

SYNCHROTRON RADIATION PERTURBATIONS IN LONG TRANSPORT LINES

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Abstract

Transverse and longitudinal emittance growths due to synchrotron radiation in long beam transport lines are calculated using a very simple analytical method. Results show that these emittance growths can be minimized when some particular optical conditions are fulfilled.

INTRODUCTION

As they travel along a beam line containing many bending magnets electrons lose energy in the form of synchrotron radiation. This radiation may be very important at high energy and degrade strongly the beam qualities (transverse and longitudinal emittances) because the energy loss of each particle differs from the average as a consequence of statistical fluctuations in the number and energy of the emitted photons.

In order to describe the behavior of the phase space ellipses in beam lines which have not necessarily a periodic structure and where the input beam is not necessarily matched to the Twiss ellipses, we present here a very simple analytical method based on a statistical point of view (rms emittance).

Such a method has been previously used in other similar works for calculating transverse emittance growth [1],[2],[3].

In addition, it will be shown that some perturbations of the output beam may be minimized by a proper choice of the beam line optical properties.

TRANSVERSE MOTION

Let us describe a collection of electrons in a (X, X') phase space, where for each particle

$$X = x - \bar{x} \quad X' = x' - \bar{x}' \quad (1)$$

x, x' being the betatron coordinates, and the symbol $\bar{\cdot}$ denotes an average over the particles.

Let us consider an arc with s_i, s_f the azimuths at its entrance and its exit, and s_e a given azimuth where a photon emission arises.

The RMS ellipse associated with the collection of electrons at the azimuth s_e is defined by

$$\begin{aligned} & \overline{X'^2(s_e)} X^2 - \overline{X X'(s_e)} X X' + \overline{X^2(s_e)} X'^2 \\ & = 4 \left[\overline{X^2} \overline{X'^2} - (\overline{X X'})^2 \right] (s_e) = \frac{1}{4} \left[\frac{S(s_e)}{\pi} \right]^2 \quad (2) \end{aligned}$$

$S(s_e)$ being the ellipse surface.

Introducing new notations

$$\begin{aligned} \gamma^*(s_e) &= 4 \frac{\overline{X'^2(s_e)}}{S/\pi} \\ \alpha^*(s_e) &= -4 \frac{\overline{X X'(s_e)}}{S/\pi} \\ \beta^*(s_e) &= \frac{\overline{X^2(s_e)}}{S/\pi} \end{aligned} \quad (3)$$

with $\beta^* \gamma^* - \alpha^{*2} = 1$, Eq.(2) becomes

$$\gamma^*(s_e) X^2 + 2\alpha^*(s_e) X X' + \beta^*(s_e) X'^2 = \frac{S(s_e)}{\pi} \quad (4)$$

or in matrix form

$$[X, X'] \sigma(s_e) \begin{bmatrix} X \\ X' \end{bmatrix} = 1 \quad (5)$$

with the following definition of the beam matrix

$$\begin{aligned} \sigma(s_e) &= \frac{S(s_e)}{\pi} \begin{bmatrix} \beta^*(s_e) & -\alpha^*(s_e) \\ -\alpha^*(s_e) & \gamma^*(s_e) \end{bmatrix} \\ &= 4 \begin{bmatrix} \overline{X^2(s_e)} & \overline{X X'(s_e)} \\ \overline{X X'(s_e)} & \overline{X'^2(s_e)} \end{bmatrix} \end{aligned} \quad (6)$$

It follows evidently that

$$\det \sigma(s_e) = \left[\frac{S(s_e)}{\pi} \right]^2 \quad (7)$$

In the absence of synchrotron radiation perturbation, the σ matrix at azimuth s_f is related to that at s_e by

$$\sigma(s_f) = T(s_f \leftarrow s_e) \sigma(s_e) \tilde{T}(s_f \leftarrow s_e) \quad (8)$$

where T is the transfer matrix between s_e and s_f , and \tilde{T} its transpose.

