Longitudinal Beam-Beam Effects for an Ultra-High Luminosity Regime

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Abstract

The high luminosity projects imply collisions of intense short bunches with micro-beta at the IP, which entails high variations of the particle energy on the IP passage, due to the opposing bunch electric fields. This effect is shown to put the limit on the attainable luminosities. The estimates are made for the stability of the incoherent and coherent longitudinal motion which enhance the required RF voltage depending on the design beta minimum at the IP. The results of the analytical treatment and the computer simulation of the off-axis particle synchrobetatron motion relate the limitations, imposed on the space charge parameter by the longitudinal beam-beam effects, and the main parameters of the machine.

I. INTRODUCTION

The new generation of colliders aims at the ultimate luminosities and consequently these projects are based on very high design values of the space charge parameter ξ and very low β-function at the interaction point (IP). This puts forward the problem of correct and complete account for the particle's energy change in the electric field of opposing bunch to be included in the analysis of the beam-beam effects.

II. LONGITUDINAL MOTION IN THE FIELDS OF OPPOSING BEAM.

First we rederive the known effect for on-axis particles [1] in more "visual" alternative way by integration of microscopic fields for a short round bunch. At the ultra-relativistic energies we have a simple approximation for the particle's electric field, which is normal to its velocity (wake-fields are neglected):

$$E_\perp = \frac{2e}{r}\cdot\delta(s-so) \qquad (1)$$

here s is the longitudinal coordinate.

To find the on-axis test particle energy change ΔE on the single passage through the opposing bunch fields, we take the projection of (1) onto the design orbit direction for arbitrary betatron displacements x, x', z, z':

$$E_1 = \frac{2e}{r^2}\cdot\delta(s-so)\cdot(x\cdot x'+z\cdot z') \qquad (2)$$

Assuming the Gaussian distribution in the short opposing bunch with the equal r.m.s. emittances εt and optical functions β, β' for round beams:

$$f= C\cdot\exp(-\frac{1}{2\beta\varepsilon}\cdot(x^2+z^2+(\beta\cdot x'-\beta'\cdot x/2)^2+(\beta\cdot z'-\beta'\cdot z/2)^2))\cdot\delta(s-s_0),$$

one can find the energy gain on the passage:

$$\Delta E = \int f\cdot E_1\cdot dS = \frac{Ne^2}{2\beta}\cdot\beta' , \qquad (3)$$

where β and β' are taken at the point s/2 where the test particle encounters the short bunch. Another derivation of (3), based on the induction electric fields of the beam due to its transverse size modulation on the low-β IP passage, is available in [1] since 1972.

For the finite length l of a Gaussian bunch we obtain:

$$\Delta E \sim \int \frac{\beta'(t/2)}{\beta(t/2)}\cdot\exp(-\frac{(t-s)^2}{2l^2})\ dt \qquad (4)$$

In the Novosibirsk ϕ-factory proposal [2] $l=0.8\cdot\beta_0$ and the difference of 10 per cent between (3) and (4) justifies the use of the infinetely short bunch approximation.

From (3) we can obtain the incoherent synchrotron tune shift:

$$\frac{\Delta\Omega^2}{\Omega^2} = \Delta U/U = \frac{NeR}{2qU}\cdot\frac{1}{\beta_0^2} \qquad (5)$$

here Π is the circumference, U and q are the RF voltage amplitude and harmonic number.

To see how important is this effect for reduction of the longitudinal focusing in the new designed machines, we present the data for VEPP-2M:

$\Delta U \sim 3$ kV (I=30 mA, β_0=5 cm, q=12, R=250 cm)

and for ϕ-factory:

$\Delta U \sim 200$ kV I=300mA, β_0=1cm, q=76, R ~500 cm).

in terms of effectively diminished voltage of the main RF ΔU.

III. COHERENT LONGITUDINAL BEAM-BEAM INSTABILITY.

To get a quick estimate we consider a dipole oscillation modes of beam-beam motion in the rigid bunch model as the disregard of the longitudinal distribution has been shown to give only a small error. For calculation of the mode tune shifts we use the simple equations:

$$s_1'' + \omega_0^2\cdot s_1 = k\cdot(s_1-s_2)$$
$$s_2'' + \omega_0^2\cdot s_2 = k\cdot(s_2-s_1)$$

where $k\sim\Delta U$ and ω_0 is the design synchrotron tune.

