

## Beam-Breakup Calculations for the DARHT Accelerator\*

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### Abstract

We have modeled an induction linac that will accelerate a 4-MeV, 3-kA beam of electrons to 16- to 20-MeV in 64 gaps. To suppress beam-breakup (BBU) instabilities induced by excitation of rf deflecting modes, the growth factor  $\Gamma$  must be kept sufficiently small (e.g.  $<3$ ). On prototype DARHT cavities, rf measurements have shown that the normally degenerate TM modes are split in frequency by the asymmetry that the two pulsed-power drive rods present to the cavity. If half the cavities had vertical and half had horizontal drive-rod orientations, the effective number of gaps would be reduced by half if there were no coupling between the modes by the solenoidal focusing and if the split modes had no overlap. The LLNL code BREAKUP was used to study BBU growth for drive rod alternation patterns of blocks of 1, 2, 4, 8, 16, 24, 32, or 64 (no alternation) for both constant and alternating polarity solenoids. For alternating polarities the optimum alternation pattern is 2 or 4, whereas for constant polarities BBU is approximately independent of pattern.

### I. INTRODUCTION

The theory of BBU for induction [1] linacs has been developed for some time. For an accelerating beam, the theory [2] shows that if the beam has steady-state radial oscillations of amplitude  $\delta r_0$  at the linac entrance, then after  $n$  successive cavities the amplitude growth  $g = \delta r_f / \delta r_0 = e^\Gamma$ , with

$$\Gamma_a = [ni(\omega Z/c)/cB_0][2/(\sqrt{(\gamma/\gamma_0)+1})] \quad (1)$$

where  $(\omega Z/c)/Q = c(\int B_y dz)^2/2U$  is the transverse cavity impedance,  $U$  is the stored energy in the cavity,  $n$  is the number of accelerating cavities,  $i$  is the current, and  $B_0$  is the rms solenoidal field. The notation  $\Gamma_a$  is used to distinguish this analytic approximation from the value calculated by the code. To reduce the centroid motion ("corkscrew") resulting from misalignments and energy spread, the total phase advance  $\phi = \int (Bdz/2B\rho)$  should be minimized [2]. For a given  $\Gamma$  this is achieved if the solenoidal fields increase with beam energy as  $B_0(\gamma/\gamma_0)^{1/2}$ . As a design criterion for DARHT (see model parameters in Table 1), we take a growth factor  $g=20$ , or  $\Gamma=3$

Table 1. DARHT Linac Model

Parameters	
$n$ , number of cells and solenoids	64
$i$ , beam current	3 kA
$B_0$ , first solenoid rms field	250 G
energy gain/cell	250 kV
length of cell	39.37 cm
$\epsilon$ , lab 4rms emittance at 20 MeV	1.5 $\mu\text{cm-mrad}$

as an upper limit. The maximum value of  $B_0$  is limited by

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beam stability considerations, a general criterion for which is that the phase advance per cell not be too large. We will use the notation  $x_B=1, 1.5, 2$  to denote field profiles that are 1, 1.5, and 2 times larger than 250 G rms in the first cell. For  $x_B=2$ , in the first magnet  $\phi_0=0.7$  rad, a reasonable upper limit for centroid motion stability and minimum corkscrew.

Since the ferrites exposed to the interior of the accelerating cavity are very lossy ( $\mu=3-20i$  at 400 MHz) [3], the cavities have low  $Q$ 's. The Briggs model [4] of a pillbox cavity with a resistive outer wall is useful in understanding the approximate parametric dependence of the cavity impedance. The impedance at the nominally  $TM_{110}$  mode resonance is  $\omega Z/c = (g/\pi b^2)Z_0\eta$ , where  $Z_0=377 \Omega$ ,  $b$  is the inner beam pipe radius,  $g$  is the accelerating gap, and  $\eta$  is a function of wall impedance  $Z_s$  and the outer cavity radius. We anticipate that  $\eta \approx 0.7-2$ , so  $\omega Z/c = (80-240)g/b^2$ .

For accelerating stress  $E_0$ ,  $nE_0g$  is the total linac voltage gain  $V$ . Then  $\Gamma \propto V/(E_0b^2B)$ , independent of the number of cells  $n$ . To minimize  $\Gamma$ , we choose the maximum  $E_0$  consistent with sparking criteria. Cost and ferrite availability have led to the choice  $b=7.5$  cm, and so  $\omega Z/c \approx 220 \Omega/\text{m}$  at  $Q=1$  for gap  $g=1.5$  cm from the Briggs model at  $x_B=2$ , with  $Q \leq 3$  required for  $370 \Omega/\text{m}$  to make  $\Gamma_a < 3$ .

