

First-, Second-, and Third-Order Achromatic Bend Systems for Free-Electron Laser Applications*

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Abstract

We evaluated and compared several achromatic bend systems for the application of transporting a spent electron beam exiting the wiggler of a free-electron laser (FEL). For high-power, efficient FELs, the design of a postwiggler electron-beam transport system is challenging because of the large double-peaked momentum distribution caused by the laser. The optics system had to be able to handle electron-beam power arbitrarily distributed between the two momentum peaks. This system included not only the achromatic bend but also the final transport that involved a beam distribution or rastering system targeting a beamstop. It was necessary to consider the quality of the beam exiting the achromat as well as the specific characteristics of the achromatic bend system itself. In this paper, we discuss the salient features of 11 achromatic bend systems, and those characteristics by which to choose one system over another.

I. ACHROMATIC BENDS

An achromatic bend for charged particle beams is defined as a bend system in which the particles in the beam arrive at their final locations (in transverse space) independent of momentum. Essentially, an achromat will produce the same output beam regardless of the momentum spread of the input beam. Typically, however, achromats are not perfect, and a particle's final transverse coordinates and divergence depend on its momentum deviation from some nominal reference momentum. This momentum dependence on final coordinates can be written as a Taylor series expansion giving the final coordinates in terms of the initial coordinates and momentum. It is convenient to use the notation found in TRANSPORT.[1] A right-handed coordinate system is used in which the Z-axis coordinate is tangent to the central ray, the X-axis lies in the horizontal bend plane, and the Y-axis is in the vertical nonbend plane. A particle's divergences X' and Y' are defined as dX/dZ and dY/dZ , respectively. Defining δ as the fractional momentum deviation from the central momentum (P_0),

$$\delta = (P - P_0)/P_0,$$

defining l as the path length difference between the reference particle and any beam particle, and defining the coordinates (X_1 to X_6) by

$$(X_1, \dots, X_6) = (X, X', Y, Y', l, \delta),$$

the Taylor series expansion can be written as

$$X_{F_i} = \sum_{j=1,6} R_{ij} X_{0_j} + \sum_{j,k=1,6} T_{ijk} X_{0_j} X_{0_k} + \sum_{j,k,l=1,5} U_{ijkl} X_{0_j} X_{0_k} X_{0_l}$$

The terms X_{0_n} ($n = j, k, l$) are the initial particle coordinates, and the terms R_{ij} , T_{ijk} , and U_{ijkl} are the Taylor coefficients that give the final particle coordinates (X_{F_i}) in terms of the initial coordinates. The parameters R_{ij} , T_{ijk} , and U_{ijkl} represent the first, second, and third order effects of the beam transport channel in transporting a beam particle with the initial conditions X_{0_n} . A first-order achromat is achieved when no first-order Taylor series term depends on δ (i.e., no nonzero R_{i5} terms in which $i = 1, 2, 3$ or 4); a second-order achromat has no terms through second order that contain δ or δ^2 (i.e., no

nonzero T_{i56} terms in which $i = 1, 2, 3$ or 4).[2] First-, second-, third-, and higher-order achromats are obtained, depending on what elements are included in the beamline and to what order corrections are made to the nonzero chromatic terms of the transfer matrices.

The first-order term, R_{56} , gives the path length difference between the reference particle and an off-momentum particle. This term is often referred to as the *isochronous* term because the flight time of a particle through the system depends on the path length. For particle beams having velocities approaching that of light, an appropriate R_{56} term is often utilized to produce beam bunching, provided that there is a proper momentum-vs-position correlation in the beam.[3]

II. FIRST-ORDER SYSTEMS

In first-order systems, the transfer-matrix chromatic terms, R_{16} and R_{26} , equal zero at the end of the achromat and hence contribute nothing to beam aberrations. Four of the first-order achromats considered in this study include the Enge Dual-270°, the Brown 2-Dipole, Leboutet 3-Dipole, and the Enge 4-Dipole.[4] Figure 1 illustrates these first-order achromats. The two Enge achromats use edge angles on the bends that serve to focus in the nonbend plane. The 2-Dipole achromat includes one quadrupole for focusing,[5] whereas the 3-Dipole system has no means for focusing. Also included with these first-order achromats is one periodic system (contiguous identical cells), which we have called the Brown-Servranckx-1 because it is identical to the Brown-Servranckx second-order achromat, but without the second-order (sextupole) elements. This achromat allowed us to compare the advantages of periodic versus nonperiodic systems and is discussed later.

