INCOHERENT BEAM-BEAM EFFECTS FOR ROUND BEAMS IN THE NOVOSIBIRSK PHI-FACTORY PROJECT

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1. Introduction. The goal luminosity in the Novosibirsk phi-factory project [1.2] is in the range 10^{33} 1/cm^{2}s and very likely will strongly depend on the beam-beam interaction. Quantitatively this limitation is described via the threshold value of the beam-beam parameter $\xi = \xi_{th}$. Once for phi-factories the luminosity $L \propto \xi^2_{th}$ any effects, which can decrease the perturbations of the beam from its the counter-moving partner and therefore increase ξ_{th}, becomes very desirable. The preliminary study, which has been performed in Refs. [3,4] indicated that from this point of view the most favorable can be the configuration, where the colliding bunches are round and the working point of the ring is placed at the linear coupling resonance line $\nu_x = \nu_z$.

Recently Ref. [5] indicated the possibility of a strong suppression of the beam-beam resonance powers for round beams, if the lengths of the colliding bunches exceed the value of the β-function at the interaction point (IP) $\sigma_s \geq \beta^*$ (so-called hour-glass effect) Nevertheless, since the calculations in Ref. [5] in fact were done within the approximation $\sigma_s \ll \beta^*$, they must be confirmed for wider region.

In this paper the influence of the indicated factors on the limitation of the beam-beam parameter ξ is discussed for the weak-strong beam case.

2. Hour-glass effect. To estimate the influence of the hour-glass effect on the beam-beam interaction we shall assume Gaussian distributions in colliding bunches. The motion of a particle from the weak beam can be described by the Hamiltonian:

$$\mathcal{H} = \mathcal{H}_0 - \mathcal{L} \quad , \tag{2.1}$$

where $\mathcal{H}_0$ is the Hamiltonian of unperturbed oscillations, while the Lagrangian $\mathcal{L}$ describes perturbations of these oscillations by fields, which are induced by the counter-moving strong beam. Assuming for the sake of simplicity one interaction point (IP) per turn, for relativistic particles one can write $\mathcal{L}$ in the form

$$\mathcal{L} = 2Ne^2\rho(s+ct)\int \frac{d^2k}{\pi\, \mathbf{k}^2}\exp\left(i\mathbf{kr} - \frac{k_x^2\sigma_x^2 + k_z^2\sigma_z^2}{2}\right) \quad ,$$
$$\mathbf{k}^2 = k_x^2 + k_z^2 \; . \tag{2.2}$$

$\rho(s)$ is the linear density in the strong bunch:

$$\rho(s) = 1/\sqrt{2\pi}\,\sigma_s \cdot \exp\left(-s^2/2\sigma_s^2\right) \quad , \tag{2.3}$$

$\sigma_{x,z,s}$ are its r.m.s. sizes. Below we shall describe the particle unperturbed oscillations by the action-phase variables [6] :

$$(x,z) = (J\beta(s))^{1/2}\cos\,(\psi + \chi(s))_{x,z}, \quad \mathbf{p}_\perp = p_s\frac{d\mathbf{r}_\perp}{ds} \; ,$$
$$s = ct + R_0\varphi \quad , \quad \varphi = \varphi_0 \sin\,\psi_\parallel \quad , \tag{2.4}$$
$$\frac{d\chi}{ds} + \frac{\nu}{R_0} = \frac{1}{\beta(s)} \; , \; \dot\psi_{x,z} = \omega_0\nu_{x,z} \; , \quad \dot\psi_\parallel = \omega_c = \omega_0\nu_c \; ,$$
$$\omega_0 = c/R_0 \quad , \; I_{x,z} = (p_s J_{x,z})/2 \; , \quad I_\parallel = R_0\varphi_0^2\nu_c\frac{\gamma Mc}{2\alpha} \; .$$