When an electron emits a photon of energy $\varepsilon > 0$ at azimuth s_e , the local change of the betatron coordinates implies

$$\delta X(s_e) = D(s_e) \frac{\varepsilon}{E} \quad \delta X'(s_e) = D'(s_e) \frac{\varepsilon}{E} \quad (9)$$

D and D' being the dispersion function and its derivative.

Since to first order the emission mechanism is independent of the electron location, we can consider, neglecting the energy spread, that each electron radiates \bar{N} photons of mean energy $\bar{\varepsilon}$ by unit of time.

Therefore the resulting beam matrix perturbation over an interval $ds_e = cdt$ is

$$d\sigma(s_e) = 4 \frac{\dot{N}}{c} \frac{\bar{\varepsilon}^2}{E^2} ds_e \begin{bmatrix} D^2(s_e) & D(s_e)D'(s_e) \\ D(s_e)D'(s_e) & D'^2(s_e) \end{bmatrix} \quad (10)$$

Since the elementary perturbation $d\sigma(s_e)$ can be transported as $\sigma(s_e)$, the total change of the σ matrix at s_f is

$$\Delta\sigma(s_f) = \int_{s_i}^{s_f} T(s_f \leftarrow s_e) \frac{d\sigma(s_e)}{ds_e} \tilde{T}(s_f \leftarrow s_e) ds_e \quad (11)$$

relation which can be written in a more practical form.

Indeed, by solving the equation of motion by the constant variation method, one has

$$\begin{bmatrix} D(s_e) \\ D'(s_e) \end{bmatrix} = T(s_e \leftarrow s_i) \begin{bmatrix} U(s_e) \\ V(s_e) \end{bmatrix} \quad (12)$$

with

$$\begin{bmatrix} U(s_e) \\ V(s_e) \end{bmatrix} = \begin{bmatrix} D(s_i) - \int_{s_i}^{s_e} \frac{T_{12}(s \leftarrow s_i)}{\rho(s)} ds \\ D'(s_i) + \int_{s_i}^{s_e} \frac{T_{11}(s \leftarrow s_i)}{\rho(s)} ds \end{bmatrix} \quad (13)$$

Therefore

$$\frac{d\sigma(s_e)}{ds_e} = 4 \frac{\dot{N}}{c} \frac{\bar{\varepsilon}^2}{E^2} T(s_e \leftarrow s_i) \begin{bmatrix} U^2(s_e) & U(s_e)V(s_e) \\ U(s_e)V(s_e) & V^2(s_e) \end{bmatrix} \tilde{T}(s_e \leftarrow s_i) \quad (14)$$

and

$$\Delta\sigma(s_f) = T(s_f \leftarrow s_i) G(s_i, s_f) \tilde{T}(s_f \leftarrow s_i) \quad (15)$$

where $G(s_i, s_f)$, depending only on the arc structure, is a matrix determinated at the arc entrance and equivalent to the total radiative perturbation :

$$G(s_i, s_f) = 4 \left(\frac{\sigma_E}{E} \right)^2 \begin{bmatrix} \langle U^2 \rangle & \langle UV \rangle \\ \langle UV \rangle & \langle V^2 \rangle \end{bmatrix} \quad (16)$$

In this relation, $\langle * \rangle$ denotes an average over the dipoles $\langle * \rangle = \frac{1}{L_{dip}} \int_{dip} (*) ds_e$

and $\left(\frac{\sigma_E}{E} \right)^2 = \frac{\dot{N}}{c} \frac{\bar{\varepsilon}^2}{E^2} \rho \Delta\theta = 1.44 \cdot 10^{-27} \frac{\gamma^5}{\rho^2} \Delta\theta$ is the classical induced energy spread with

- γ the Lorentz factor
- ρ the bending radius in meter
- $\Delta\theta$ the turn angle of the arc in radian.