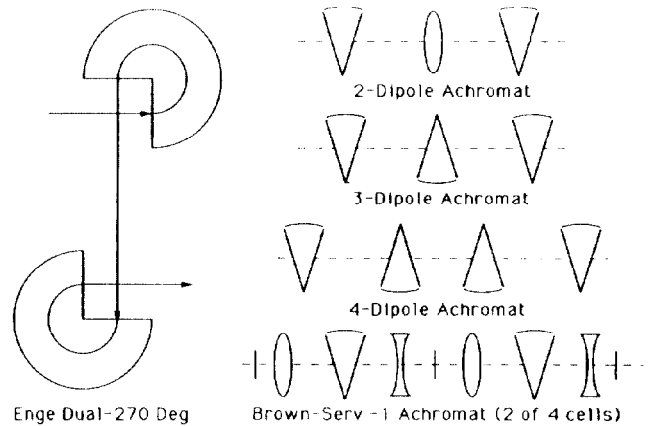


Figure 1. First-order achromats.

III. SECOND-ORDER SYSTEMS

Second-order systems not only have first-order chromatic terms equal to zero, but also the second-order transverse chromatic terms are zero (e.g., T_{116} , T_{126} , T_{156} , and all other T_{ijk} terms where j and/or $k = 6$).[2] This is accomplished by the addition of sextupole elements. The second-order systems that we considered include the familiar periodic Brown-Servranckx,[6] the periodic Blind-Neri and Modified Blind-Neri,[7] and the semiperiodic Stair-Step[8]. Figure 2 illustrates these second-order systems.

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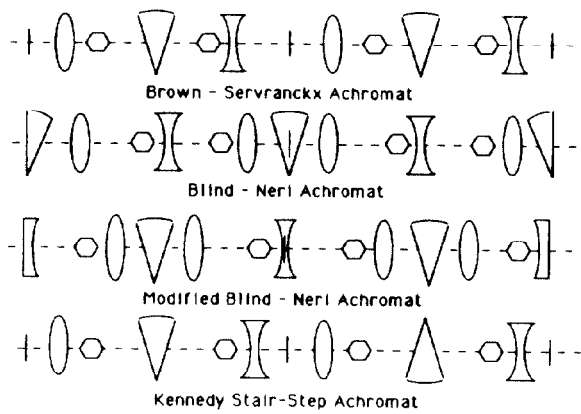


Figure 2. Second-order achromats (2 of 4 cells each).

IV. THIRD-ORDER SYSTEMS

In like manner, third-order achromats include octupoles to eliminate the third-order transverse chromatic terms (the U_{ijkl} terms where j and/or k and/or $l = 6$). The two third-order achromats included in this study are the Dragt 5-Cell[9] and the Neri 7-Cell,[10] both illustrated in Fig. 3.

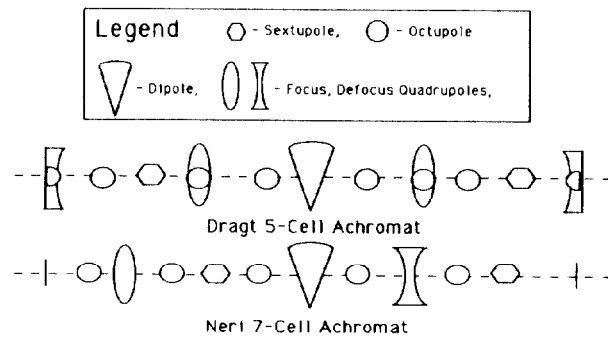


Figure 3. Third-order achromats (1 cell each).

V. THE FEL APPLICATION

The transport of the electron beam exiting the FEL wiggler required the design of an S-shaped bend that would offset the beam centerline 8.6 meters over a distance of about 30 meters. The S bend was made up of two achromats (of opposing bend angle) for 9 of the 11 types included in this study. We chose a bend angle of 32° for each achromat (the Enge Dual-270° was an exception) and compared the various characteristics, some of which are listed in Table I below. It was important to maintain minimal beam loss of the high-power beam as well as a good quality beam for a beamstop targeting system. It was also important to select a transport system that would have sufficient room for the required diagnostics, would be as insensitive to misalignments and tuning errors as possible, and in addition, have the least amount of hardware and the lowest possible cost.

