A Study of Misalignment Effects of the ANL-APS Electron Linac Focusing System*

A. Nassiri, G. Mavrogenes
Argonne National Laboratory
9700 South Cass Avenue
Argonne, IL 60439

Abstract

We present an analytical treatment of quadrupole misalignment effects for the Argonne Advanced Photon Source (APS) $200-MeV$ electron linac. The results of numerical modeling with TRANSPORT [1] are discussed.

I. Introduction

The APS electron linac focusing system consists of three magnetic quadrupole triplets to guide and focus a 1.7 A beam with final energy of 200 MeV on a positron production target. Adequate positron yield requires a small electron beam emittance which means that a beam size of less than 3 mm must be achieved at the positron production target [2]. An off-centered electron beam at the tungesten target will have at least two effects. First, it will cause positron capture into the pulsed solenoid located immediately after the target to be deteriorated. Second, it will produce wakefields in the disk-loaded accelerating structures leading to emittance growth and beam breakup [3]. Here, we will only discuss the first effect and give a criterion on the beam centroid variation in terms of positron capture into the pulsed solenoid.

II. Magnet Displacement Errors

The equation of motion in the transverse plane in the presence of a net displacement error δ, is

$$\frac{d^2x}{ds^2} + \kappa^2 x = \kappa^2 \delta \qquad (1)$$

where $\kappa^2 = \frac{\mu^2}{D^2}$ and μ is the phase advance without space charge and D is the length of the period. If the beam is centered when it enters a misaligned magnet, then the solution to equation (1) is obtained as

$$x(s) = \delta[1 - \cos\ \kappa(s - s_i)] \qquad (2)$$

and

$$x'(s) = \kappa\delta\ \sin\ \kappa(s - s_i). \qquad (3)$$

At the end of the misaligned magnet period, the offsets of the beam centroid are given by

$$x_{cf} = \delta[1 - \cos \kappa D] \qquad (4)$$

and

$$x'_{cf} = \kappa\delta \sin \kappa D \qquad (5)$$

where $D = s_f - s_i$.

*Work supported by U.S. Department of Energy, Office of Basic Energy Sciences under Contract No. W-31-109-ENG-38.

The beam offset produced is equivalent to an injection error for the motion through the rest of the focusing system. Thus, equations (4) and (5) can be used as initial conditions to describe the trajectory in the perfectly aligned section beyond the displaced magnet [4]. The resulting equation is

$$x = \delta[1 - \cos\ \kappa(s_f - s_i)]\cos\ \kappa(s - s_f) + \kappa\delta \sin\ \kappa(s_f - s_i)\sin\ \kappa(s - s_f) \qquad (6)$$

or

$$x = \delta[\cos\ \kappa(s - s_f) - \cos\ \kappa(s - s_i)] \qquad (7)$$

with $\kappa = \frac{\mu}{D}$ and $s_c = \frac{(s_i + s_f)}{2}$, equation (7) can be written as

$$x = 2\delta \sin(\frac{\mu}{2})\sin(\frac{\mu}{D})(s - s_c) \qquad (8)$$

If two adjacent magnets are misaligned by δ_1 at position s_1 and by δ_2 at position s_2, the centroid trajectory downstream of the magnets can be written as

$$x = 2\delta_1 \sin\frac{\mu}{2}\sin\frac{\mu}{D}(s - s_1) + 2\delta_2 \sin\frac{\mu}{2}\sin\frac{\mu}{D}(s - s_2) \qquad (9)$$

For n successive misaligned magnets, one has

$$x = \sum_{i=1}^{n} 2\delta_i \sin(\frac{\mu}{2})\sin(\frac{\mu}{D})(s - s_c) \qquad (10)$$