Here $p_s = \gamma Mc$ is the momentum of the synchronous particle, $2\pi R_0$ is the perimeter of the orbit, α is compaction factor of the ring, we also assume zero dispersion in the interaction region. After the transformation, which is generated by eq.(2.4), $\mathcal{H}_0$ takes the form (independent variable $\vartheta_s = ct/R_0$)

$$\mathcal{H}_0 = \nu_x I_x + \nu_z I_z + \nu_c I_\parallel \tag{2.5}$$

and therefore all the variations of amplitudes ($J\alpha$) are caused by the perturbation $\mathcal{L}$:

$$J'_\alpha = \frac{dJ_\alpha}{d\vartheta_s} = \frac{\partial V}{\partial\psi_\alpha} \; , \quad \alpha = x,z \; , \quad V(J,\psi,\vartheta_s) = \frac{2R_0}{p_s c}\,\mathcal{L} \; . \tag{2.6}$$

The conventional analysis indicates the fundamental role of nonlinear resonances:

$$q_x\nu_x + q_z\nu_z + q_c\nu_c = \vec q\vec\nu = n \tag{2.7}$$

for the beam-beam instability ($q_{x,z,c}$ and n are integer numbers). In the first approximation of the perturbation theory the powers of those resonances are determined by Fourier-harmonics of $\mathcal{L}$ in phases and time:

$$\mathcal{L} = \sum_{\vec q, n} \mathcal{L}_{\vec q, n}\;(\vec J)\,\exp\,(\,i\vec q\vec\psi - in\vartheta_s\,) \quad , \tag{2.8}$$

Since the perturbations of the particle motion will be as stronger as higher are the amplitudes $\mathcal{L}_{\vec q,n}$ the relative importance of various effects can be evaluated by the comparison of these values calculated for the particular conditions.

Following this philosophy let us first estimate the dependence of powers of betatron resonances:

$$q_x\nu_x + q_z\nu_z = n \tag{2.9}$$

on the bunch length σ_s of the round counter-moving bunch for the synchronous particle. We have to calculate the integrals assuming $\varphi = 0$:

$$V_{\vec q,n} = \int_0^{2\pi}\frac{d\psi_x d\psi_z}{(2\pi)^2}\,e^{-iq_x\psi_x - iq_z\psi_z}\int_0^{2\pi}\frac{d\vartheta_s}{2\pi}\,e^{in\vartheta_s}\,V(J,\psi,\vartheta_s) \tag{2.10}$$

The powers of the betatron resonances (and their widths) will be then determined by $|V_{\vec q,n}|$. One can rewrite eq.(2.10) in the form

$$V_{q,n} = Y\cdot V^{(0)}_{q,n} \; , \tag{2.12}$$

where

$$V^{(0)}_{q,n} = 4\xi\varepsilon\int\frac{d^2k}{\pi\mathbf{k}^2}\,J_q\left(k_x(J_x/\varepsilon)^{1/2}\right)\,J_{q_z}\left(k_z(J_z/\varepsilon)^{1/2}\right)e^{-\mathbf{k}^2/2} \tag{2.13}$$

determine the powers of the beam-beam resonances in the thin lens approximation ($\sigma_s \ll \beta$), while the factors

$$Y = 2R_0\int_0^{2\pi} d\vartheta_s\;\rho(2\vartheta_s)\,\exp\left(in\vartheta_s + i(q_x+q_z)\chi(\vartheta_s)\right) \tag{2.14}$$

describe the dependence of those powers on the bunch length as well as on the phase advance of betatron oscillation on the IR. Below we shall call Y as the resonance power suppression factors. Here we used $\sigma k_\alpha \to k_\alpha$. Since in the vicinity of the (IP) one has

$$\chi_\alpha(\vartheta) = -\nu_\alpha\vartheta + \mathrm{arctg}(s/\beta^*_\alpha) \quad , \tag{2.16}$$

where β^* is the value of β-function at the (IP), eq.(1.14) with the Gaussian linear density reads

$$Y = 2\int_{-\infty}^{\infty}\frac{du}{(2\pi)^{1/2}}\,\exp(-2u^2)\,\cos\left[\,q_b\mathrm{arctg}\left(\frac{\sigma_s}{\beta^*}u\right)\right] \quad ,$$
$$q_b = |q_x + q_z| \; ,$$

or, using [7]

$$T_q(x) = \cos(q\,\arccos(x)) \; , \quad \zeta = \sigma_s/\beta^* \; ,$$

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$$Y(\zeta) = \sqrt{2/\pi} \int_{-\infty}^{\infty} du\, e^{-2u^2} T_{q_b}\left(\frac{1}{\sqrt{1+(\zeta u)^2}}\right) . \quad (2.17)$$

