© 1991 IEEE. Personal use of this material is permitted. However, permission to reprint/republish this material for advertising or promotional purposes or for creating new collective works for resale or redistribution to servers or lists, or to reuse any copyrighted component of this work in other works must be obtained from the IEEE.

On the Low-Beta Optics for Round and Flat Beams in Colliders

D. Möhl
PS Division, CERN, CH-1211 Geneva 23

Abstract

Low-beta insertions for round and flat beams are analyzed. It is found that the potential advantage in luminosity with round beams is offset by the difficulty to have a small beta in both planes. Scaling laws are discussed which provide a general overview of the performance of low-beta insertions.

I. INTRODUCTION

Ideally the use of round beams with equal horizontal and vertical sizes, $\sigma_v = \sigma_h$ should gain a factor of two in the luminosity of head-on colliders. This is due to the factor $1 + \sigma_v/\sigma_h$ in the luminosity limit as set by the beam-beam effect [1].

This gain can, however, be offset by other difficulties related to round beams like: i) lower tolerable beam-beam tune shifts although this remains a controversial subject [2, 3], ii) shorter luminosity lifetime [4], and iii) difficulties of the required low beta in both planes.

To quantify point iii) above, which is the object of the present note, we will determine the concurrent maximum beta values and related to this the contribution of the insertion to the chromaticity Q' = dQ/(dp/p).

Both parameters depend on the quadrupole strength and on the minimum distance available between the first lens and the interaction point. For the purpose of comparison we shall define "standard conditions". It will then emerge that the maximum beta and the chromaticity are typically 3 to 5 times larger in round than in flat beams, "all other parameters equal".

II. ASSUMPTIONS

We take fixed low-beta values: $\beta_v^* = 1$ cm; $\beta_h^* = 50$ cm for the flat and $\beta_v^* = \beta_h^* = 1$ cm for the round beam. We assume a first quadrupole starting at d = 50 cm from the interaction point. The space between the quadrupoles is $d_1 = 10$ cm. Scaling to other low-beta values and other geometries will be discussed below.

A doublet is used to match the flat and a triplet to match the round beam. For the calculation the quadrupoles have the same absolute strength $|k| = |(\partial B/\partial x)/B\rho|$. Their lengths are adjusted to optimize the maximum beta. To be specific the additional condition is imposed that both beta functions have zero derivative at the exit. Different slopes, as they will be required for matching to a given storage ring lattice, modify the results of the comparison very little. In all cases the tracking routine of the program MAD [5] has been used to determine the parameters of the "optimized" insertion. Examples for k = 8 m^{-2} are given in Figs. 1 and 2.

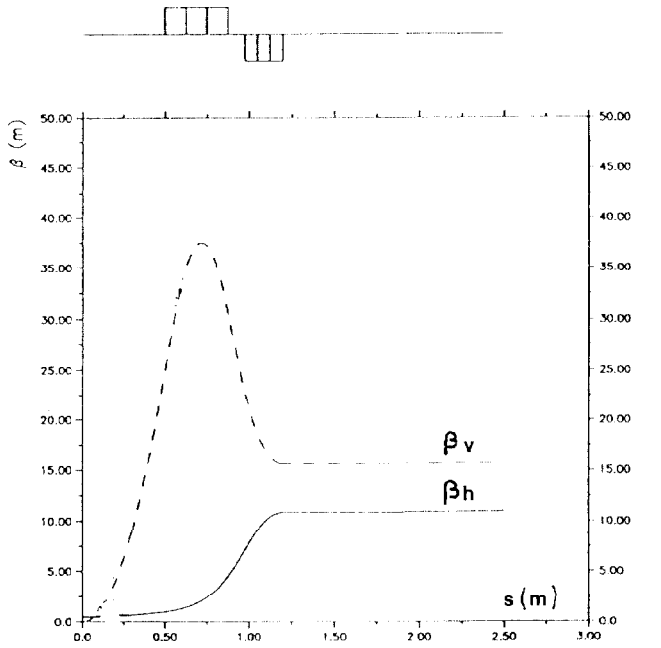

Figure 1. Flat-beam low-β insertion with $\beta_h^* = 50$ cm, $\beta_v^* = 1$ cm and k = 8 m^{-2}. The distance to the first quadrupole is d = 0.5 m, the inter-quadrupole space is 0.1 m.