If the misalignment errors are known, one can calculate both the displacement and slope of the beam centroid at the end of the n magnet period. If the misalignment errors are random, the displacement and slope of the centroid trajectory through a system of n magnets are given as

$$x = \sin\ (\frac{\mu}{D})s\ \left[\sum_{i=1}^{n} 2\delta_i\ \sin(\frac{\mu}{2})\cos(\frac{\mu}{D})s_i\right] - \cos(\frac{\mu}{D})s\ \left[\sum_{i=1}^{n} 2\delta_i\ \sin(\frac{\mu}{2})\sin(\frac{\mu}{D})s_i\right] \qquad (11)$$

and

$$x' = \frac{\mu}{D}\cos(\frac{\mu}{D})s\ \left[\sum_{i=1}^{n} 2\delta_i\ \sin(\frac{\mu}{2})\cos(\frac{\mu}{D})s_i\right] + \frac{\mu}{D}\sin(\frac{\mu}{D})s\ \left[\sum_{i=1}^{n} 2\delta_i \sin(\frac{\mu}{2})\sin(\frac{\mu}{D})s_i\right] \qquad (12)$$

The square of the amplitude of the beam oscillation after passage through n magnets is

$$d^2 = x^2 + (\frac{x'}{\kappa})^2 = \left[\sum_{i=1}^{n} 2\delta_i \sin(\frac{\mu}{2})\cos(\frac{\mu}{D})s_i\right]^2 + \left[\sum_{i=1}^{n} 2\delta_i \sin(\frac{\mu}{2})\sin(\frac{\mu}{D})s_i\right]^2 \quad (13)$$

After simplifying, this can be written as

$$d^2 = 4\sin^2\frac{\mu}{2}\left[\sum_{p=1}^{n}\delta_p^2 + \sum_{q=1}^{n}\sum_{p\neq q}^{n}\delta_p\delta_q \cos\frac{\mu}{D}(s_p - s_q)\right] \quad (14)$$

For a set of n independent, identically distributed random errors, the average of the distribution for each magnet is

$$<\delta_i>= 0 \qquad \text{for i= 1,..., n}$$

and

$$<\delta_i^2>=<\delta^2> \quad \text{for i= 1,..., n}$$

Each possible set of the n magnets will give a different value for d. Taking the average value over all sets gives

$$<\delta_p\delta_q>= \begin{cases} 0 & p \neq 0 \\ \delta^2 & p = q \end{cases}$$

so

$$<d^2>= 4\sin^2\frac{\mu}{2}n<\delta^2> \quad (15)$$

or the rms value is

$$d_{rms} = 2\delta_{rms}\sqrt{n}\sin(\frac{\mu}{2}) \quad (16)$$

with

$$\delta_{rms} = (<\delta^2>)^{\frac{1}{2}}$$

III. Magnet Rotation Errors

If there are random errors in rotation of each magnet about the axis of the focusing channel, there will be transverse amplitude growth. The change in transverse amplitude (say in x, x' phase space) is calculated by considering the Courant-Snyder invariant, W_x, defined as [5]

$$\frac{W_x}{\beta\gamma} = \gamma_x x^2 + 2\alpha_x x x' + \beta_x x'^2 \quad (17)$$

The change in W_x for λ successive magnets with rotation error ϵ_λ, is [5]

$$\delta W_x \cong \sum_{\lambda=1}^{n} 2(\beta\gamma)(\alpha_x{}^\lambda x_\lambda + \beta_x{}^\lambda x'_\lambda)\kappa_\lambda \ell_\lambda \epsilon_\lambda y_\lambda \quad (18)$$

The change in radial amplitude is a maximum for the case $A_x = A_y = \frac{d}{\sqrt{2}}$, where $\beta\gamma A_{x,y}^2 = \beta_{max} W_{x,y}$. Expressing x, x', y, y' in terms of the amplitude and phase of the transverse oscillations, one obtains [5]

$$\frac{\delta d_n}{d_n} \cong \frac{1}{2}\sum_{\lambda=1}^{n}\kappa_\lambda \ell_\lambda \epsilon_\lambda (\beta_x{}^\lambda \beta_y{}^\lambda)^{\frac{1}{2}} \sin(\phi_o + 2\lambda\mu) \quad (19)$$

where ϕ_o is the sum of the x and y starting phases, μ is the phase advance of either oscillation, and the subscript or superscript λ refers to the value at the λ magnet. There is a value of ϕ_o for which equation (19) is maximum. For this value and with the assumption of uncorrelated errors ϵ_λ, with rms value ϵ, one has [5]

$$(\frac{\delta d_n}{d_n})_{max} \cong \frac{\epsilon}{2}\left[\sum_{\lambda=1}^{n}\frac{\beta_x{}^\lambda\beta_y{}^\lambda}{f_\lambda^2}\theta_\lambda^4\right]^{\frac{1}{2}} \quad (20)$$