The Briggs model provides guidance; the AMOS code [5] models cavities more generally using a wave impedance boundary condition at the ferrite surface. Recent cavity impedance measurements [6] with ferrites in position led to code modifications to treat the ferrites as having an effective volume magnetic-conductivity,  $\sigma_m = \omega\mu''$ , with  $\mu''$  being the imaginary part of the permeability.

Each cell is connected to two power cables with drive rods  $180^\circ$  apart. An important result of the measurements was that the rods split the normally degenerate  $TM_{110}$  modes in frequency by  $\approx 20\%$ , and the impedance curves for the two planes did not overlap substantially (Fig. 1). Mode splitting

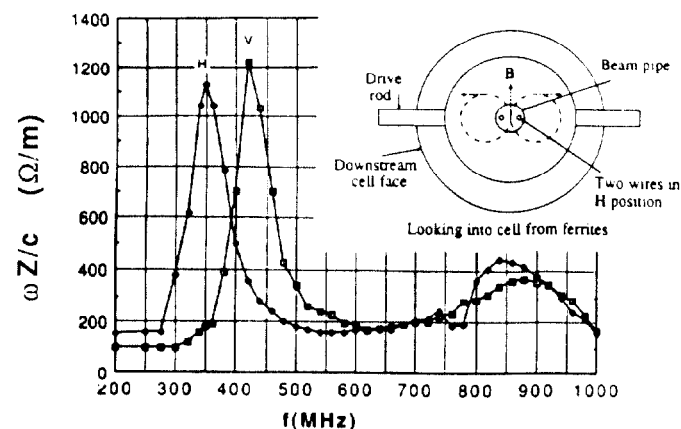


Fig. 1 Vertical and horizontal impedances of test cavity

led to the idea of a stagger tuning, for which the azimuthal orientation of the drive rods would be alternated in some pattern between horizontal (H) and vertical (V) alignment, reducing  $\Gamma$  by half in the absence of mode overlap. For constant solenoid polarity, beam rotation by the solenoids might be expected to average the impedances and make growth results insensitive to cell orientation patterns. Since each solenoid rotates the beam by  $\approx 40^\circ$ , coupling x- and y- motion, it is not clear that simple alternation of the orientation would be the best strategy. The use of alternating polarities may be advantageous in minimizing corkscrew motion at high  $B_0$ .

## II. CALCULATIONS

The BREAKUP [7] code was modified so that each cell could have independent modes for horizontal or for vertical motion. The notation  $\Gamma_+$  and  $\Gamma_{\pm}$  will be used to denote the growth factors calculated for constant and for alternating polarity solenoids, respectively. The mode parameters given in Table 2 are based on measurements [6] on the first cell design.

Table 2 Cavity Parameters

Mode	f(MHz)	Q	$\omega Z/c(\Omega/m)$	$[\omega Z/c]_{\max}(\Omega/m)$
1a	357	5	673	$\approx 417$
1b	430	6	1173	$\approx 677$
2a	890	3	354	-
2b	860	3.75	253	-

With stagger tuning,  $\Gamma$  should be approximately proportional to the average vertical and horizontal impedance  $\omega Z/c = \omega(Z_H + Z_V)/2c$  (Fig. 2). The maximum occurs near 430 MHz,

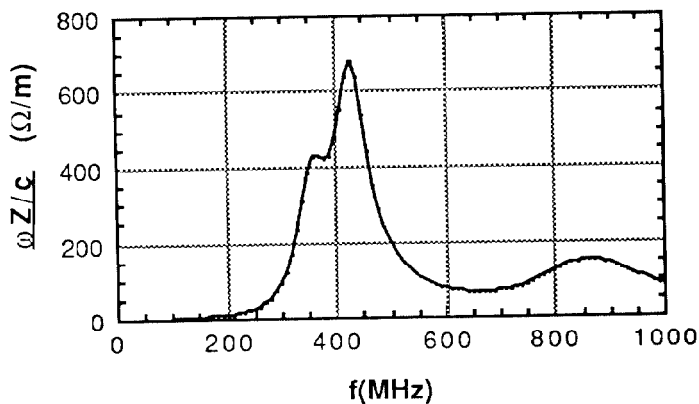


Fig. 2 Average of vertical and horizontal impedance for modes of table 2

the frequency chosen for the transverse motion of the input beam for most of the BBU calculations. Patterns labeled 1, 2, ...64 have cell drive-rod orientation alternating every cell, in blocks of two, ...or blocks of 64 (no alternation), with the first cell always having H orientation.

The beam at injection was given transverse oscillations  $\delta r_0 \sin \omega t$  in either or both planes. The solenoids in the linac could be given random tilts  $\delta \theta$  ( $3\sigma$  values) with random orientation. The beam centroid was traced in time down the linac in x-y space as well as in x-x' and y-y' spaces.

