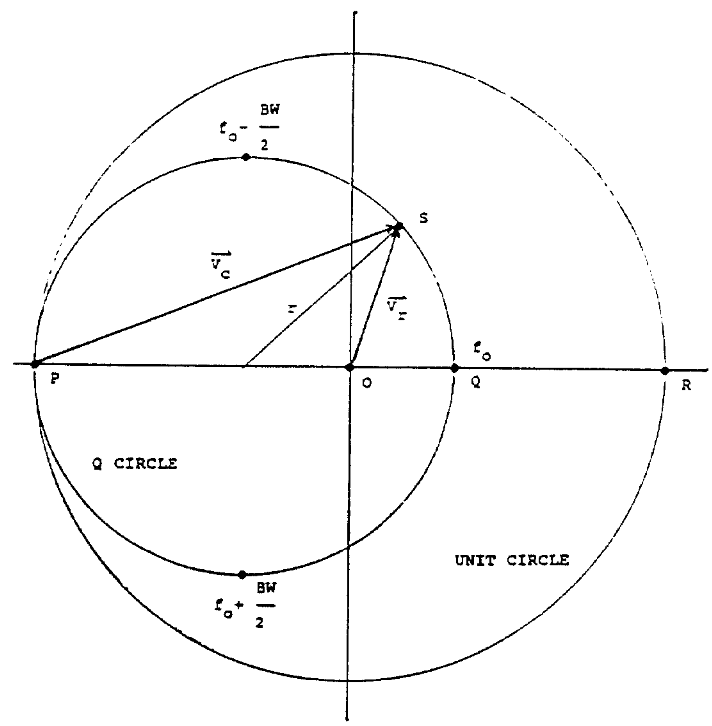

Figure 2. Voltage phasors for a cavity resonant at f_o, having a bandwidth BW. Point S moves clockwise around the Q circle as f increases.

Note in particular that the reflected voltage phasor V_r is the complex sum of the voltage V_c from inside the cavity and the negative unit phasor representing the reflection from the coupling aperture itself:

$$V_r = V_c - 1 \quad . \tag{1}$$

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In order to optimize efficiency at full beam current, accelerating cavities are made to be overcoupled when no beam is present; for such cavities, the standing wave ratio on resonance equals the coupling coefficient β. In Fig. 2, the forward voltage phasor is a vector of unit length (i.e., the length PO) from left to right. The standing wave ratio σ (on resonance) is obviously the ratio of the length PQ to the length QR.

$$\beta = \sigma = \frac{\overline{PQ}}{\overline{QR}} = \frac{2r}{2-2r} \ . \tag{2}$$

This can be inverted to obtain the Q circle radius in terms of the coupling coefficient.

$$r = \frac{\beta}{1+\beta} \ . \tag{3}$$

The angle ψ between the real axis and phasor V_c is called the tuning angle, and is given in terms of the difference between resonant frequency f_0 and drive frequency f by

$$\tan\psi = \frac{2(f_0 - f)}{\text{Bandwidth}} = \frac{2Q_L(f_0 - f)}{f} \ , \tag{4}$$

where Q_L is the loaded Q of the cavity. The tuning angle is therefore a measure of the resonant frequency error.

Reflected Voltage Phasor

In Fig. 2, V_c is readily seen to be given by

$$V_c = r + re^{i2\Psi} \ . \tag{5}$$

Thus, Eqns. (1), (3), and (5) can be combined to give the reflected voltage phasor:

$$V_r = \frac{\beta e^{i2\Psi} - 1}{\beta + 1} \ . \tag{6}$$

Reflected Phasors at the Hybrid Input Port

Assuming an ideal quadrature hybrid, each of the two cavity feed waveguides will have a forward voltage amplitude reduced by $1/\sqrt{2}$ from the forward voltage in the hybrid input. Each of the reflections from the cavities produces a reverse wave at the input and load ports which is reduced by another factor of $1/\sqrt{2}$, so that the overall normalization factor for reflections in the input port is 1/2. Furthermore, the reflection from the second cavity, as seen in the hybrid input port, undergoes a phase shift of 180° which is equivalent to a sign change. Thus, normalized to the forward wave in the feed guide from the klystron to the hybrid, the reverse voltage phasor due to the cavity pair is

$$V_{rh} = \frac{V_{r1} - V_{r2}}{2} = \frac{\beta_1 e^{i2\Psi_1} - 1}{2(\beta_1 + 1)} - \frac{\beta_2 e^{i2\Psi_2} - 1}{2(\beta_2 + 1)} \ , \tag{7}$$

where subscripts 1 and 2 refer to cavities 1 and 2.

