Calculation of Required Tuner Accuracy and Bandwidth With and Without Fast Feedback

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Abstract

An expression for the resonance-controller time constant is given in terms of the allowed tuner error and rate of frequency sweep. A cavity tuner-control with this time-constant guarantees the resonance program is tracked to the desired accuracy. Expressions are given for the permissible tuner frequency error, in terms of the reactive to resistive power ratio, for an accelerating cavity operating with or without fast feedback. An expression is given for the resonance frequency sweep rate for arbitrary drive frequency program and relative beam-loading. The results are applied to the TRIUMF-KAON [1] Accumulator and Booster rings. The constraint imposed on the time constant by accumulation in the former is not severe; a passband of a few hundred hertz is sufficient. However, it is found that in the absence of fast feedback the Booster tuner control must have a passband that extends to 30 kHz. This paper is an abridged version of an unpublished design note [2].

I. Controller Time Constant

We suppose that the cavity resonance frequency is to follow a predetermined program (given by equation 2 below) to within some accuracy $\Delta\omega_0$. In the general case of an arbitrary frequency tuning rate $\dot{\omega}_0(t)$, and varying allowed tuning error $\Delta\omega_0(t)$, the controller time constant required to track the frequency program is:

$$\Delta T \leq \left|\frac{\Delta\omega_0(t)}{d\omega_0/dt}\right|_{\min} . \tag{1}$$

The bandwidth of the tuner control is the inverse of ΔT. The value of $\Delta\omega_0(t)$ depends on how much reactive power can be delivered, as shown below.

II. Resonance Program

Let ω_0 = resonance frequency, and ω = drive frequency. Let I_b = beam current component at the radio frequency, and ϕ_b the rf phase of the bunch centre. Let $I_0 = V/R$ be the gap voltage divided by cavity shunt resistance, and Q = cavity quality factor. The resonance frequency program $\omega_0(t)$ is given by:

$$\frac{\omega_0^2 - \omega^2}{\omega\omega_0/Q} = \frac{I_b}{I_0}\cos\phi_b \equiv \tan\Psi_0 . \tag{2}$$

Here Ψ_0 is the tuning angle, and $\tan\Psi_0$ is the beam loading ratio. For brevity, this ratio will be denoted ρ.

III. Allowed Tuning Error

From (2) we may find the relation between incremental changes $\Delta\Psi_0$ and $\Delta\omega_0$:

$$\sec^2\Psi_0\Delta\Psi_0 = \left[\frac{2Q}{\omega} - \frac{\tan\Psi_0}{\omega_0}\right]\Delta\omega_0 . \tag{3}$$

Now from (2) provided $\rho \ll 2Q$

$$\omega_0 \approx \omega\left[1 + \frac{\rho}{2Q} + \frac{1}{2}\left(\frac{\rho}{2Q}\right)^2\right] .$$

$$\text{Hence} \qquad \frac{\Delta\omega_0}{\omega_0} \approx \frac{\sec^2\Psi_0}{2Q}\Delta\Psi_0 . \tag{4}$$

Expressions for $\sec^2\Psi_0\Delta\Psi_0$, consistent with accurate generation of the accelerating voltage across the cavity gap, are given in reference [2]. We suppose that the cavity phase and amplitude controls have bandwidth much greater than the cavity tuner and its power supply, so that the generator current exactly compensates the tuning error $\Delta\Psi_0$. Then, as shown in the appendix,

$$\sec^2\Psi_0\Delta\Psi_0 \approx (-)\tan\Delta\phi_g[1 + (I_b/I_0)\sin\phi_b] .$$

Here $\Delta\phi_g$ is the phase difference between the generator current and the gap voltage. $\tan\Delta\phi_g$ happens to be the ratio of reactive power to resistive power delivered to the cavity. The maximum permissible tuning error occurs when maximum allowable reactive power is delivered by the amplifiers and power tube. Thus

$$\sec^2\Psi_0|\Delta\Psi_0| = (1 + \tan\Psi_0\tan\phi_b)\left[\frac{\text{reactive power}}{\text{resistive power}}\right]_{\max} \tag{5}$$

Combining relations (4) and (5) gives the permissible tuning error :

$$\frac{\Delta\omega_0}{\omega_0} \approx \frac{(1 + \tan\Psi_0\tan\phi_b)}{2Q}\left[\frac{\text{reactive power}}{\text{resistive power}}\right]_{\max} . \tag{6}$$

This is for the case of an rf-cavity in isolation, as treated in the Appendix. If fast feedback [3] around the amplifier is used, then the expression is modified according to the loop gain $H = A_0R$.

$$\frac{\Delta\omega_0}{\omega_0} \approx \frac{(1 + H + \tan\Psi_0\tan\phi_b)}{2Q}\left[\frac{\text{reactive}}{\text{resistive}}\right]_{\max} \tag{7}$$

Since H can be large compared with $\tan\Psi_0\tan\phi_b$, there can be a significant increase in the allowed tuning error.