Apparently, the tune of the antiphasal mode is left unchanged because in this mode the bunches always collide exactly at the IP where $\beta'=0$ and hence $\Delta E=0$

The cophasal mode tune goes down with intensity:

$$\omega = \omega_0\sqrt{1-\frac{2\Delta U}{U_0}}$$

Hence we have to double the lower limit for the RF voltage as determined by the incoherent motion in the previos Section (above 400 kV for the ϕ-factory).

The similar calculations are easily done for the compensated collision of 2x2 bunches and show that at equal intensities two of the four mode tunes are fixed, one goes up and one goes down similarly to the cophaphasal mode tune.

In the above formulas we used the design values of β_0. The focusing by the opposing bunch at $\xi\sim 0.1$-0.2 can compress β_0 2 or 3 times, but flattering of its minimum results in only 20 or 30 per cent growth of maximum β'/β. Moreover, for the strong-strong collision regime the concept of β needs revision, as the self-consistent beam sizes must be dependent on the particles longitudinal coordinate.

IV. FULL DESCRIPTION OF SYNCHROBETATRON MOTION.

In Section II we disregarded the betatron motion of the test particle, passing the opposing bunch fields on axis. For the off-axis test particle eq. (2) can be rewritten in the same form:

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$$E_1 = \frac{2e}{r^2}\cdot\delta(s-so)\cdot((x-x_0)\cdot(x'+x'_0)+(z-z_0)\cdot(z'+z'_0))$$

here x, x', z, z' are the betatron motion variables of the opposing bunch particle, while x_0, x'_0, z_0, z'_0 are those of the test one. Then integrating over the distribution we obtain:

$$\Delta E = \int f\cdot E_1 dS = \frac{Ne^2}{2\beta}\cdot\beta' - \qquad (6)$$

$$\frac{1}{r_0^2}\cdot\frac{Ne^2}{\beta}\cdot\left(1-\exp\left(-\frac{r_0^2}{2\sigma^2}\right)\right)\cdot\left(x_0\left(x'_0+\frac{\beta'}{2\beta}\cdot x_0\right)+z_0\left(z'_0+\frac{\beta'}{2\beta}\cdot z_0\right)\right)$$

The new additional term in the right-hand side of (6) results from the opposing bunch field projection onto the test particle velocity inclined by the betatron motion angle x'_0. For the particles close to the aperture limit this can contribute up to ± 0.5 Mev to their energy thus deteriorating the expected beam lifetime in the ϕ-factory project.

At first glance this energy gain related to the particle's angle at IP seems to be averaged out by the relatively fast betatron motion. However the betatron oscillation phase is strongly modulated by the synchrotron motion. This results in coupling of these two degrees of freedom.

Normalization of the betatron variables in (6) yields:

$$\Delta E=\int f\cdot E_1 dS= \frac{Ne^2}{2\beta}\cdot\beta' + \frac{Ne^2}{\beta}\cdot\left(1-\exp\left(-\frac{\varepsilon\cdot\cos^2\varphi}{2\varepsilon_t}\right)\right)\cdot\frac{\sin\varphi}{\cos\varphi} \qquad (7)$$

where ε, φ are the action and phase variables of the betatron motion. For simplicity the betatron motion is treated unidimensionally, that is relevant in the ϕ-factory lattice with round beams and for zero angular momentum though the 2D generalization is straightforward.

To the first order in ξ the changes of the betatron action and phase variables for one turn are:

$$\Delta\varepsilon = 16\pi\xi\cdot\varepsilon t\left(1-\exp\left(-\frac{\varepsilon\cdot\cos^2\varphi}{2\varepsilon t}\right)\right)\cdot\frac{\sin\varphi}{\cos\varphi} \qquad (8)$$

$$\Delta\varphi = 8\pi\xi\cdot\frac{\varepsilon t}{\varepsilon}\left(1-\exp\left(-\frac{\varepsilon\cdot\cos^2\varphi}{2\varepsilon t}\right)\right) + \omega_B + \frac{\Delta s}{2\beta} + 2\pi\mathcal{C}\cdot\Delta E \qquad (9)$$

$$\Delta s = -\alpha\cdot\Delta E \qquad (10)$$

here α is the momentum compaction factor, $\mathcal{C}$ is the chromoticity and $\omega_b = 2\pi\{\nu_b\}$. To complete the one-turn we involve eq.(7) complemented with the RF term, omitting the first for simplicity and interpreting ΔE in eqs.(7) and (10) as a dynamical variable.

