Tune Modulated Beam-Beam Resonances in the Tevatron

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Introduction

During the 88-89 Tevatron collider run, 6 antiproton bunches were colliding head-on with 6 proton bunches at 12 crossing points symetrically distributed around the ring. Typical intensities were 7×10^{10} and 2.5×10^{10} particles per bunch for protons and antiprotons respectively. The normalized transverse proton emittance (95 % definition) was typically 25 π mm-mr on both planes and the antiproton transverse emittance was typically 18 π mm-mr. The working point (unshifted horizontal and vertical tunes) was near 19.41.

Given the working point and typical tune spreads of about 0.024, it is unavoidable that the 12^{th} order resonance will affect some of the antiprotons. This resonance was certainly felt by the $a > 2\sigma$ particles in the 88-89 run when proton and antiproton emittances were equal. The situation improved when the proton emittance was increased. The question of whether or not the 12^{th} will be harmless in future collider runs remains, since the beam-beam tune shift parameter will be much higher than it was in the 88-89 Run.

In this paper, we answer this question by applying the analytic theory of tune-modulated beam-beam resonances to the 88-89 run and to the main injector upgrade scenario. The analytic theory neglects long-range tune shifts and resonances.

Threshold Condition

For a complete discussion of the theory we refer the reader to References [1] and [2]. Here we repeat the threshold condition for synchrobetatron sideband overlap,

$$\left(|\sum^{m} \vec{\xi}_{Ni}| \cdot \sum^{m} |\vec{\xi}_{Ni}| \right)^{1/2} >$$
$$\frac{1}{4}(\pi q)^{1/4}(Q_s)^{3/4}\left[\frac{\alpha}{N^{3/2}V_N(\alpha)D'(\alpha)}\right]^{1/2} \quad (1)$$

where N is the order and $\alpha \equiv a/\sigma$ is the normalized amplitude of the resonance, and m is the number of head-on beam-beam interactions. It is supposed that, owing to an external modulating source, the perturbed betatron tune is given by

$$Q = Q_0 + q\ sin(2\pi Q_s t) \quad (2)$$

where Q_0 is the unperturbed betatron tune, q is the amplitude of the modulation (modulation depth), Q_s is the modulation tune, and t is the turn number. The resonance analysis is done at a particular point in the ring and "time" for the purposes of this analysis is discretized. One source of tune modulation is ripple in the current supplied to some of the guide field magnets. A more systematic source is the chromatic tune variation due to energy oscillations (synchrotron oscillations). As the momentum of the particle changes the effective focusing strength also changes, resulting in tune modulation. The detuning function $D(\alpha)$ and the resonance width function $V_N(\alpha)$ are explained below. The $\vec{\xi}_{Ni}$ are the so called resonance vectors

$$\vec{\xi}_{Ni} = \xi_i\ exp(jN\phi_i) \quad (3)$$

where ξ_i and ϕ_i are the tune shift parameter and the betatron phase of the i'th collision, and $j \equiv \sqrt{-1}$. Properly speaking, $\vec{\xi}_{Ni}$ is a phasor since time (number of turns) is eliminated from this expression.

Detuning Function and Resonance Width

We restrict our attention to one transverse dimension. Moderate amplitude nonresonant oscillations in a second dimension appear to have little influence on chaotic behaviour in the first dimension[2].

Consider a collider with a single beam-beam collision per turn. The betatron tune of a test particle depends on its amplitude, according to

$$Q(\alpha) = Q_0 + \xi D(\alpha) \quad (4)$$

where $D(\alpha)$ is the so called "detuning function" and ξ is the "beam-beam tune shift parameter" which is equal to the tune shift experienced by a small amplitude particle. If colliding bunches have Gaussian transverse charge distributions of the same size (round beams), the detuning function has the exact analytic form[3]

$$D(\alpha) = 4\alpha^{-2}\left[1 - exp(-\alpha^2/4)I_0(\alpha^2/4)\right] \quad (5)$$

Here I_0 is a modified Bessel function. A beam-beam resonance of order N is present if the tune is equal to a rational fraction n/N at some amplitude α_N. The resonance islands have a full width given by

$$\Delta\alpha = 4\left[\frac{V_N(\alpha)}{\alpha D'(\alpha)}\right]^{1/2} \quad (6)$$