Once $T_q(1) = 1$, eq.(2.17) and eq.(2.12) yield the result of the thin lens approximation for resonance powers of short bunches ($Y \to 1$, $\zeta \ll 1$). In the inverse case of very long bunches ($\zeta \gg 1$), due to $T_q(0) = (-1)^q$ [7], the modulation of betatron phases over (IP) becomes too fast and asymptotically does not affect the values of resonance powers:

$$Y(\zeta) \to (-1)^{q_b} , \quad \zeta \gg 1 . \quad (2.18)$$

This result generally disagrees with predicted in the paper [5] exponential decay of $Y(\zeta)$. Nevertheless, analytical calculations from [5], which were performed in the approximation $\zeta \ll 1$, may indicate that the enlargement of the bunches interaction region (IR) and the corresponding enhancement of the betatron phase advance during the collision can suppress the powers of betatron beam-beam resonances in some intermediate region $\sigma_s \sim \beta^*$.

The results for some initial values of q_b are presented in Fig.1. These indicate the significant suppression of beam-beam resonances at least in the region $2 \le \zeta \le 4$. In wider region the behaviour of Y generally depends on q_b. From Fig.1 one can see that the region of strong suppression is as wider as higher is q_b. This tendency obviously is caused by the increase in the betatron phase advance per (IR), when q_b increases. Since eq.(2.17) yields the suppressing factor for both one and two dimensional resonances this may cause the paradoxical situation, when coupling resonances $q_x \cdot q_z < 0$ of higher order can be suppressed less than the lower order ones, if

$$|q_x|, |q_z| \gg 1 \quad \text{and} \quad |q_x| \simeq |q_z| \ .$$

3. Synchrobetatron resonances. The deviation of a particle in phase φ from the synchronous one generally distorts these dependencies. In the bunched beam this deviation periodically depends on ϑ_s with the frequency ν_c and therefore both modifies betatron resonances $q_c = 0$ and excites synchrobetatron resonances $q_c \ne 0$. The powers of these resonances can be written in the form of eq.(2.12) but with

$$Y_{q_c}(\zeta,\varphi_0) = \int_0^{2\pi} \frac{d\psi_c}{2\pi} e^{iq_c\psi_c} G(\zeta,\varphi_0 \sin\psi_c) , \quad (3.1)$$

$$G(\zeta,\varphi) = \sqrt{2/\pi}\int_{\infty}^{\infty} du \exp\left[-2\left(u - \frac{\varphi}{2\sigma_s}\right)^2 + i(q_x+q_z)\mathrm{arctg}(\zeta u)\right] \quad (3.2)$$

This expression can be easily calculated only for the case of the short IR ($\sigma_s \ll \beta^*$), when it yields

$$Y_{q_c}(\zeta,\varphi_0) = \exp\left[-\frac{1}{2}\left(\frac{q_b\sigma_s}{2\beta^*}\right)^2\right] J_{q_c}\left(q_b\frac{\varphi_0}{2\beta^*}\right) . \quad (3.3)$$

For other regions of parameters ($\sigma_s \simeq \beta^*$) integrals in eq.(3.1), eq.(3.2) should be calculated numerically. Dependencies of resonance powers on β^* are shown as examples in Figs.2,3,4 and 5. These results indicate the suppression of powers only for resonances of higher orders and strong dependence of this suppression on the amplitudes of synchrotron oscillations of the particle. A more close inspection of the last dependence (see Fig.6 and 7) shows that in the case of interest the perturbations of particles with small amplitudes of synchrotron oscillations are strongly suppressed, while for particles with high φ_0 ($\varphi_0 \gg \sigma_s$) the perturbation can reach the nominal value ($Y \to 1$). This means that in the region ($\sigma_s \simeq \beta^*$) the influence of beam-beam effects on the life time of the beam will be mainly determined by the tails of its longitudinal distribution.