In the ϕ-factory project $\{\nu_b\} \ll 1$, therefore we can transform the mapping (7-9) into a differential equation. At $\mathcal{C} = 0$ and with the substitutions $\varphi = \Phi + \mathrm{atan}\,\frac{s}{2\beta}$, $\delta E = -2\Delta E$, the eqs. (7-10) become a canonical set with the three-terms Hamiltonian:

$$H = H_b + H_c + H_l$$

$$H_c = 16\pi\varepsilon_t\dot{\xi}\cdot\int_0^a\frac{1-\exp(-x^2/2\varepsilon t)}{x}dx \qquad (11)$$

$$H_b = \omega_b\cdot\varepsilon$$

$$H_l = b\cdot\left(\frac{1}{2}(\delta E)^2 - 2\cdot\frac{Uo\lambda}{\alpha}\cos\frac{s}{\lambda}\right)$$

$$a=\sqrt{\varepsilon}\cdot\cos\left(\Phi + \mathrm{atan}\,\frac{s}{2\beta o}\right),\quad b = -\frac{1}{2}\alpha$$

Thus, at low space charge parameters the motion is described with the canonical set (7-10) and the found Hamiltonian can be used for an analytical estimate of the beam life time. However at $\xi \geq 0.1$ the approximations are no more valid and more reliable conclusions are expected from the numerical simulation of these effects.

Note, that the positive chromaticity can compensate for the betatron phase modulation by the synchrotron motion (see eq.(8)). Besides, the modulation is weakened with smaller α, however this option is restricted by the $|Z|/n$ criterion for the energy spread blow-up due to dynamic bunch lengthening.

V. DISCUSSION OF THE SIMULATION RESULTS

For complete numerical simulation of the 3D motion we used the half-turn mapping where the finite bunch length was represented by division of its space charge into a dosen of thin lenses placed over the effective bunch length with the strength modulated according to β-function variation (see [3] for details) with the addition of (6) to the particle energy change in each thin lens. We could see a significant difference in the beam size dependence on ξ as compared to the case neglecting these longitudinal beam-beam effects [3]. In Fig.1 one can see the enhancement of the betatron beam size above $\xi \simeq 0.2$ while for no energy change in collision the size is practically constant in much wider range of ξ [3].

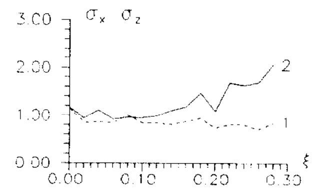

Fig.1. Normalized RMS beam sizes vs. ξ (solid line is full formulas simulation result, dots corresponds to the simulation without energy change).

Fig.2 shows the motion with large initial amplitudes ($A/\sigma \sim 5$) traced for 1000 turns (negligible damping) at $\xi = 0.2$. Both amplitudes are strongly modulated and the betatron motion looks chaotically. However the Hamiltonian (11) remains constant within 2 or 5 per cent, thus a urging the explanation of this "chaoticity" by complicated coupling. The apparent picture of this coupling is shown in Fig.3 where tracking with too low betatron tune $\{\nu\}$=0.01 reveals the betatron phase modulation. The correlation between the synchrotron and betatron motions is especially dramatic, when the betatron phase advance over one turn is compensated by the longitudinal motion: $\omega_\beta = \frac{\alpha E}{2\beta}$ Then the betatron phase advance is stopped and reversed resulting in strong changes of the both amplitudes.

Though the complexities of this Hamiltonian system are challenging and its exhaustive analysis is problematic we can make simple assumptions concerning statistical properties of betatron and longitudinal motion.

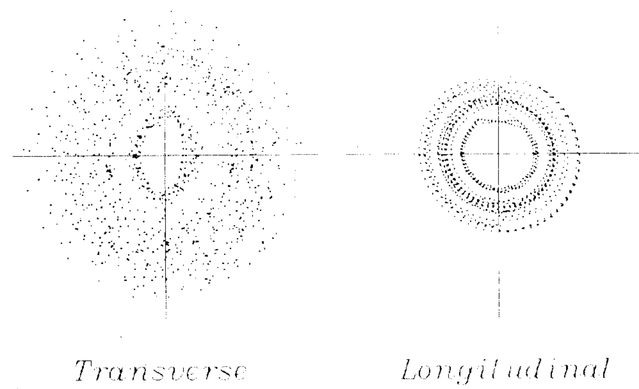

Fig.2. Betatron and synchrotron phase-space trajectories. One-dimensional betatron motion case, $\xi = 0.2$, $\{\nu_b\} = 0.05$. Initial amplitudes are: $A_b = 5\sigma_b$, $A_s = 5\sigma_s$.

