# Analysis of Eddy Currents in the Walls of the Ferrite Tuned RF Cavity for the TRIUMF Kaon Factory Booster Synchrotron

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# Abstract

In the perpendicular biased ferrite tuned cavity of the proposed TRIUMF Kaon Factory Booster Synchrotron, magnetizing flux passes through the cavity walls. If special care is not taken to minimize eddy current loss in the walls, the dissipated power would be excessive and the magnetic fields set up by the eddy currents would disturb the magnetic field being applied. By electrically isolating the cooling structure from the cavity walls and introducing slots in the walls it is possible to bring to an acceptable level both the power loss and the maximal temperatures. Based on the measurements, an analytical model – essentially 3D – was derived and the eddy currents were predicted using the circuit analysis program PSpice. The calculated surface current and power distribution agree with measurements. PSpice can now be used to determine the effect of design changes on the eddy current and power distribution.

# I. INTRODUCTION

The rf cavity for the TRIUMF Kaon Factory Booster Synchrotron requires a frequency swing of 46 MHz to 61 MHz at a repetition rate of 50 Hz [1]. This will be accomplished using a tuner containing yttrium garnet ferrite where the magnetizing bias field is perpendicular to the rf magnetic field. The ac magnetizing flux passes through the walls of the resonator and special care must be taken to minimize the induced eddy currents [2].

In order to remove the heat resulting from rf and eddy current losses in the cavity walls, and rf losses in the ferrite, the construction of the cavity includes large stainless steel cooling wheels and a cylindrical copper cooling jacket [2]. The power dissipation due to ec losses in the cavity is determined by the relationship between induced emf's, conductivity of materials employed and the geometric configuration.

Previously PE2D has been used to evaluate eddy current loss in the different sections of the cavity [3]. Since electromagnetic software capable of 3D eddy current analysis was not available at TRIUMF, PSpice was used to simulate the rf membrane and cooling jacket in 3D. In order to check the quality of the equivalent circuit, 2D simulations were initially performed and compared with the results of measurements. The predictions compared well, and hence a 3D equivalent circuit was simulated. The PSpice software is very flexible: the equivalent circuit is set up such that it is relatively easy to make changes (e.g., changing resistivity and introducing slots).

# II. THEORY & MEASUREMENTS

The equivalent circuit utilized is based on Faraday's and Kirchoff's laws. The Faraday law of electromagnetic induction, states that a time (t) varying magnetic flux  $(\phi)$  induces an emf (e). In the case of axial symmetry the emf is induced uniformly along the length  $(\ell)$  of any circle whose center lies on the axis. It follows that:

$$e = -\frac{d\phi}{dt} = \oint_0^{2\pi} \overline{E}_r \cdot d\overline{\ell} = E_r \times 2 \times \pi \times r \tag{1}$$

hence

$$E_r = \frac{e}{2 \times \pi \times r} = -\frac{1}{2 \times \pi \times r} \times \frac{d\phi}{dt}$$
(2)

where  $E_r$  represents the azimuthal density of the emf [i.e., the electric field strength] at radius r.

The above was used to obtain a first estimate of the eddy current loss in the walls of the rf resonator for the TRIUMF Kaon Factory Booster cavity [4], and also used to determine the ratings for the power supply for perpendicular biasing of the ferrite in the rf cavity.

For each given radius r the EMF<sub>r</sub> is scaled with  $\text{EMF}_{r_{max}} \times r^2/r_{max}^2$ . To map experimentally the eddy currents in the ac field, the resonator walls were put in the stripped (without yoke) magnetization coil (Fig. 1).



Figure 1: Experimental set-up for measurement of current and temperature distribution

Measurements of the temperature and currents were taken

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at different points for various configurations and conditions. Since the measurement loop included two parts with practically equal and opposite emf, the measured voltage was proportional to the actual surface current (Fig. 2). The temperature was measured using a calibrated thermocouple after the system had reached thermal equilibrium.



Figure 2: Measurement probe: measured voltage is proportional to surface current.

# III. EQUIVALENT CIRCUIT

The equivalent circuit utilized for simulating eddy currents in the walls of the ferrite tuned rf cavity, represents  $\frac{1}{8}$ of the disk, i.e. a sector. Each sector is sub-divided into six sub-sectors (Fig. 3). Each of the disk sub-sectors is mathematically modelled using either 2D or 3D "cells". Each cell has its cylindrical coordinates and is characterized by its azimuthal emf and its impedance in the azimuthal, radial and axial direction. The representation of a sub-sector consists of 95 cells; 5 cells for each of 19 rows (Fig. 3). PSpice [6] utilizes nodal analysis methods so it is advantageous to simulate a cell which minimizes the number of circuit nodes [7].