Since the beam envelope radius R, calculated with the envelope code ETRACEM [8], decreases by approximately a factor of 2.5 down the linac, g would be about 0.4 in the absence of growth.

The x-x' or y-y' phase space that is traced out by the beam at the exit can be used to calculate an emittance growth factor  $\delta \epsilon/\epsilon$  for determining permissible BBU growth. The centroid motion at the linac exit is found to be approximately given by  $x = \delta x_f \sin \omega t$  and  $x' = \delta x_f k \cos \omega t$ , and the beam phase-ellipse boundary is given approximately by  $x = R \sin \alpha$  and  $x' = k R \cos \alpha$ , with  $0 \leq \alpha \leq 2\pi$ . Then  $\epsilon = k R^2$ , with  $k = B/2B\rho = 0.0045 x_B/\text{cm}$ . For a uniform distribution within the boundary, the smearing of the final emittance by centroid motion leads to a time-averaged rms growth factor  $\delta \epsilon/\epsilon \approx 2(\delta x_f/R)^2$ . For  $\delta \epsilon/\epsilon < 10\%$ , then  $\delta x_f = g x_0 < 0.22R$ . For  $x_B = 2$ ,  $R = 7$  mm, requiring  $g x_0 < 1600 \mu\text{m}$ , or  $g < 20$  for  $x_0 = 80 \mu\text{m}$ .

When the mode asymmetry was eliminated by giving mode 1b the parameters of mode 1a, then  $\omega Z/c = 1173 \Omega/m$  at 430 MHz. We calculated  $\Gamma_{\pm} = 12.9$  and  $\Gamma_+ = 11.2$  for  $x_B = 1$ , about  $0.6\Gamma_a$ . Using the modes of Table 2 again, growth was calculated for orientation patterns 1, 2, 4, 8, 24, 32, 64. Pattern 24 was chosen because the phase advance in the first 24 cells is approximately the same as in the last 40. The input beam was given a transverse amplitude of  $25 \mu\text{m}$  at 430 MHz in the H plane. As expected,  $\Gamma_+$  was fairly insensitive to the pattern (Fig. 3), ranging from 7.5 to 5.5, but  $\Gamma_{\pm}$  ranged from

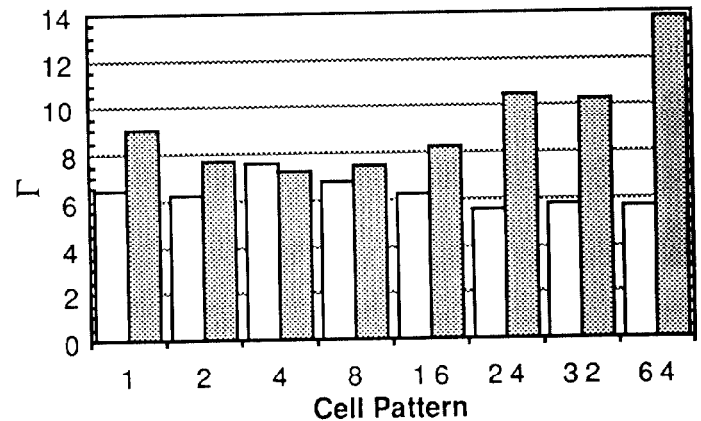


Fig. 3  $\Gamma(1X)$  vs mode pattern for constant- (open) and for alternating-polarity (shaded) solenoids

13.8 to 7. Pattern 4 gave the smallest  $\Gamma_{\pm}$  but the highest  $\Gamma_+$ . Pattern 2 was chosen for most of the rest of the studies because it had low growth for both cases and because of mechanical assembly considerations. The constant polarity solenoid tune had the lowest growth. Pattern 64 gave very high  $\Gamma_{\pm}$ . Since beam rotation for alternating polarities through the linac changes by only about  $\pm 20^\circ$  for  $x_B = 2$ , this is not surprising. The values of  $\Gamma_+$  and  $\Gamma_{\pm}$  were about  $0.62\Gamma_a$ .

Changing the plane of the transverse oscillation from H to V reduced  $\Gamma_{\pm}$  (Fig. 4) by 2-25%. Since  $x_B \Gamma$  should be constant according to Eq.(1), the ratios  $\Gamma(1X)/1.5\Gamma(1.5X)$  and  $\Gamma(1X)/2\Gamma(2X)$  were calculated for alternating polarities and are seen to be close to unity with a few exceptions (Fig. 5). There

was no evident dependence of  $\Gamma$ , within  $\pm 1\%$ , on  $\delta\theta$  in the range 0- to 5-mrad in any of the calculations. We anticipate that DARHT misalignments will not exceed these values and