VI. ACHROMATS COMPARED FOR FEL APPLICATION

The ability to diagnose the wiggler performance by measuring the momentum distribution of the beam in the postwiggler transport was a system requirement. Because resolving power is directly proportional to the R_{16} term, we evaluated this characteristic by comparing the maximum values of the R_{16} terms that were computed at every element location for each achromat. Table I lists the maximum value of the R_{16} term (or "dispersion index" for each achromat. As can be seen, the nonperiodic first-order achromats and the Stair-Step achromat have the highest dispersion indices, and are therefore the best systems for momentum resolution.

For some applications, it was important to know how isochronous (or how nonisochronous) an achromat was. Again, we were able to evaluate this characteristic because the value of the R_{56} term at the end of each achromat provided a direct index of isochronicity. Table I shows that the 2-Dipole, the Brown-Servranckxs, and the Neri 7-cell achromats are the most isochronous, while the Stair-Step and Dual-270° could be useful for their bunching or debunching capability (most nonisochronous).

We determined the sensitivity of each achromat by observing the change (from zero) in the matrix elements, R_{16} and R_{26} , at the end of the achromat with a 1% change in the field of any component or element group in the system or in a cell of a periodic system. Table I shows the worst case result of

TABLE I. COMPARISON OF ACHROMATS

ACHROMAT	Type*	Order	Dimensions		Dispersion (Max R_{16}) (m†)	Isochronicity, R_{56} (m†)	Sensitivity		Beam Radius		Hardware dip/other‡
			length (m)	offset (m)			R_{16} (m†)	R_{26} (rad†)	X_{max} (cm)	Y_{max} (cm)	
Dual 270°	NP	1 st	-	8.60	4.969	-28.386	0.4552	0.0492	43.32	0.84	2/-
2-Dipole	NP	1 st	11.41	3.27	1.497	-0.011	0.0006	0.0027	14.00	1.10	2/1
3-Dipole	NP	1 st	8.31	2.38	1.462	0.704	0.0070	0.0048	13.83	0.47	3/-
4-Dipole	NP	1 st	15.00	4.30	2.372	2.490	0.0511	0.0062	23.58	0.48	4/-
Brown-Serv.-1	P	1 st	15.00	4.30	0.679	-0.099	0.0063	0.0156	5.93	1.17	4/8
Brown-Serv.	P	2 nd	15.00	4.30	0.679	-0.099	0.0063	0.0156	5.90	1.13	4/16
Blind-Neri	P	2 nd	15.00	4.30	0.739	-0.203	0.0011	0.0054	6.50	0.45	5/20
Mod Blind-Neri	P	2 nd	15.00	4.30	0.743	-0.197	0.0007	0.0157	7.54	0.22	4/21
Stair-Step	SP	2 nd	30.00	8.60	5.461	5.768	0.2171	0.0652	78.80	0.29	4/16
Dragt 5-Cell	P	3 rd	15.00	4.30	0.671	-0.165	0.0008	0.0135	9.86	0.22	5/62
Neri 7-Cell	P	3 rd	15.00	4.30	0.563	-0.100	0.0073	0.0173	7.06	0.92	7/63

*Type: NP = nonperiodic, SP = semiperiodic, P = periodic

†Units are per fraction momentum spread

‡Includes quadrupoles, sextupoles, and/or octupoles

this change. The Dual-270° and the Stair-Step appear to be the most sensitive of the achromats.

The beamline aperture requirements and the requirement to minimize beam losses were other important factors to consider. Periodic systems (Type P) have the advantage when considering the ability to control beam size; and, if a matched beam is provided the beam size in the nonbend plane can be kept very small. Nonperiodic first-order achromats have either little or no focusing control in the nonbend plane. The 2- and 3-Dipole achromats would require additional focusing in a drift space required between the two opposing achromats to make the specified S-bend configuration. The beam radius data in Table I, generated for comparison purposes, lists the maximum value of the beam radius in the transport system determined by tracking 10,000 particles through each achromat at the design momentum and with energies 6.0% above and below the design using the computer code MARYLIE to third order.[11] We also tracked a beam with an emittance six times larger than the nominal beam to allow for beam jitter and found the bend-plane beam size relatively unaffected because the beam behavior was dominated by dispersion.