From eq.(15) it follows that if the input beam is characterized by a matrix $\sigma(s_i)$ corresponding to

an emittance $\frac{S}{\pi}(s_i) = [\det \sigma(s_i)]^{1/2}$, the phase space ellipse at the arc exit is determined by

$$\sigma(s_f) = T(s_f \leftarrow s_i) [\sigma(s_i) + G(s_i, s_f)] \tilde{T}(s_f \leftarrow s_i) \quad (17)$$

$$\frac{S}{\pi}(s_f) = [\det (\sigma(s_i) + G(s_i, s_f))]^{1/2} \quad (18)$$

In addition to the emittance blow up, synchrotron radiation induces a modification of \bar{x} , the center of gravity of the distribution.

At the arc exit, $\overline{x(s_f)}$ is given by

$$\begin{bmatrix} \overline{x(s_f)} \\ \overline{x'(s_f)} \end{bmatrix} = T(s_f \leftarrow s_i) \left[\begin{bmatrix} \overline{x(s_i)} \\ \overline{x'(s_i)} \end{bmatrix} + \frac{\sigma_E}{E} \begin{bmatrix} \langle U \rangle \\ \langle V \rangle \end{bmatrix} \right] \quad (19)$$

When developing all these expressions, numerous interesting results arise and emphasize the importance of the optical properties of the considered beam line.

Without entering here into the details of calculations (see [4]), the main results are summarized below :

- Generally, from Eq.(18) $\frac{S(s_f)}{\pi} \geq \frac{S(s_i)}{\pi} + \left(\frac{\Delta S}{\pi} \right)_0$ where $\left(\frac{\Delta S}{\pi} \right)_0$ is the emittance growth for a zero initial emittance. Equality, i.e. the minimum emittance growth, only occurs when $\sigma(s_i)$ is proportional to $G(s_i, s_f)$, that means a particular matching of the input beam.

- If now the arc is composed of n identical and achromatic cells with the same transfer matrix M so that $T_{arc} = M^n = [I]$, then the phase-space ellipse associated with the radiative perturbation $\left(\frac{\Delta S}{\pi} \right)_0$ is matched to the Twiss ellipse of the cell at the arc exit. In this case, $\left(\frac{\Delta S}{\pi} \right)_0$ is given by the classical formula

$$\left(\frac{\Delta S}{\pi} \right)_0 = 1.44 \cdot 10^{-27} \frac{\gamma^5}{\rho^2} \Delta\theta \langle \mathcal{H} \rangle \quad (20)$$

where

$$\langle \mathcal{H} \rangle = \frac{1}{L_{dip}} \int_{arc} \frac{ds}{\beta_x} [D^2 + (\alpha_x D + \beta_x D')^2]$$

β_x, α_x being betatron functions derived from the Twiss ellipse.

- If the arc is composed of two symmetrical subsystems with transfer matrices $M_1 = M_2 = [-I]$, then $\overline{x(s_f)} = \overline{x(s_i)}$ and $\overline{x'(s_f)} = \overline{x'(s_i)}$, i.e., an initially centered beam is also centered at the arc exit. It is due to the fact that all shifts of the center of gravity of the distribution are two by two equal and π phase spaced.

LONGITUDINAL MOTION

A similar approach has been developed to investigate the effect of synchrotron radiation on longitudinal emittance [5]. With a collection of electrons in the (ℓ, δ) phase space the beam matrix at azimuth s_f is

$$\sigma(s_f) = 4 \begin{bmatrix} \overline{\ell^2(s_f)} & \overline{\ell\delta(s_f)} \\ \overline{\ell\delta(s_f)} & \overline{\delta^2(s_f)} \end{bmatrix} \quad (21)$$