4. On the suppression of coherent beam-beam instability. The results, which have been discussed in previous sections, can be used to predict the behavior of coherent beam-beam oscillations. In the simplest case, when the rigid bunch model is used to describe coherent oscillations, one may directly apply the predictions of the weak-strong calculations to the strong-strong case [8]. For instance, this means that the amplitudes of longitudinal coherent oscillations of bunches becomes of primary importance for the strength of coherent beam-beam instability. If they are small, which generally corresponds to normal conditions, coherent beam-beam resonances will be well suppressed in the region $\sigma_s \simeq \beta^*$. On the contrary, if these amplitudes occasionally exceed σ_s, coherent beam-beam resonances can reach the nominal values

$$Y_{coh} \to 1.$$

This fact can be very important for those schemes, where the realization is expected to be limited by coherent beam-beam instabilities, for instance, for so-called 4-beams compensated schemes [9] and for asymmetric colliders with different perimeters of rings [10], [11].

5. Computer simulations. To simulate the hour-glass effect within the weak-strong beam approximation in tracing code the counter moving strong beam was replaced by ten equidistantly spaced thin lenses. Then the motion of a particle was traced during 10^6 turns. To avoid the recalculation of arcs transformation matrixes describing the synchrotron motion of the particle the following mapping procedure was used. Just after the transformation from the center of one IP to the center of the next initial conditions for the particle were recalculated to the beginning of the interaction region :

$$y_{in} = \left(s - \sum_{j=1}^{10} l_j\right)/2 , \quad s = R_0\varphi , \quad x_{in} = x + p_x y_{in} , \quad (5.1)$$

where x and p_x are the values after the transformation on the arc (the same transformation for vertical motion). Then, beam-beam kicks were described by the formulae

$$\beta_j = \beta_0^* + y_j^2/\beta_0^* , \quad \sigma_j^2 = \sigma_0^2\beta_j/\beta_0^* , \quad j = 1, 2, \ldots, 10 ,$$

$$(p_x,p_z)_{j+1} = (p_x,p_z)_j - (x,z)_j \frac{4\pi\xi(j)}{\beta_j} \mathcal{F}\left(\frac{x_j^2+z_j^2}{2\sigma_j^2}\right) ,$$

$$(x,z)_{j+1} = (x,z)_j + (p_x,p_z)_j l_j , \quad y_{j+1} = y_j + l_j . \quad (5.2)$$

Before the transformation in the next arc coordinates of the particle were recalculated using

$$x_{in} = x - p_x\left(s + \sum_{j=1}^{10} l_j\right)/2 . \quad (5.3)$$

Here $\xi(j)$ are local beam-beam parameters, which satisfy the normalization condition

$$\sum_j \xi_j \equiv \xi_0 = \frac{Nr_e\beta_0^*}{4\pi\gamma\sigma_0^2} \quad \text{and} \quad \mathcal{F}(t) = \frac{1-\exp(-t)}{t} . \quad (5.4)$$

The transportation of particles between IP was described by the transformation (for short we write it only for horizontal oscillations)

$$\begin{pmatrix} x \\ p_x \end{pmatrix}_f = \lambda_x \begin{pmatrix} \cos(\pi\nu) & \beta\sin(\pi\nu) \\ -\sin(\pi\nu)/\beta & \cos(\pi\nu) \end{pmatrix}_x \cdot \begin{pmatrix} x \\ p_x \end{pmatrix}_{in} + \sqrt{(1-\lambda_x^2)(\varepsilon\cdot\beta)} \begin{pmatrix} \hat{r}_x \\ \hat{r}_p/\beta \end{pmatrix}$$

Here $\hat{r}_i$ are Gaussian random numbers with the dispersion $\langle\hat{r}_i^2\rangle \equiv 1$ and a zero average value $\langle\hat{r}_i\rangle \equiv 0$, $\lambda_\alpha = \exp(-\delta_\alpha/2)$, where $\delta_x, \delta_z, \delta_s$ are radiation damping decrements between two interaction points.