Finally, the amount of hardware required for each achromat is listed in the last column of Table I. The number of dipoles is listed on the left-hand side of the slash (/), followed by the sum of the "other" elements such as quadrupoles, sextupoles, and/or octupoles. The overall cost is generally proportional to the amount of hardware. This is certainly true when comparing periodic systems in which the hardware is similar. Note that although the Dual-270° contains only two dipoles, the dipoles are large, 270° bends that require large apertures and would therefore be costly to produce.

VII. ACHROMAT SELECTION FOR FEL APPLICATION

Based on our achromat comparison study, we eliminated the Dual-270°, the 4-Dipole, and the Stair-Step achromats because of the high sensitivity factor and the large aperture requirement. We did not consider the third-order achromats good alternatives because of the high cost that would be associated with the amount of hardware. Although the remaining second-order achromats were similar, the Blind-Neri was the one of choice because of its lower overall sensitivity. Of the remaining first-order systems, sensitivity considerations favored the 2-Dipole achromat.

However, up to now we have compared achromats based only on characteristics of the achromats alone, independent of beam. To address beam quality, one must consider both the input beam and the desired output beam. The given input and the desired output determine which coefficients or matrix elements of the achromat are important to observe. In our application in which the input beam had a large momentum spread, the chromatic terms had the most significant impact on beam quality. For the first-order achromats, the second-order terms such as T_{166} and T_{256} contributed the most to beam aberration. Because we needed to keep beam displacement at the exit of the achromat to a minimum for the FEL beamstop targeting system, the T_{166} term was the most significant. Table II, which lists the T_{166} term for each of the first-order achromats, shows that the largest value of this term is associated with the 2-Dipole system. Thus, for

Table II

Beam Quality Considerations:

Dominant Matrix Elements Contributing to Beam Aberrations for a Dispersion-Dominated Beam

ACHROMAT	Type	Chromatic T_{166}	Geometric T_{111}
Dual-270°	NP	1.9893	0.2106
2-Dipole	NP	2.9736	0.0482
3-Dipole	NP	1.5482	0.1772
4-Dipole	NP	1.7913	0.3074
Brown-Serv.-1	P	0.5140	0.0000

example, for a 10% momentum spread, there would be a ~ 3-cm displacement in the 2-Dipole system, a ~ 1.5-cm displacement in the 3-Dipole system, and a ~ 0.5-cm displacement in the Brown-Servranckx-1 system. For beam quality, the periodic Brown-Servranckx-1 was the best choice.

In addition to the T_{166} term, we looked at the geometric terms that would have the greatest impact on beam position in the bend plane. For our application, the T_{111} term was the most significant. Note that for the Brown-Servranckx-1, the T_{111} term is zero. Brown and Servranckx[12] have shown that in a periodic lattice having n identical cells with $n > 3$ and having a total phase shift of $2m\pi$, all second-order geometric aberrations cancel. Although we compared the T_{111} terms in Table II, we found that, for our application, the effect of this term on beam quality was negligible.

In final analysis, our selection narrowed to three systems: the Blind-Neri, the Brown-Servranckx-1, and the 3-Dipole. Of these three, the Blind-Neri had the highest beam quality and lowest sensitivity, but because the Brown-Servranckx-1 had an acceptable level of both quality and sensitivity for half the hardware, the Brown-Servranckx-1 was preferable to the Blind-Neri. The 3-Dipole system remained a possible choice because of its advantage in dispersion; however, if the wiggler diagnostic requirements could be satisfied with the Brown-Servranckx-1 achromat, it would be the best choice of all systems considered for this application.

VIII. CONCLUDING COMMENTS

We have attempted to discuss the various aspects of achromatic bend systems and those characteristics by which to choose one system over another. We have also tried to point out some of the complexities involved when considering the resultant beam quality. The choices we made were based on a specific application that dictated specific requirements. This study was an interesting one, and one that shed light on the advantages of different achromatic bend systems through the comparison process.

IX. ACKNOWLEDGMENTS

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X. REFERENCES

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