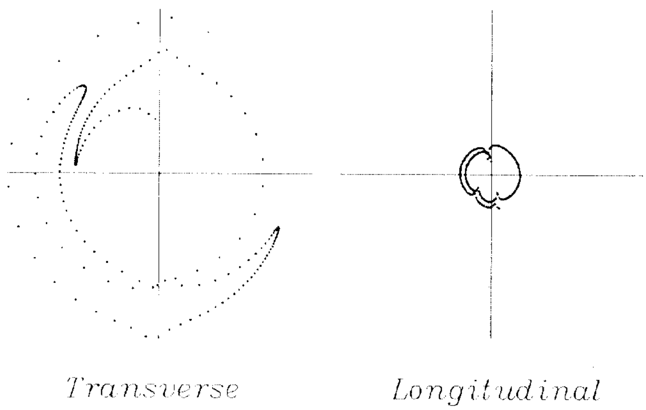

Fig.3. Same as Fig.2 except $\{\nu_b\} = 0.01$.

At low ξ the coupling term H_c in $H = H_b + H_c + H_l$ is negligible and $\bar{H}_l/\bar{H}_b \sim 3$ hence the temperature is 3 times higher for the longitudinal degree of freedom. Therefore at stronger coupling (with higher ξ) we expect predominant enhancement in the betatron beam sizes while the energy spread change is insignificant. Of course, with strong coupling these speculations are only qualitatively valid, but still can be useful as they are confirmed by the distribution of the particle excursions traced on 10^6 turns. In Fig.4 these histograms are shown for $\xi = 0.1$ and $\xi=0.2$, in the latter case one can see the sub-threshold blow up of the betatron size with the energy spread unchanged, while at higher ξ the motion is totally destroyed and

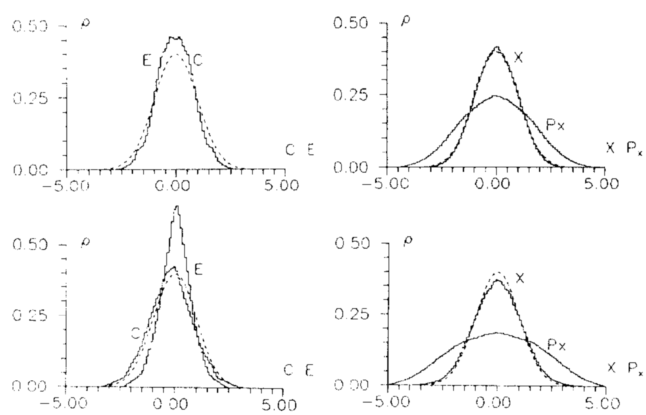

Fig.4. Synchrotron (left) and betatron (right) distributions for $\xi = 0.1$ (above) and $\xi = 0.2$ (below). The curves C,E are the longitudinal and energetic and X, P_x are the betatron coordinate and angle distributions (dots is unperturbed normal distribution).

the particle is lost before 10^6 turns.

VI. CONCLUSIONS

The effect of longitudinal electric field of the opposing bunch was originally studied for on-axis particles. For the ϕ-factory design parameters this effect results in serious reduction of longitudinal focusing thus necessitating the RF overvoltage well above 400 kV to override the longitudinal instability onset and to oppose the incoherent synchrotron tune shift.

In the present paper even more serious effect for off-axis particles has been revealed: they may gain ~±0.5 MeV energy over one IP passage. The gain is not completely averaged out due to strong modulation of the betatron phases by the synchrotron motion across the micro-β IP. The above analysis of this dynamics may explain the new simulation of the beam-beam interaction which now involves this effect: the transverse beam size blows up without a noticeable changes in the longitudinal degree of freedom and this limits the attainable space charge parameter $\xi_o \sim 0.2$.

Anyhow the scanty knowledge on these new effects urges provisions in the ϕ-factory design for wide-range operating variations of the β-function at the IP, the chromaticity, the momentum compaction factor i.e. the parameters which predominantly affect the dynamics of the longitudinal beam-beam effects.

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