On-line Correction of Aberrations in Particle Spectrographs

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Abstract

A new method is presented that allows the reconstruction of trajectories and the on-line correction of residual aberrations that limit the resolution of particle spectrographs. Using a computed or fitted high order transfer map that describes the uncorrected aberrations of the spectrograph under consideration, it is possible to determine a pseudo transfer map that allows the computation of the corrected data of interest as well as the reconstructed trajectories in terms of position measurements in two planes near the focal plane.

The technique is only limited by the accuracy of the position measurements and the accuracy of the transfer map. In practice the method can be expressed as an inversion of a pseudo transfer map and implemented in the differential algebraic framework. The method will be used to correct residual high aberrations in the S800 spectrograph which is under construction at the National Superconducting Cyclotron Laboratory at Michigan State University.

1 Introduction

Efficient modern high-resolution mass spectrographs usually offer rather large phase space acceptances. One such spectrograph is the S800 currently under construction at Michigan State University's National Superconducting Cyclotron Laboratory [1, 2]. Such large acceptance high resolution spectrographs usually require a careful consideration and correction of aberrations. But because of the large phase space acceptance, effects of rather high orders contribute. This makes the correction process often considerably more difficult and complex, and sometimes even prevents a complete correction of aberrations in the conventional sense.

It is often possible to circumvent or at least alleviate these problems by using additional information about the particles. In particular, one often measures not only their final position but also their final angle by means of a second detector. With this additional information it is to some degree possible to retroactively construct the whole trajectory of the particle. This information can be used both for the numerical correction of the quantities of interest, but it also reveals additional properties like the initial angle, which is of course of interest in the study of many nuclear processes.

In the past such trajectory reconstruction techniques were quite involved, often requiring extensive ray tracing and the storage of large arrays of ray data and extensive interpolation. In this paper, we present a rather direct and efficient method based on differential algebraic (DA) techniques.

In recent years we have shown that maps of particle optical systems can be computed to much higher orders than previously possible using DA methods [3, 4, 5, 6]. Furthermore, the techniques also allow the accurate treatment of very complicated fields that can be treated only approximately otherwise. In our particular case, these include the fringe fields of the large aperture magnets required for such particle spectrographs. So for the first time there is now the possibility to really compute all the aberrations that comprise a modern high resolution spectrograph without having to rely on tedious ray tracing.

On the practical side this requires high order codes for the computation of highly accurate maps for realistic fields. The new code COSY INFINITY [7, 8, 9, 10] allows such computations in a very powerful language environment. It also has extensive and general optimization capabilities, supports interactive graphics and provides ample power for customized problems, and it provides all the necessary tools for efficient trajectory reconstruction.

In the next section, we will discuss an important algorithm for this task, the inversion of transfer maps. Section 3 outlines the use of map inversion techniques for the purposes of trajectory reconstruction. Section 4 provides an outlook for the practical application in connection with the S800 spectrograph.

2 Inversion of Transfer Maps

At the core of the operations that follow is the need to invert transfer maps in their DA representation. Though at first glance this appears like a very difficult problem, we will see that indeed there is a rather elegant and closed

*Supported in Part by the U.S. National Science Foundation, Grant Number PHY 89-13815
0-7803-0135-8/91$01.00 ©IEEE

algorithm to perform this task.

We begin by splitting the map A_n into its linear and nonlinear parts:

$$A_n = A_{1n} + A_{2n}. \tag{1}$$

Furthermore, we write the sought for inverse as M_n.

$$A^{-1n} = M_n \tag{2}$$

Composing the functions, we obtain

$$\begin{aligned} (A_1 + A_{2n}) \circ M_n &= E_n \Rightarrow \\ A_1 \circ M_n &= E_n - A_{2n} \circ M_n \Rightarrow \\ M_n &= A_1^{-1} \circ (E_n - A_{2n} \circ M_{n-1}). \end{aligned} \tag{3}$$

Here "o" stands for the composition of maps. In the last step use has been made of the fact that knowing M_{n-1} allows us to compute $A_{2n} \circ M_n$. The necessary computation of A_1^{-1} is a linear matrix inversion.

Equation (3) can now be used in a recursive manner to compute the M_i order by order.

3 Trajectory Reconstruction

The result of the computation of the transfer map of the system allows us to relate final quantities to initial quantities and parameters. In our case, the relevant quantities are the positions in x and y directions as well as the measures of slopes p_x/p_0, p_y/p_0 and the energy of the particles under consideration. Usually the initial x, which is determined by the target thickness or a subsequent slit, is kept small to provide a minimal entrance width. So the final positions and slopes are primarily determined by the energy, and to higher orders also by the initial y position and the initial slopes.