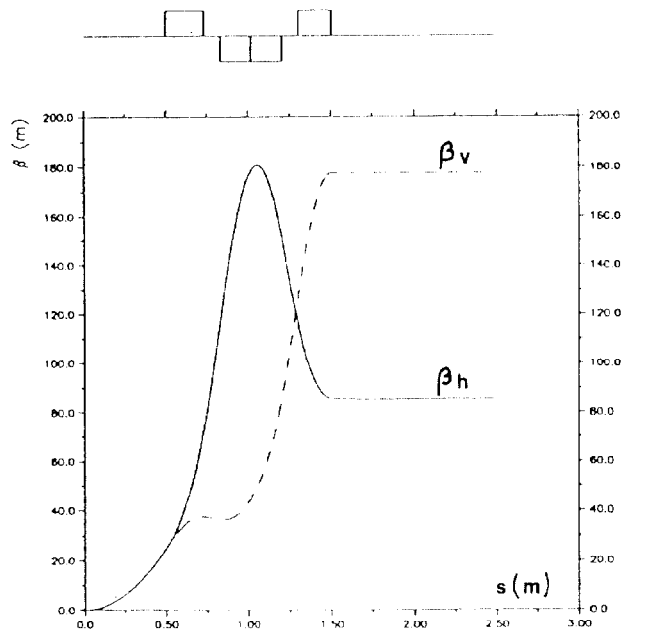

Figure 2. Round-beam low-β insertion with $\beta_h^* = \beta_v^* = 1$ cm and k = 8 m^{-2}. The distance to the first quadrupole is d = 0.5 m, the inter-quadrupole space is 0.1 m.

0-7803-0135-8/91$01.00 ©IEEE

III. RESULTS

The maximum beta values are given in Fig. 3 as a function of the quadrupole strength. The available strength depends on the beam energy, the aperture and the technology used. Assuming a gradient $\partial B/\partial x$ in the range of 20 T/m ("normal quadrupole") to 200 T/m ("superconducting quadrupole") we expect typical k values as indicated in Table 1.

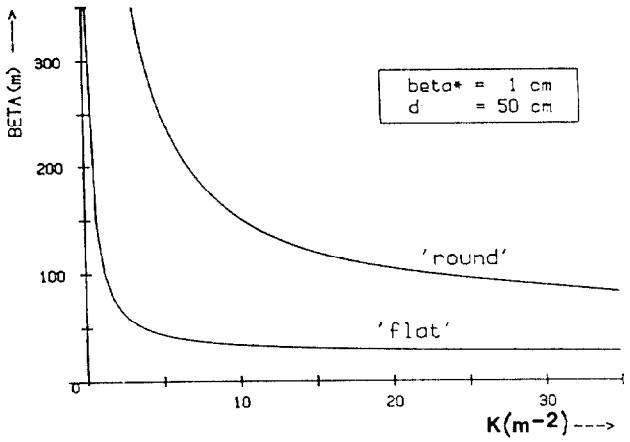

Figure 3.- Maximum β vs. quadrupole strength.

Table 1
Typical values of quadrupole strength k for different energies

Energy	e+e- machine	Quadrupole strength k for B' = 20 to 200 T/m
5.3 GeV	B-factory (symmetric)	1.2 - 12 m^{-2}
2.0 GeV	T-C factory	3.0 - 30 m^{-2}
0.5 GeV	φ factory	12.0 - (120) m^{-2}

One concludes from Fig. 3 that for the k values between 5 and 30 m^{-2} the peak round beam betas are a factor of 5 to 3 larger. For smaller k the difference is even more pronounced. In addition, both betas are large whereas for the flat beam the horizontal beta remains small.

There are several difficulties related to such large betas. The beam size increases with the square root and the effect of errors and stray fields of the quadrupole increases (roughly) linearly with the beta values.

To preserve the geometrical and dynamical acceptance one has to limit the peak betas and this requires larger betas at the interaction point such that the factors of 3 to 5 override the luminosity gain.

One important "error" of the quadrupoles comes from the momentum dependence of their strength $k = k_0/(1 + \Delta p/p)$. The corresponding contribution to the chromaticity

$$Q' = \frac{\Delta Q}{\Delta p / p} = -\frac{1}{4\pi}\int k_0 \beta \, ds$$

is typically 2 to 3 times larger for a round than for a flat beam (Fig. 4).

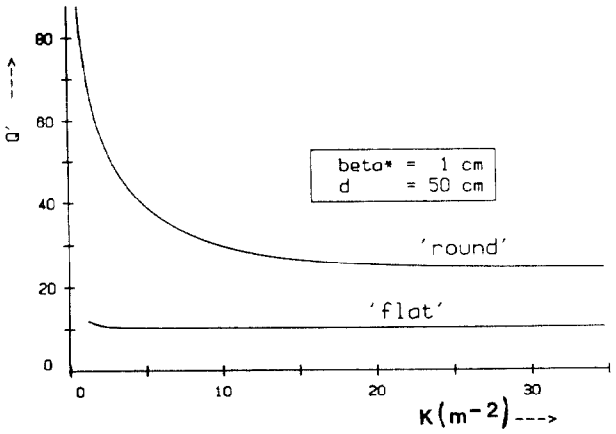

Figure 4. Chromaticity contribution of the full insertion vs. quadrupole strength.

IV. UNEQUAL ENERGY BEAMS

To assess the important case of unequal energies [6], a rough design of the optics for a 3 GeV against 9 GeV collider has been done using 4 quadrupoles common to both beams, with $|k| = 8\ m^{-2}$ at 9 GeV. Results indicate the same relative difference between round and flat beam conditions as for the symmetric cases discussed above, namely a factor of 4 difference in the maximum betas and a factor of 3 in the chromaticities.