For round beams the "resonance width function" $V_N(\alpha)$ is (even order only) [3]

$$V_N(\alpha) = \int_0^{\alpha} \frac{8}{\alpha}\ exp(-\alpha^2/4)\ I_{N/2}(\alpha^2/4)d\alpha \quad (7)$$

Tune Modulation

Tune modulation causes a family of synchrobetatron sideband resonances to appear, at time-averaged tunes of

$$Q(\alpha) = n/N + p\ Q_s/N \quad (8)$$

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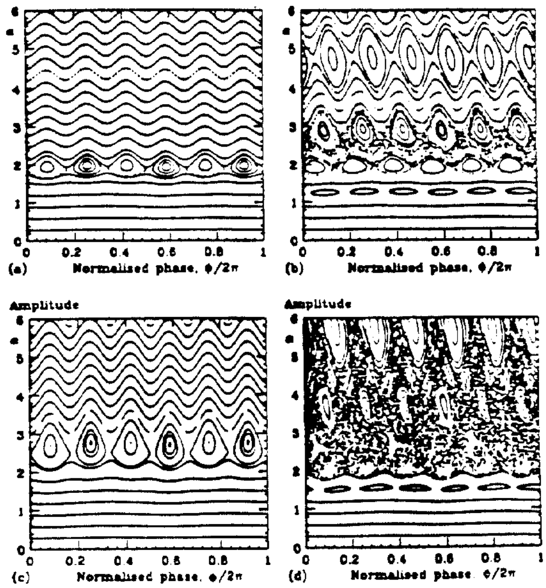

Figure 1: Simulated trajectories tracked for 2000 modulation periods, with $Q_s = 0.005$ and an unshifted tune of 0.331, near a sixth-order beam-beam resonance. The two left figures have no tune modulation, while the two right figures have modulation amplitude $q = 0.001$. The two top figures have a tune shift parameter of $\xi = 0.0042$, while the two bottom figures have a value $\xi = 0.0060$. Side bands $p = +1, 0, -1$, and -2, visible in (b) at increasing amplitudes, overlap and are submerged in a chaotic sea in (d).

where p is an integer. This situation is depicted in Fig.(1)(a,b) where the sideband islands surround the betatron islands. The full width of the p^{th} sideband is given (if the sidebands do not overlap) by

$$\Delta\alpha_{wp} = 4\left[\frac{V_N(\alpha_p)J_p(Nq/Q_s)}{\alpha_p D'(\alpha)\mid_{\alpha=\alpha_p}}\right]^{1/2} \quad (9)$$

Here J_p is the p^{th} integer order Bessel function, and α_p is the betatron amplitude corresponding to this sideband. The magnitude of J_p is of the order of

$$J_p(Nq/Q_s) \approx (Q_s/\pi Nq)^{1/2} \quad (10)$$

if

$$\frac{n}{N} - q < Q(\alpha_p) < \frac{n}{N} + q \quad (11)$$

and very small if condition 11 is violated. The physical interpretation of this condition is as follows. Because of the tune modulation, the "instantaneous" tune varies between $Q(\alpha) - q$ and $Q(\alpha) + q$. For the resonance to have effect, this tune must cross n/N. So, if $Q(\alpha) < (n/N - q)$ or $Q(\alpha) > (n/N + q)$, the tune never reaches the resonance condition and the sidebands are suppressed. Sidebands are separated in amplitude from each other by

$$\Delta\alpha_s \equiv \frac{(Q_s/N)}{Q'(\alpha)} = \frac{Q_s}{N\xi D'(\alpha)} \quad (12)$$

As the beam-beam tune shift parameter ξ is increased, the sidebands remain constant in size while their separations decrease. When $\Delta\alpha_s < \Delta\alpha_{wp}$, the sidebands overlap and a chaotic layer is formed in phase-space flow as shown in Fig.(1)(d). In other words, there is overlap if

$$\xi > \xi_{max} \equiv \frac{1}{4}(\pi q)^{1/4}(Q_s)^{3/4}\left(\frac{\alpha}{N^{3/2}V_N(\alpha)D'(\alpha)}\right)^{1/2} \quad (13)$$

which is very similar to Equation 1 but needs to be generalized to multiple collisions.