In the full transfer map we now set x_i to zero and consider the following submap:

$$\begin{pmatrix} x_f \\ a_f \\ y_f \\ b_f \end{pmatrix} = \mathcal{S} \begin{pmatrix} a_i \\ y_i \\ b_i \\ d \end{pmatrix} \tag{4}$$

This map relates the quantities which can be measured in the two planes to the quantities of interest. The map $\mathcal{S}$ is not a regular transfer map, and in particular its linear part does not have to be a priori invertible. In a well designed particle spectrograph, the linear part has the following form:

$$\begin{pmatrix} x_f \\ a_f \\ y_f \\ b_f \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 & * \\ * & 0 & 0 & * \\ 0 & * & * & 0 \\ 0 & * & * & 0 \end{pmatrix} \cdot \begin{pmatrix} a_i \\ y_i \\ b_i \\ d_i \end{pmatrix} \tag{5}$$

where a star denotes an entry that is not zero. Since the system is imaging, clearly (x,a) vanishes, and all the other

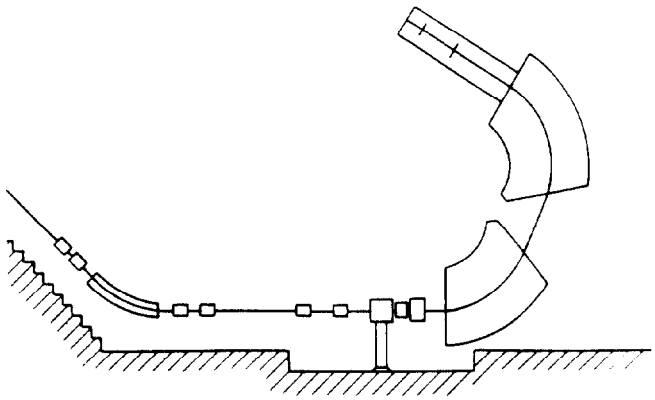

Figure 1: The vertical layout of the S800 spectrograph

zero terms vanish because of midplane symmetry. (x,d) is maximized in spectrograph design, and (a,a) cannot vanish in an imaging system because of symplecticity. In fact, to reduce the effect of the finite size entrance slit, (x,x) is minimized within the constraints, and so (a,a) = 1/(x,x) is also maximized.

Because of symplecticity, (y,y)(b,b)-(y,b)(b,y) = 1, and so we obtain for the total determinant of S:

$$|\mathrm{S}| = (x,d) \cdot (a,a) \neq 0, \tag{6}$$

besides being nonzero, the size of the determinant is also a good measure of the quality of the spectrograph: the larger the better.

So certainly the linear matrix is invertible, and according to the last section, this entails that the whole nonlinear map $\mathcal{S}$ is invertible to arbitrary order, and thus it is possible to compute the initial quantities of interest to arbitrary order.

A closer inspection of the algorithm shows that in each iteration, the result is multiplied by the inverse of the linear matrix S. Since the determinant of this inverse is the inverse of the original determinant and is thus quite small, this entails that the originally large terms in the nonlinear part of the original map are more and more suppressed. So clearly even with trajectory construction, the original investment in the quality of the spectrograph, which is determined by its dispersion and its x demagnification, directly influences the quality of the trajectory reconstruction.

4 The Correction of Aberrations in Spectrographs

The proposed superconducting magnetic spectrograph, the S800 [1] shown in fig. 1, for the National Superconducting Cyclotron Laboratory will allow the study of heavy ion reactions with magnetic rigidities of up to 1.2 GeV/c. It will have an energy resolution of one part in 10000 with a

Table 1: The S800 Spectrograph

Drift	l = 60 cm
Quad	l = 40 cm, G_{max} = 21 T/m , d = .01 m
Drift	l = 20 cm
Quad	l = 40 cm, G_{max} = 6.8 T/m, d = .02 m
Drift	l = 50 cm
Dipole	r = 2.6667 m, $B_{max} = 1.5T$, ϕ = 75 deg, ϵ_1 = 0 deg, ϵ_2 = 30 deg
Drift	l = 140 cm
Dipole	r = 2.6667 m, $B_{max} = 1.5T$, ϕ = 75 deg, ϵ_1 = 30 deg, ϵ_2 = 0 deg
Drift	l = 257.5 cm

large solid angle of about 20 msr and an energy acceptance of about 10 percent.

The spectrograph will be used in connection with the new K1200 Superconducting Cyclotron for beams of protons up to Uranium with energies of 2 to 200 MeV/u. It will provide unique opportunities for research in various areas, including the study of giant resonances, charge exchange, direct reaction studies and fundamental investigations of nuclear structure [11].

The S800 consists of two superconducting quadrupoles and two 75 degree dipoles with y-focusing edge angles. Table 1 lists the parameters of the system. The settings of the quadrupoles shown here correspond to particles of 193.04 MeV, mass 100 and charge 50. Standard optics notation is used.

After a careful measurement of the crucial fringe fields of the dipoles, we will be using COSY to determine the high order properties of the map of the spectrograph. The computation of the map $\mathcal{S}$ from the resulting transfer map can be performed directly within the COSY environment, and so can the inversion of the map $\mathcal{S}$. Altogether, a correction map $\mathcal{S}$ is found, the nonlinearity of which is determined by the nonlinearity of the original map and the quality in the spectrograph measured by (x,d)/(x,x). It is anticipated that the correction map can be used for an on line determination of the relevant data without having to store the raw two plane position measurements.

In closing we would like to note that the method can also be employed for spectrographs for which no sufficient field measurements are known. To this end, one has to perform experimental ray tracing and fit the resulting data with a polynomial type transfer map. Also in this case, the inversion can be done in the map picture resulting in a rather compact representation of the data necessary for correction.

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