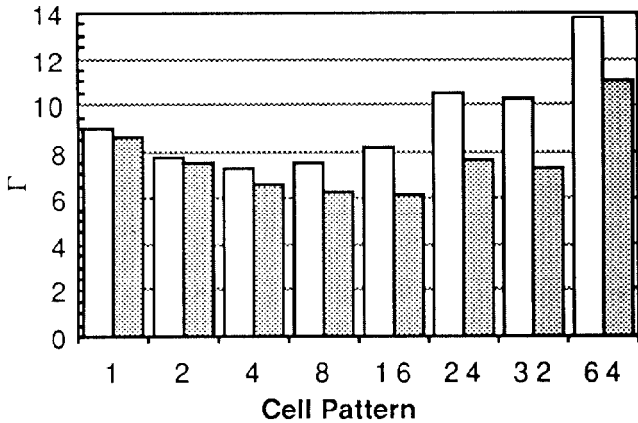


Fig.4  $\Gamma_{\pm}(1X)$  vs mode pattern for initial beam oscillation in H- (open) and V- (shaded) planes

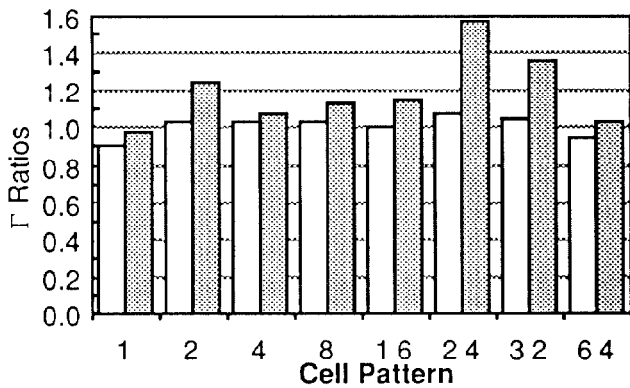


Fig. 5 Ratios  $\Gamma(1X)/1.5\Gamma(1.5X)$  (open) and  $\Gamma(1X)/2\Gamma(2X)$  (shaded) vs mode pattern for alternating polarities

that with steering corrections applied, the effective  $\delta\theta$  will be much less. To test the parametric dependence of  $\Gamma$  on  $\omega Z/c$ , we set the input oscillation frequency at 357 MHz, for which frequency  $\omega Z/c=417 \Omega/m$ , 62% of the 430-MHz value. The results (Table 3) follow the predicted scaling with B better at 430 MHz than at 357 MHz.

Table 3  $\Gamma$  for the two dominant frequencies vs  $x_B$

$x_B$	$\Gamma_{430}$	$x_B\Gamma_{430}$	$\Gamma_{357}$	$x_B\Gamma_{357}$	$(\Gamma_{357}/\Gamma_{430})$
1	7.7	7.7	7.8	7.8	1.01
1.5	5	7.5	4	6	0.8
2	3.1	6.2	2.2	4.4	.71

The beam from the REX injector [9] has transverse motion of 100  $\mu m$  at 235 MHz and of 25  $\mu m$  at 470 MHz at approximately the linac injection point. BREAKUP calculations showed that for  $x_B=1$ , damped oscillations with amplitudes  $\approx 1$  cm are excited at either 357 MHz or 430 MHz, making it difficult to estimate the steady-state gain. For  $x_B=2$ , the transient damps quickly. The gains are  $\approx 0.5$  and 1.7, or  $\Gamma_{\pm} \approx -0.7$  and 0.4, as compared with  $\Gamma_a \approx 0.15$  and 3,

respectively. Thus there is essentially no gain at 235 MHz, considering the beam compression factor.

If the beam has no initial transverse oscillation, magnet and beam alignment errors will cause transient excitation of BBU oscillations at 430 MHz. The maximum peak-peak amplitude at the linac exit during the "flat top" part of the beam pulse for random angular errors  $\delta\theta$  from 1- to 5-mrad in the absence of initial beam motion. A good approximation to the results was  $\delta r_{max}(cm) \approx (0.072 \pm 0.015)\delta\theta/x_B$ , with the uncertainty being due to the scatter inherent in statistical misalignment errors. Applying the emittance growth criterion,  $\delta\theta < 4$  mrad is required for  $x_B=2$ . This emittance criterion is probably too strict, as the transient damps with a time constant  $\tau(ns) \approx 24/x_B$ .

### III. CONCLUSIONS

The cell-orientation pattern 2 appears to be the best choice for DARHT from BBU growth and mechanical design considerations for alternating polarities. The code-calculated values of  $\Gamma$  are approximately 60% of the analytically-calculated value  $\Gamma_a$ , and the parametric dependence is approximately the same. Solenoid misalignment do not contribute significantly to growth. For the DARHT design parameters and the expected cell impedances, growth is less than 20 at the 430-MHz resonance, and considerably lower at those frequencies for which the REX injector beam has measurable transverse oscillation. Growth is lower for constant- than for alternating-polarity solenoids.

### IV. References

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