The generalized $\Delta\alpha_s$ and $\Delta\alpha_{wp}$ are [2]

$$\Delta\alpha_s = \frac{Q_s}{N\cdot\sum^m|\vec{\xi}_{Ni}|\cdot D'(\alpha)} \quad (14)$$

$$\Delta\alpha_{wp} = 4\left[\frac{|\sum^m\vec{\xi}_{Ni}|\cdot V_N(\alpha)J_p(Nq/Q_s)}{\sum^m|\vec{\xi}_{Ni}|\,\alpha D'(\alpha)}\right]^{1/2} \quad (15)$$

Using the overlap condition $\Delta\alpha_s < \Delta\alpha_{wp}$ and Equations 14,15 and rearranging we obtain Equation 1, the threshold equation. Given the order of the betatron resonance N, the particle amplitude α, the tune modulation frequency Q_s and depth q, the threshold equation tells whether the beam-beam strength parameter ξ is large enough to cause an overlap of sideband resonances.

Summing the Beam-Beam Resonance Vectors

We are now ready to calculate the terms on the left hand side of Equation 1. The calculation of $|\sum^m\vec{\xi}_{Ni}|$ requires the knowledge of phases at crossing points. There is typically a several percent error in the lattice functions, and it is difficult to know the phases exactly enough at the crossing points. We simply take the root mean square average of the resonance vectors $\vec{\xi}_{Ni}$, namely, we approximate

$$|\sum^m\vec{\xi}_{Ni}| \approx (m)^{1/2}\xi \quad (16)$$

and the other summation is easier since the phase information is not needed.

$$\sum^m|\vec{\xi}_{Ni}| = m\xi \quad (17)$$

Critical Resonances

The threshold condition, Eq.(1), defines the highest order betatron resonance that allows side-band overlap in the presence of tune modulation. From here on we shall call these critical resonances. Critical resonances are calculated graphically from Fig.(2) where the right hand side of Eq.(1) is plotted for different N. We ignore long-range interactions in the summation of beam-beam resonance vectors (left hand side of the threshold equation) but do not ignore odd-resonances (right hand side of the threshold equation) since they are not fully suppressed by the symetry of the beam-beam interaction.

The curves in Fig.(2) have been calculated using realistic Tevatron parameters. For instance, a chromaticity of $\Delta Q/(\Delta p/p) = 5$ and $\sigma p/p = 1.5\times10^{-4}$ was used, assuming that the source of the tune modulation was synchrotron oscillations at a frequency of 37 Hz in the Tevatron at 900 GeV. These numbers translate into $Q_s = 0.00075$ and $q = 0.00075$.

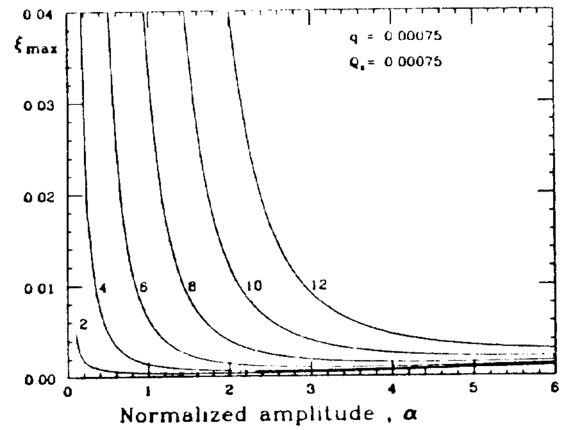

Figure 2: Right hand side of Eq.(1) **plotted** for various N. Chromaticity = 5 units.