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This happens, not because feedback alters the time constant of the tuner control, but because feedback makes the gap voltage insensitive to the exact tuning angle.

So far we have assumed that no error is allowed in the cavity voltage. In fact there is only a small gain in the allowed tuner error unless large voltage errors can be tolerated. If the reactive-to-resistive power ratio is unity, and the gap voltage has errors $\Delta\phi_V$ and ΔV, then

$$\sec^2\Psi_0\Delta\Psi_0/\tan\Delta\phi_g \approx$$

$$1+\frac{\mathrm{I_b}}{\mathrm{I_0}}\left[\sin\phi_b+(\Delta\phi_V+\Delta V/V_T)(\cos\phi_b-\sin\phi_b)\right]. \quad (8)$$

IV. Resonance Frequency Sweep Rate

Solve relation (2) for ω_0 in terms of $\omega(t)$ and $\rho(t)$. For the case $\rho \ll Q$ we find the approximation $\omega_0 \approx \omega[1+\rho/2Q]$. Take the time derivative, and neglect terms ρ/Q as insignificant, to give:

$$\dot\omega_0 \approx \dot\omega+\frac{\omega}{2Q}\dot\rho = \omega_\infty\left[\frac{d\beta}{dt}+\frac{\beta}{2Q}\frac{d\rho}{dt}\right]. \quad (9)$$

ω_∞ = drive frequency if $\beta \equiv$ unity; and β is the particle-beam speed divided by the speed of light.

Now from (2) $$\frac{1}{\rho}\frac{d\rho}{dt}=\left[\frac{\dot I_b}{I_b}-\tan\phi_b\dot\phi_b-\frac{\dot I_0}{I_0}\right]. \quad (10)$$

Equations (1), (6), (7), (9) and (10) are sufficient to determine the time constant ΔT.

V. KAON Booster

The Booster is a fast-cycling synchrotron with biased sinusoidal magnet excitation. The injection and extraction energies are 0.5 GeV and 3 GeV, respectively. Detailed numerical evaluations would be needed to find the minimum value $\Delta T_{\min}$ during the acceleration cycle. We will be content with a good estimate, and guess that the $\Delta T_{\min}$ occurs near mid-cycle. The magnet cycling angular frequency is Ω, assuming single frequency excitation; and the time range is $0 \le \Omega t \le \pi$. We assume all quantities vary in a sinusoidal manner, that is:

$\beta(t) = \check\beta + (1\text{-}\cos\Omega t)(\hat\beta-\check\beta)/2$
$\mathrm{I}_b(t) = \check{\mathrm{I}}_b + (1\text{-}\cos\Omega t)(\hat{\mathrm{I}}_b-\check{\mathrm{I}}_b)/2$, and likewise $\mathrm{I_0}(t)$.
$\phi_b(t) = (1\text{-}\cos 2\Omega t)\hat\phi_b$, so $\dot\phi_b$ = zero at mid-ramp.

We shall denote the value of x at mid-ramp by $\bar x$, an average value by $\langle x\rangle$, and the change from start to finish of ramping by Δx. Substituting the above time variations into equation (9) gives:

$$\dot\omega_0 \approx \omega_\infty\frac{\Omega}{2}\left\{\Delta\beta+\frac{\bar\beta.\tan\bar\Psi_0}{Q}\left[\frac{\Delta\mathrm{I_b}}{\langle\mathrm{I_b}\rangle}-\frac{\Delta\mathrm{I_0}}{\langle\mathrm{I_0}\rangle}\right]\right\}.$$

For the KAON Booster ring, the change in relativistic β is substantial: $\Delta\beta \sim 0.23$; whereas $(\tan\Psi_0/Q) < 0.006$ so we may drop the second term leaving $\dot\omega_0 \approx \omega_\infty\Delta\beta\Omega/2$. Finally, we may estimate the control time-constant to be $\Delta T =$

$$\frac{\Delta\omega_0}{\dot\omega_0} \approx \frac{(1+H+\tan\bar\Psi_0\tan\bar\phi_b)}{\bar Q\Omega}\frac{\langle\beta\rangle}{\Delta\beta}\left[\frac{\text{reactive}}{\text{resistive}}\right]_{\max}.$$

Substitute values appropriate to the Booster: $\langle\beta\rangle = 0.854$, $\Delta\beta = 0.234$, $H = 65$, $\bar Q = 4000$, $\tan\bar\Psi_0 = 16$, $\tan\bar\phi_b = 1/\sqrt{3}$, and $\Omega = 2\pi\times 50$ Hz. Arguably the power ratio can rise, for short periods, as high as unity. In this case the *time constant* is $\Delta T = 0.2$ milli-second, which means the controller must pass frequencies up to 5 kHz with only -3 dB attenuation.