IV. Numerical Simulations

Misalignments of the electron linac focusing triplets were studied using TRANSPORT [1]. Quadrupole triplets were randomly misaligned (assuming a Gaussian distribution) with $\sigma_{x,y} = 0.3$ *mm* and $\sigma_{x',y'} = 2.0$ *mrad*. Three cases were considered: (1) lateral displacement, (2) rotation around the transverse axes, and (3) quadrupole field gradient errors.

Ten statistically independent sets of misaligned triplets were produced for each type of misalignment. The perturbed trajectories were then calculated using TRANSPORT [1]. The rms deviations of the beam centroid along the electron linac were calculated. Figures 1 and 2 illustrate the beam centroid fluctuation due to displacement and rotation errors respectively.

V. Summary and discussion

The capture of the positrons into the pulsed solenoid located after the positron production target depends on how well the electron beam is centered on that target. If the electron beam spot is displaced from the center of the target by $\delta\rho$, in order to have a 90% capture, $\delta\rho$ should be $\sim 0.1d$ ($d =$ *beam diameter*). This criterion sets the limits on the misalignment tolerances. Equation (16) shows that the standard deviation is proportional to rms error and grows as $n^{\frac{1}{2}}$. Figures 1 and 2 indicate this relation. The simulations also show that the random quadrupole displacement errors have stronger effects than the rotation errors. This is to be expected when the quadrupoles are arranged as symmetric triplet. The magnetic field gradient variation on quadrupoles is specified to be better than 0.1%. Simulations indicate that no significant beam degradation occurs at this tolerance level. Simulations indicate that a parallel displacement error tolerance of 0.3 *mm* and an axial rotation error tolerance of 2.0 *mrad* are acceptable (see Figures 3 and 4).

VI. Acknowledgement

A. Nassiri has greatly benefited from discussion with Ed Crosbie on the perturbation effects in beam focusing systems. Thanks to Rabinder Koul for useful conversations.

VII. References

[1] David C. Carey, "Third Order TRANSPORT with MAD Input,"TM-1546, FNAL, 1988.

[2] A. Nassiri, W. Wesolowski, and G. Mavrogenes , "The Injector Linac For The ANL 7 GeV Advanced Photon Source," in *Proceedings of the 1990 Linear Accelerator Conference*, p. 611 , September 1990.

[3] R. D. Ruth, "The Next Linear Collider," SLAC-PUB 5406, SLAC, February 1991.

[4] M. Reizer, "Theory And Design Of Charged Particle Beam," University Of Maryland, 1989.

[5] R. Gluckstern, "Transverse Beam Quality In Linacs," in *Proceedings of Sixth International Conference On High Energy Accelerators*, p.157, 1967.