In these simulations zero chromaticity and dispersion function at the interaction point as well as real radiation damping decrements were used. The working point (ν_x, ν_z) was chosen on the main coupling resonance near an integer resonance. During the tracing the ensemble averaging of the coordinates:

$\sqrt{<x^2>}$, $\sqrt{<z^2>}$ and $\sqrt{|<x\cdot z>|}$ have been calculated.

Figs 8 ÷ 11 present the results of numerical simulations. The threshold value of ξ_{th} is decreased with the increase in the length of the weak bunch (Fig.8a) and has the maximum corresponding to the length of the strong bunch $\sigma_s/\beta_0^* \sim 1$ (Fig.8b). With a simultaneous variation of the weak and the strong bunch lengths the maximum threshold value ξ_{th} lies in the length region $\sigma_s/\beta_0^* \sim 0.2 \div 0.4$ (Fig.8c). In the synchrotron tune range $0 \le \nu_s \le 0.05$ a strong diffusion in the betatron phase space occurs only at high values of the space charge parameter $\xi_{th} \sim 0.3$ (Fig.9). Figs 11 present the results of rough estimates of the longitudinal dynamic aperture A_s/β_0^* and, hence, the life time. For a given value of ξ_* the maximum amplitude of synchrotron oscillations A_s/β_0^* is determined by the condition under which the particle in the betatron phase space does not leave the boundaries of the dynamic aperture $15\cdot\sigma_{x,z}$.

For very short bunches in the approximation $\sigma_s \ll \beta_0^*$ one could hardly hope for attaining the value of $\xi \sim 0.1$ for a space charge parameter due to the supposed short life time of the beam. In this case the role of synchrobetatron resonances is very important. As a result of energy exchange between the longitudinal and transverse motions for the particles with $A_s \ge 5\cdot\sigma_s$ a fast growth of betatron oscillation amplitudes is observed when the particles leave the boundaries of the dynamic aperture at the level of $15\cdot\sigma_{x,z}$. It is the distribution of these particles that determines the beam life time. With the bunch length increase σ_s/β_0^* for $\xi \le 0.2$ a monotonous growth of the longitudinal aperture A_s/β_0^* is observed. It is explained by an increasing factor of synchrotron resonance suppression. For higher values of the space charge parameter $\xi \ge 0.3$ the influence of betatron resonances becomes predominant. Therefore, from beam lengths of $\sigma_s/\beta_0^* \ge 0.8$ the diffusion in the betatron space is sharply increased.

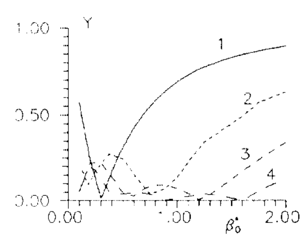

Fig.1. Betatron resonance powers suppression factors for the synchronous particle; 1. $q_b=2$, 2. $q_b=4$, 3. $q_b=6$, 4. $q_b=8$; $\sigma_s=1$.

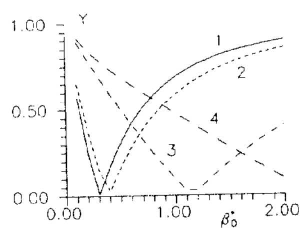

Fig.2. The betatron resonance power suppression factors vs β_0^*; $q_b=2$, $q_c=0$, 1. $\varphi_0=0$, 2. $\varphi_0=1$, 3. $\varphi_0=4$, 4. $\varphi_0=8$; $\sigma_s=1$.

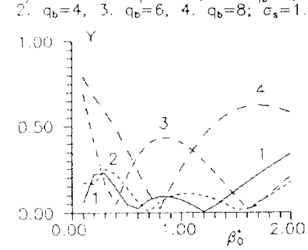

Fig.3 The same as in Fig.2, but $q_b=6$.

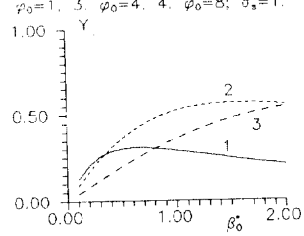

Fig.4. The synchrobetatron resonance power suppression factors vs β_0^*; $q_b=2$, $q_c=1$ 1. $\varphi_0=1$, 2. $\varphi_n=4$, 3. $\varphi_n=8$; $\sigma_s=1$.