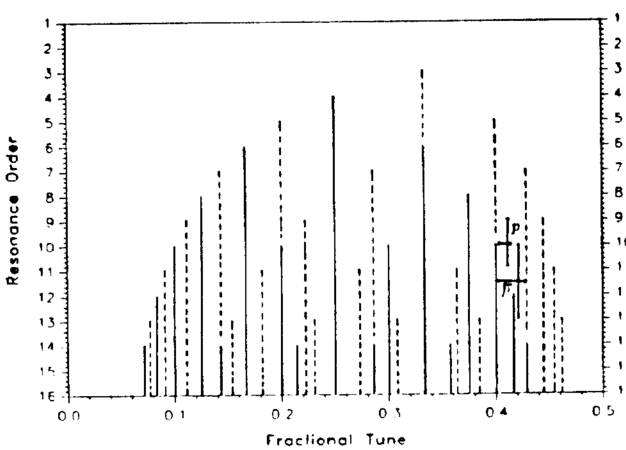

Figure 3: Bed-of-Nails plot for the 88-89 Collider Run.

Bed-of-Nails Plots

Critical resonances are displayed in Fig.(3) and Fig.(4). In these figures, nails have different heights representing the order of the resonance. Even-order resonances are shown by solid lines and odd-order resonances by dashed ones. Lower order resonances are represented by taller nails. The tune spread is shown by the horizontal error bar, the position of which carries crucial information. Its vertical position indicates the critical resonance for $\alpha = 2.5$. The vertical error bar shows the range of critical resonances for particles in the range $\alpha = 2$ to $\alpha = 3$. The point where the horizontal and the vertical error bars cross each other roughly gives the beam-beam shifted average tune. The working point (unshifted tune) is near the left edge of the horizontal bar.

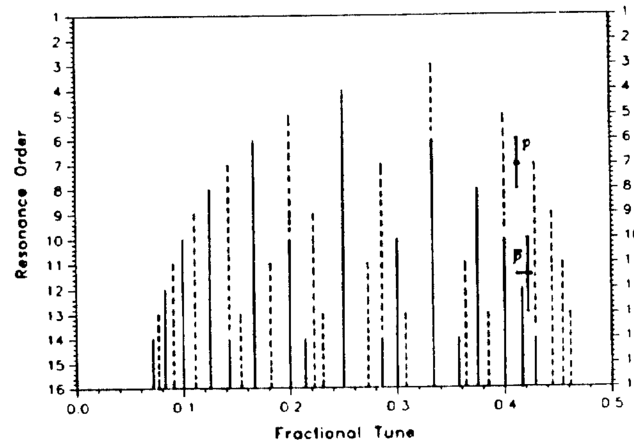

Figure 4: Bed-of-Nails plot for the Collider Run with the Main Injector.

Conclusions

Fig.(3) depicts the situation in the 88-89 collider run. The analytic theory of tune modulated beam-beam resonances correctly predicts the lack of importance of the 12^{th} order resonance, since it only just affects $\alpha = 2.6 - 3.0$ antiprotons. For $\alpha = 2$ antiprotons the theory predicts no trouble from 12^{th}, which was the case when proton emittance was increased artificially, effectively making all antiproton amplitudes $\alpha \equiv a_{\bar{p}}/\sigma_p < 2$. Protons in the 88-89 run were comfortably away from the 12^{th} since the antiproton intensity was low and the beam-beam tune shift per crossing experienced by protons was small.

Examining Fig.(4) we find that for the Collider Run with the Main Injector, the horizontal error bar (tune spread) is smaller and the vertical error bar (range of critical resonances for amplitudes $\alpha = 2-3$) is approximately the same compared to those of the 88-89 run. Having a smaller tune spread is an important improvement since we gain freedom to adjust the working point. The size of the vertical error bar being equal to that of the 88-89 run is also good news since it means that the 12^{th} order resonance will only affect the antiprotons in the transverse tails.

References

[1] S.Peggs and R.Talman; "Nonlinear Problems in Accelerator Physics"; Ann.Rev.Nucl.Sci., 36,p:287-325 (1986).

[2] S.Peggs; "Hadron Collider Behaviour in the Nonlinear Numerical Model EVOL"; Particle Accelerators, 17, p:11-50 (1985)

[3] L.Evans, CERN SPS/DI(MST)/Note/81-2 (1981); L.Evans, CERN SPS/83-38 (DI-MST) (1983); L.Evans and J.Garayte, CERN SPS/82-8 (DI-MST) (1982).