If the cavity were not equipped with fast feedback around the amplifier, then $H = 0$ and the time constant is significantly smaller : $\Delta T = 30$ micro-second which is equivalent to a 30 kHz passband.

Of course, if less reactive power is allowed than the unity ratio assumed here, then the controller time constant must diminish proportionately.

VI. KAON Accumulator

The Accumulator is a storage ring. Current is accepted from a cyclotron by charge-exchange multi-turn injection for 2×10^4 turns, and transfer is bucket-to-bucket so that the beam never debunches. The synchronous phase angle is zero so that $\tan\phi_b = 0$. Consequently, there is no change in drive frequency ($\dot\omega = 0$) and the resonance frequency program derives entirely from beam-loading. Borrowing from equations (9) and (10) the tuning rate becomes

$$\dot\omega_0 \approx \frac{\omega}{2Q}\dot\rho = \frac{\omega_\infty\beta}{2Q}\frac{\mathrm{I_b}}{\mathrm{I_0}}\left[\frac{\dot I_b}{I_b}-\frac{\dot I_0}{I_0}\right]. \quad (11)$$

The gap voltage is constant in time and so $\dot I_0 = 0$. In this case, dividing equation (4) by (11) gives :

$$\frac{\Delta\omega_0}{\dot\omega_0} \approx \frac{\sec^2\Psi_0}{(\dot{\mathrm{I}}_b/\mathrm{I_0})}\Delta\Psi_0.$$

In the Accumulator the beam current follows a linear ramp $\mathrm{I_b}(t) = \check I_b + \Delta I_b(t/T_r)$, where T_r is the duration of beam filling. Substituting from equation (9), the controller time constant becomes

$$\Delta T_{\min} = \frac{\mathrm{I_0}(1+H)}{\Delta\mathrm{I_b}}\left[\frac{\text{reactive power}}{\text{resistive power}}\right]\times T_r.$$

For the Accumulator, values are : $\mathrm{I_0} = 0.34$ A, $\Delta\mathrm{I_b} = 2.5$ A, $H = 20$ and $T_r = 20$ ms ; giving $\Delta T_{\min} = 58$ ms for unity power ratio. If there is no fast feedback ($H = 0$) the *time constant* becomes 2.7 ms, implying a -3 dB frequency of 370 Hz. This is the minimum recommended value, since one must also consider the need to periodically reset the tuner to the zero beam-load frequency.

VII. Conclusion

The constraints on tuner-control imposed by A ring accumulation are likely much less severe than those deriving from beam-injection transient compensation and periodic tuner resetting.

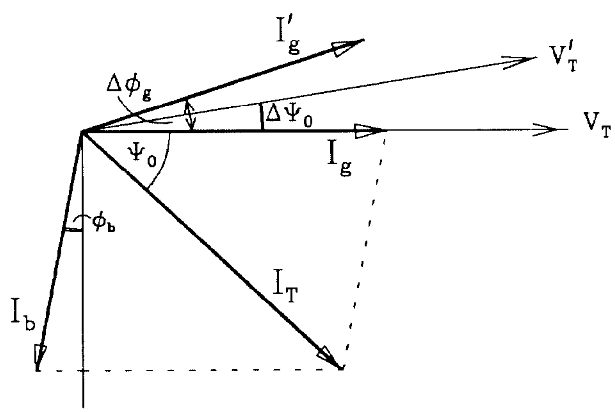
The B ring tuner-control is likely dominated by the substantial change in radio frequency (46-61 MHz), and is probably not feasible without 'fast feedback' around the amplifier.

Appendix: Effect of Tuner Error for Cavity

The conventions used for the phasor diagram follow Pedersen [4]. The cavity-gap voltage (V) lies along the real axis of the complex plane, as does the generator current ($\mathrm{I_g}$), and the beam image-current vector is given by:

$$\mathrm{I_b}e^{-j(\phi_b+\pi/2)} = (-)\mathrm{I_b}[\sin\phi_b + j\cos\phi_b].$$

All quantities are assumed sinusoidally varying according to $e^{j\omega_c t}$ with ω_c the drive frequency.