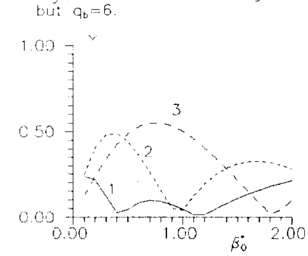

Fig.5. The same as in Fig.4, but $q_b=6$.

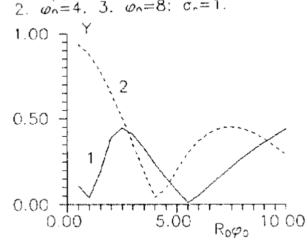

Fig.6. The betatron resonance power suppression factors vs φ_0; $q_b=6$, $q_c=0$, 1. $\beta_0^*=1$, 2. $\beta_0^*=10$; $\sigma_s=1$.

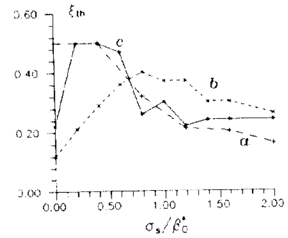

Fig.8. Threshold value of ξ vs beam length σ_s/β_0^* : a. only weak beam length is varied, b. only strong beam length is varied, c. weak and strong beam lengths are varied concurrently; $\sigma_s/\beta_0^*=0.8$, $\nu=0.08$, $\nu_s=0.02$.

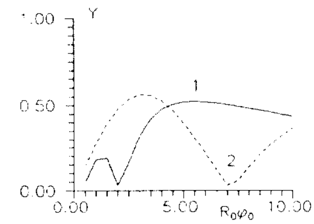

Fig.7. The same as in Fig.6, but $q_c=1$.

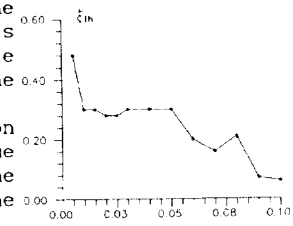

Fig.9. Threshold value of ξ vs synchrotron tune ν_s: $\sigma_s/\beta_0^*=0.8$, $\nu=0.08$.

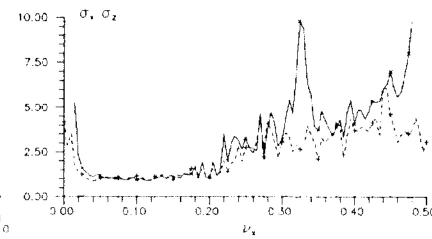

Fig.10. The coordinate averages σ_x (solid line) and σ_z (dashed line) as functions of the horizontal betatron tune ν_x : $\xi=0.2$, $\sigma_s/\beta_0^*=0.8$; $\nu_z=0.08$, $\nu_s=0.02$.

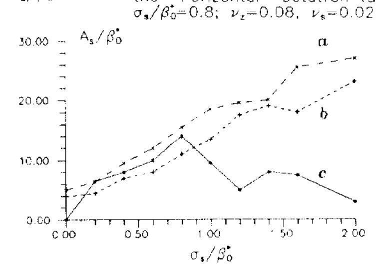

Fig.11. Longitudinal dynamic aperture A_s in the units β_0^* as function of the beam length σ_s/β_0^*: a. $\xi=0.1$, b. $\xi=0.2$, c. $\xi=0.3$; $\nu=0.08$, $\nu_s=0.02$.

6. Conclusion. On the grounds of the theoretical study and computer simulation the following conclusion can be made :

At an incoherent interaction of round beams with $\sigma_s \sim \beta_0^*$ a power suppression of betatron resonances is observed for particles with small longitudinal amplitudes $A_s \ll \sigma_s$ and an essential reduction of this effect for $A_s \ge 5\sigma_s$. As a result, the tune-shift parameter can achieved $\xi_0 \sim 0.2$ without a noticeable increase in the transverse beam size. A practical limit of the tune-shift parameter may be at a level of $\xi_0 \sim 0.1 \div 0.2$ due to a strong decrease in the beam life time ($\tau \le 100$ s).

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