V. SCALING

From the "beam envelope equation", for $w = \sqrt{\beta}$ (Eq. (3.2) of Ref. [7]) the usefulness of the following scaled variables can be derived [7]:

$$W = w\sqrt{\beta^*}/d$$
$$S = s/d$$
$$K = kd^2$$
$$Q'(k) = Q'\beta^*/d$$

To the extent that the nonlinear $1/W^3$ term of the envelope equation can be neglected in the region of the quadrupoles, the equation has unique solutions in these variables (see Appendix) from which the actual beta values, chromaticities and quadrupole parameters can be derived.

We can therefore generalize Figs. 3 and 4 by using $\beta_{max}/(d^2/\beta^*)$ and $Q'/(d/\beta^*)$ as universal ordinates and kd^2 as abscissa.

VI. CONCLUSIONS

Round beams look inferior to flat beams in e^+e^- head-on colliders where the vertical emittance can be made small and where the design of the low beta insertion has to be pushed to the limit in order to obtain highest luminosity.

Scaling laws have been derived which are convenient to assess the performance of a large class of low beta insertions.

CREDIT AND ACKNOWLEDGEMENTS

Thanks are due to Yves Baconnier, Roberto Giannini, and Leonid Rivkin for helpful discussions and suggestions.

Analytical expressions for low-beta insertions are discussed in Ref. [9]. M. Donald has drawn my attention to the work of A. Hutton [10] who discusses additional advantages of flat beams and to that of V.A. Lebedev [11] who compares two different low beta designs for a 4 GeV against 7 GeV collider.

REFERENCES

[1] M. Sands, "The Physics of Electron Storage Rings", SLAC-Report-121, 1970.

[2] S. Krishnagopal and R. Siemann, "Simulation of Round Beams", in: *Proc of the 1989 IEEE Particle Acc. Conf.*, Chicago, 1989, p. 836.

[3] K. Hirata, "Round Beams and Flat Beams in the Phi-Factory", INFN (Frascati), Internal Report ARES-15 (15.12.1989).

[4] M. Donald, "Round versus Flat Beams, Integrated Luminosity Considerations", SLAC Internal Report, B-Factory Collider Note ABC1, 1990.

[5] H. Grote and C. Iselin, "The MAD Program", CERN SL/90-13 (AP).

[6] A.A. Garren et al., "An Asymmetric B-Meson Factory at PEP", in: *Proc. of 1989 IEEE Particle Acc. Conf.*, Chicago, 1989, p. 1847.

[7] E.D. Courant and H.S. Snyder, "Theory of the Alternating Gradient Synchrotron", Annals of Phys., Vol. 3, pp. 1-48, (1958).

[8] D. Möhl, "On the Low-Beta Optics for Round and Flat Beams in Colliders", CERN-PS/AR/Note 90-15, 1990.

[9] B. Autin, "Analytical Model of Low-Beta Insertions for Flat Beams at the Interaction Point", CERN PS/90-75 (AR), and *Proc. of the Workshop on B-Factory*, Tsukuba, Japan, 1990.

[10] A. Hutton, "The Advantages of Flat Beams in Head-on Collisions", in: *Proc. of the Workshop on Beam Dynamics Issues of High Luminosity Asymmetric Collider Rings, AIP Conf. Proc. 214*, 1990, p 284.

[11] V.A. Lebedev, "Comparison of Luminosities for Round and Flat Beams", Ibid. p. 289.

APPENDIX

Expressed in the scaled variables the beam envelope equation takes the form

$$W''(S) + K(S)W(S) - \left(\frac{\beta^*}{d}\right)^2 \frac{1}{W^3(S)} = 0$$

In the region of the quadrupoles the $1/W^3$ term can usually be neglected, especially for the vertical plane, where $(\beta^*/d)^2 \ll 1$. Near the low-beta point (s = 0) the envelope has the well-known solution [1]:

$$\beta = \beta^* + \frac{s^2}{\beta^*}$$

Again, under the condition that $(\beta^*/d)^2 \ll 1$, this leads to the boundary conditions W(1) = 1, W'(1) = 1 at the entrance of the first quadrupole. For given K(S) the solution of the linear equation is then uniquely determined. Corrections for finite β^*/d can be computed if necessary including values of the spacing between the quadrupoles different from those obtained from the unscaling. The scaled chromaticity can be expressed as

$$Q' \frac{\beta^*}{d} = \left(\frac{1}{4\pi}\right) \int K W^2 \, dS$$

It is fixed once K(S) and W(S) are given.

Using the scaling relations, the comparison between round and flat beams can be further refined. Equal vertical beta values β_y^* and equal lens strength k were assumed above for simplicity. Instead, one may want to reduce β^* for the flat beam by a factor of 2 to make up for the luminosity handicap. By virtue of the scaling this doubles the maximum betas and the chromaticity. Yet - as implied above - the increased values remain favorable compared to the round beam case (cf. Figs. 3 and 4).

The assumption of equal k may be pessimistic with respect to the flat beam. This beam has in fact a smaller vertical emittance whereas its horizontal emittance is similar to that of the equivalent round beam, except for a factor of 2 [10]. In conjunction with the smaller beta values at the quadrupoles this leads to a more favorable aperture requirement.