To estimate the perturbation matrix $\Delta\sigma(s_f)$, we have to calculate the trajectory lengthening at s_f due to photon emission at s_e , having in mind the relations (9) :

$$\begin{aligned} d\ell &= \int_{s_e}^{s_f} \frac{\delta x(s)}{\rho(s)} ds \\ &= \frac{\varepsilon}{E} \left[D(s_e) \int_{s_e}^{s_f} \frac{T_{11}(s \leftarrow s_e)}{\rho(s)} ds \right. \\ &\quad \left. + D'(s_e) \int_{s_e}^{s_f} \frac{T_{12}(s \leftarrow s_e)}{\rho(s)} ds - \int_{s_e}^{s_f} \frac{T_{16}(s \leftarrow s_e)}{\rho(s)} ds \right] \\ &= \frac{\varepsilon}{E} [D(s_e)T_{51}(s_f \leftarrow s_e) \\ &\quad + D'(s_e)T_{52}(s_f \leftarrow s_e) - T_{56}(s_f \leftarrow s_e)] \end{aligned} \quad (22)$$

Averaging over the particles and integrating over the azimuth s_e , one can obtain the trajectory lengthening $\overline{\Delta\ell}$, or the bunch lengthening characterized by $[\overline{(\Delta\ell)^2}]^{1/2}$, or more generally the perturbation matrix

$$\begin{aligned} \Delta\sigma(s_f) &= 4 \begin{bmatrix} \overline{(\Delta\ell)^2} & \overline{\Delta\ell\delta} \\ \overline{\Delta\ell\delta} & \overline{\delta^2} \end{bmatrix} \\ &= 4 \left(\frac{\sigma_E}{E} \right)^2 \begin{bmatrix} \langle (DT_{51} + D'T_{52} - T_{56})^2 \rangle & - \langle DT_{51} + D'T_{52} - T_{56} \rangle \\ - \langle DT_{51} + D'T_{52} - T_{56} \rangle & 1 \end{bmatrix} \end{aligned} \quad (23)$$

And the beam matrix at the arc exit is

$$\sigma(s_f) = \sigma_0(s_f) + \Delta\sigma(s_f) \quad (24)$$

where $\sigma_0(s_f)$ is the beam matrix in absence of radiative perturbation.

In the most general case, synchrotron radiation induces therefore a bunch lengthening which is proportional to the induced energy spread and which is eventually enhanced by the transverse motion (T_{51}, T_{52} terms).

Analysis of eq.(23) shows that this lengthening can be minimized when some optical conditions are fulfilled. For instance :

- if the structure is achromatic, then,

$$DT_{51} + D'T_{52} \equiv 0 \quad (25)$$

which means that $\Delta\sigma(s_f)$ does not depend on transverse motion.

- if, in addition, the structure is symmetric and isochronous, then

$$\langle T_{56} \rangle = 0 \quad (26)$$

and therefore, eq.(23) can be reduced to the following expression :

$$\Delta\sigma(s_f) = 4 \left(\frac{\sigma_E}{E} \right)^2 \begin{bmatrix} \langle (T_{56})^2 \rangle & 0 \\ 0 & 1 \end{bmatrix} \quad (27)$$

In this last case, it is interesting to notice that the output ellipse is erect ($\Delta\sigma_{12} = 0$) and therefore that the bunch has a minimum length.

It has to be noticed that these analytical results have been confirmed by numerical computations with the computer code DYNAC developed at LNS [6],[7],[8].

CONCLUSION

The analytic formalism developed above shows that the beam matrix representing synchrotron radiation perturbations are proportional to $(\sigma_E/E)^2 \sim \gamma^5/\rho^2$.

Therefore, the induced emittance growths increase drastically with energy for a given bending radius.

In order to reduce beam degradations while conserving reasonable bending radius, our results suggest that the use of optical structures such as isochronous and symmetric achromats is highly desirable.

In addition matching of the input beam to the Twiss ellipses of the achromat cell is necessary.

In this case, bunch lengthening is minimized and

transverse emittance growth is only governed by the $\langle \mathcal{H} \rangle$ function which evidently has to be as small as possible.

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