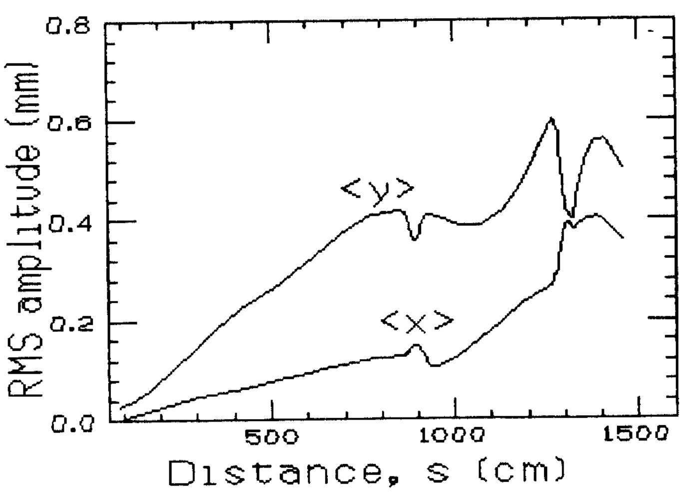

Figure 1. Fluctuation of beam centroid due to random displacement error

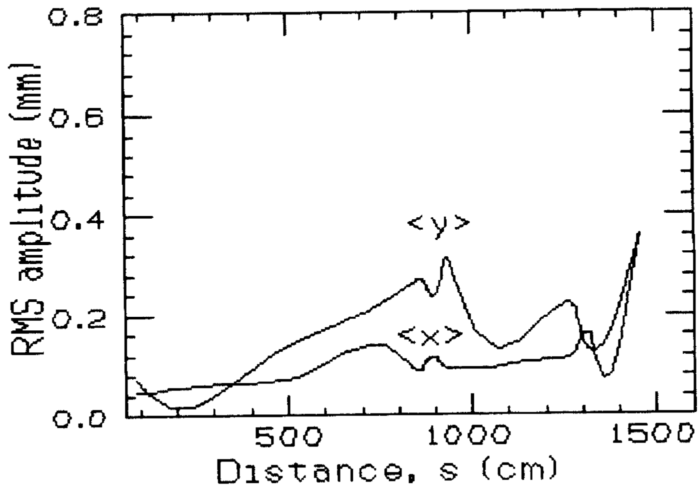

Figure 2. Fluctuation of beam centroid due to random rotational error

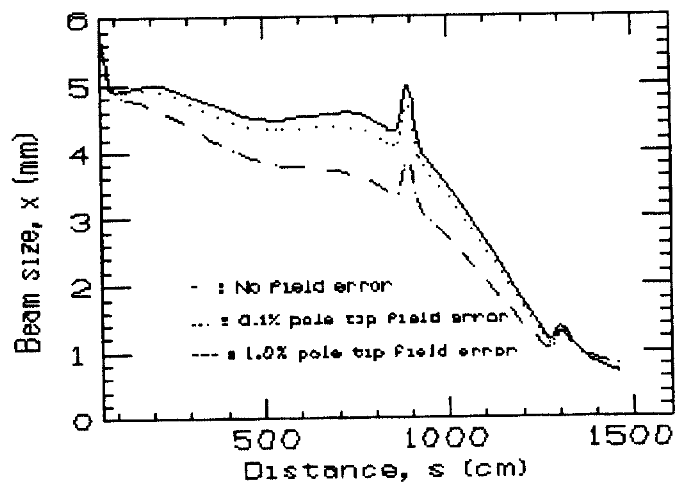

Figure 3. Beam size (x) profile variation due to field gradient error

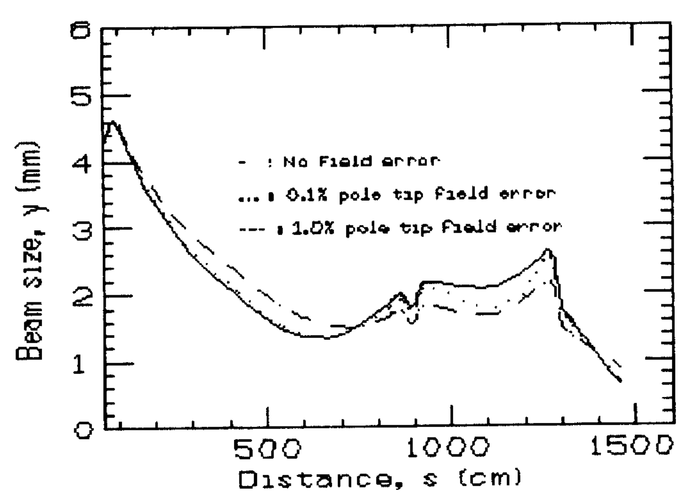

Figure 4. Beam size (y) profile variation due to field gradient error