III. CASE I: TUNING MISMATCH WITH EQUAL COUPLING

At this point we make the assumption that the cavity coupling coefficients are practically equal. The modulus of the reflected phasor can be shown to be given by

$$|V_{rh}| = \frac{\beta \sin(\delta\Psi)}{\beta + 1} \ , \tag{8}$$

where

$$\delta\Psi = |\Psi_1 - \Psi_2| \ .$$

Relation to Input Standing Wave Ratio

The standing wave ratio in the hybrid input is

$$\sigma_h = \frac{1 + |V_{rh}|}{1 - |V_{rh}|} = \frac{1 + \beta\left[1 + \sin(\delta\Psi)\right]}{1 + \beta\left[1 - \sin(\delta\Psi)\right]} \ . \tag{9}$$

With [β] as a parameter, Eqn. (9) is plotted over the range from 1 to 1.6 (covering the range of typical allowable klystron load mismatches) in Fig. 3.

Equation (9) may be inverted to find the allowed tuning angle difference for cavities of a given coupling:

$$\delta\Psi = \sin^{-1}\left[\frac{1+\beta}{\beta}\ \frac{\sigma_h - 1}{\sigma_h + 1}\right] \ . \tag{10}$$

IV. CASE II: COUPLING MISMATCH WITH EQUAL TUNING

We now assume the cavities have equal tuning angles ψ, but different coupling coefficients. Equation (7) reduces to

$$V_{rh} = \frac{e^{i2\Psi}+1}{2}\left[\frac{1}{\beta_2+1}-\frac{1}{\beta_1+1}\right] . \tag{11}$$

The modulus of this phasor is maximized for cavities resonant at the driving frequency ($\psi = 0$), for which case

$$V_{rh} = \frac{1}{\beta_2+1} - \frac{1}{\beta_1+1} . \tag{12}$$

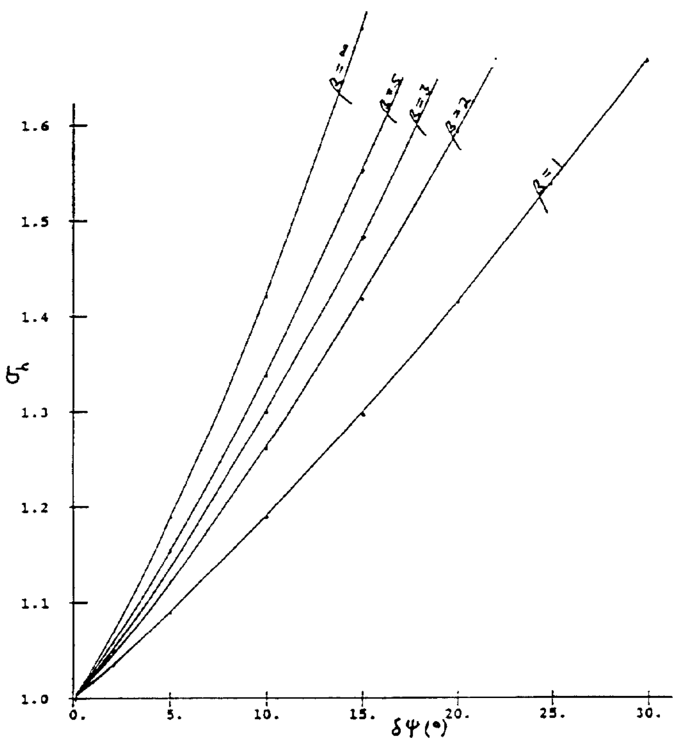

Figure 3. Standing wave ratio at hybrid input plotted against tuning angle difference for various coupling coefficients.

If we define coupling mismatch $\delta\beta$ as the difference between coupling coefficients,

$$\delta\beta = \beta_2 - \beta_1 ,$$

then the reflected voltage phasor modulus can be expressed in terms of this difference:

$$V_{rh} = \frac{-\,\delta\beta}{(\beta_1+1)^2 + \delta\beta\,(\beta_1+1)} . \tag{13}$$

This result can be used to find the coupling mismatch corresponding to a specified standing wave ratio. If $\beta_2 > \beta_1$, then

$$\delta\beta_+ = \frac{(\sigma_h - 1)(\beta_1+1)^2}{2 - \beta_1(\sigma_h - 1)} ; \tag{14a}$$

if $\beta_2 < \beta_1$, then

$$\delta\beta_- = \frac{(\sigma_h - 1)(\beta_1+1)^2}{\beta_1 - \sigma_h(\beta_1+2)} . \tag{14b}$$

V. REFERENCE

[1] E. L. Ginzton, Microwave Measurements, New York: McGraw-Hill, 1957 ch 10.