A. Steady-State Conditions

The cavity is detuned by an amount $\tan\Psi_0 = 2Q(\omega_0-\omega_c)/\omega_0$. We use the linearized approximation for the cavity impedance: $R\cos\Psi_0 e^{j\Psi_0}$. The total desired voltage is:

$$V_T = \mathrm{I_0}R = \frac{R}{1-j\tan\Psi_0}\left[\mathrm{I_b}e^{-j(\phi_b+\pi/2)} + \mathrm{I_g}e^{j\phi_g}\right] . \quad (A1)$$

Hence we find the steady-state values $\mathrm{I_g}$ and Ψ_0 in terms of $\mathrm{I_b}$ and ϕ_b by taking the real parts

$$\mathrm{I_g}\cos\phi_g = \mathrm{I_0} + \mathrm{I_b}\sin\phi_b \quad (A2)$$

and taking the imaginary parts

$$\mathrm{I_0}\tan\Psi_0 = \mathrm{I_b}\cos\phi_b - \mathrm{I_g}\sin\phi_g . \quad (A3)$$

To minimize the generator current ($\mathrm{I_g}$) we put $\phi_g \equiv 0$.

B. Small Perturbation

Suppose the tuning angle changes by a small amount $\Delta\Psi_0$. To compensate the change ($\Delta V_T e^{j\Delta\phi_V}$) in the total voltage, the $\mathrm{I_g}$ vector has to change in magnitude and phase to $\mathrm{I'_g} = (\mathrm{I_g}+\Delta\mathrm{I_g})e^{j\Delta\phi_g}$ as shown in the phasor diagram. The beam current vector does not move. Let us assume the generator compensation is perfect so that old and new voltages are identical. Hence:

$$R\cos\Psi_0 e^{j\Psi_0}[\mathrm{I_b}e^{-j(\phi_b+\pi/2)} + \mathrm{I_g}e^{j\phi_g}] =$$

$$R\cos(\Psi_0+\Delta\Psi_0)e^{j(\Psi_0+\Delta\Psi_0)}[\mathrm{I_b}e^{-j(\phi_b+\pi/2)} + (\mathrm{I_g}+\Delta\mathrm{I_g})e^{j(\phi_g+\Delta\phi_g)}].$$

Set $\phi_g = 0$ and solve for $\Delta\phi_g$ to give $\tan\Delta\phi_g = Y/X$ with

$$\begin{aligned} Y &= [1-\tan\Psi_0\tan\Delta\Psi_0]\mathrm{I_b}\cos\phi_b \\ &\quad -[\mathrm{I_b}\cos\phi_b + (\mathrm{I_g}-\mathrm{I_b}\sin\phi_b)\tan\Delta\Psi_0] \\ X &= [1-\tan\Psi_0\tan\Delta\Psi_0]\mathrm{I_b}\sin\phi_b \\ &\quad +[(\mathrm{I_g}-\mathrm{I_b}\sin\phi_b) - \mathrm{I_b}\cos\phi_b\tan\Delta\Psi_0] . \end{aligned}$$

Use the steady state conditions to eliminate $\mathrm{I_g}$ and $\mathrm{I_b}\cos\phi_b$, to give :

$$\tan\Delta\phi_g = \frac{\tan(\Psi_0) - \tan(\Psi_0+\Delta\Psi_0)}{1+(\mathrm{I_b}/\mathrm{I_0})\sin\phi_b} .$$

Expand $\tan(\Psi_0+\Delta\Psi) \approx \tan\Psi_0 + \sec^2\Psi_0\Delta\Psi_0$ as a Taylor series. To first order in $\Delta\Psi_0$, we find

$$\tan\Delta\phi_g = \frac{(-)\sec^2\Psi_0}{1+(\mathrm{I_b}/\mathrm{I_0})\sin\phi_b}\Delta\Psi_0 . \quad (A4)$$

Note that because $\sec^2\Psi_0$ can be very large (maybe 10^4), $\tan\Delta\phi_g$ can be large even though the tuner error is small $|\Delta\Psi_0| \ll 1$. Consequently, it is correct to retain the tangent term rather than replace with the first order term $\Delta\phi_g$.

C. Significance of $Tan\Delta\phi_g$

Since we have insisted there is no error in the gap-voltage magnitude or phase,
$\frac{1}{2}V(I_g+\Delta I_g)\sin\Delta\phi_g$ is the reactive power, and
$\frac{1}{2}V(I_g+\Delta I_g)\cos\Delta\phi_g$ is the resistive power.
Hence the ratio of reactive to resistive power is simply $\tan\Delta\phi_g$. The sign of $\tan\Delta\phi_g$ indicates that when $\Delta\Psi_0 > 0$ the generator current lags behind the gap voltage, and when $\Delta\Psi_0 < 0$ it leads.

VIII. References

[1] *Kaon Factory Study Accelerator Design Report*; TRIUMF, Vancouver, B.C., 1990.

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