The equivalent circuit utilized neglects reactive components of the cell impedances, reactive coupling between the cells and the effect of skin depth upon effective resistance: this can be justified because of the low frequencies.

Power dissipation within each cell is calculated using the PSpice Analog Behavioral Model (ABM) option [6]. The cells are simulated as subcircuits; a subcircuit call causes the referenced subcircuit to be inserted into the circuit with the given nodes replacing the argument nodes in the subcircuit definition. A row of cells is defined as another subcircuit: a subcircuit representing a sub-sector is constructed from 19 calls of a row subcircuit. The equivalent circuit for a sector is constructed by calling the sub-sector subcircuit six times.

As indicated above, the values of both the azimuthal and radial resistors of a cell, as well as that of the dc voltage source, are dependent upon the geometrical position of the cell and the materials employed. Equations for cell element



Figure 3: Representation of sector by cells

values are specified, within the equivalent circuit, using function definitions [7].

# A. Boundary Conditions

The east end of each row of cells in a sub-sector is connected to the west end, of the same row, of an adjacent sub-sector via a resistor, e.g., the east end of row 13 of sub-sector 2 is connected to the west end of row 13 of subsector 3 via a resistor. The value of the 'boundary condition' resistors is defined using a Parameter Definition. An open-circuit (i.e., slot) between sub-sectors is simulated by defining the appropriate resistors to have a large value relative to other resistor values in the equivalent circuit. Similarly a high conductivity path between sub-sectors is simulated by defining the appropriate resistors to have a low value relative to other resistor values in the mathematical model.

#### B. Analysis

PSpice version 4.04p was utilized for all the simulations reported. A dc sweep is performed: a fictitious voltage source is swept through one value only, hence the bias point for the circuit is calculated only once. The output file is transferred from a p.c., where the analysis is performed, to a VAX where post processing is performed.

2D modelling of the slotted disk, using a Thevenin equivalent circuit for the cells, requires about 12 MB of memory and 34 hours of CPU time on a 20 MHz 80386 based p.c. [7]. As a result of a 16 MB memory limitation with the p.c. used, and in order to simulate the cooling jacket in 3D, it was necessary to significantly reduce the number of circuit nodes; this was achieved by replacing the Thevenin equivalent azimuthal series connected voltage source and resistor by its Norton equivalent. The equivalent azimuthal current is the sum or difference of the current through the current source and parallel resistor [7].

Replacing the Thevenin equivalent by its Norton equivalent circuit resulted in a reduction in circuit memory requirements by a factor of 4.4 and a decrease in CPU time by a factor of 39. The Thevenin and Norton equivalent circuits result in virtually identical predictions for power dissipations and current flow patterns [7].

To simulate the effect of the water cooling jacket upon eddy current loss in the rf membranes, the 18th and 19th rows of each of the six sub-sectors are represented in 3D; this is achieved by modelling 7 layers of 3D cells in axial direction. The inner 17 rows of the sub-sectors are simulated in 2D, and thus their description and element values are unchanged from the 2D equivalent circuit. The values of the cell elements for the 3D analysis are again specified using equations which are coded using function definitions.

# IV. RESULTS

Scaling between the calculations and measurements can be carried out by comparing predicted cell current with the corresponding measured voltage drop [7]. Predicted radial current distribution (Fig. 4) and power loss distribution (Fig. 5) agree well with the measured temperature distribution [2,7], though it is difficult to compare them directly. Nevertheless the places where maximal temperatures are measured [2] coincide with the maximum predicted power density. For example, the maximum temperature measured occurs in the same place as the predicted value of maximum power.

## V. CONCLUSION

For low exciting frequency PSpice can be used with confidence to predict the eddy current loss and power dissipa-



Figure 4: Predicted radial current distribution in sub-sector 1: 3D simulation



Figure 5: Predicted power loss distribution due to radial, azimuthal and axial currents in sub-sectors 1, 2 & 3: 3D simulation

tion: the predictions should be a worst case as the equivalent circuit simulated neglects both the reactive component of induced emf and the skin depth in the walls.

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