

Electron Density Enhancement in a Quasi Isochronous Storage Ring

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Abstract

We establish the condition for stable single particle motion in a storage ring with very small momentum compaction, and very short bunch length, the Quasi Isochronous ring. We discuss how this condition can be achieved and its applications to colliders and synchrotron radiation sources.

Introduction

The six dimensional phase-space density of an electron beam in a storage ring is determined by the emission of synchrotron radiation, and by the transverse and longitudinal focusing forces determining the particle trajectories. In the simplest case of uncoupled horizontal, vertical and longitudinal motion, the phase space volume occupied by the beam can be characterized by the product of its three projections on the single degree of freedom planes, the horizontal, vertical, and longitudinal emittances.

To minimize the beam phase space volume we can minimize the transverse and longitudinal emittances. In the case of the transverse emittances this problem is very important for synchrotron radiation sources, and has been studied by several authors [1]. The results have been used to build high brightness synchrotron radiation sources like the ALS at Berkeley and the APS at Argonne.

A method to minimize the longitudinal emittance, and produce electron bunches with a short pulse length, small energy spread and large peak current has been proposed and discussed recently [2] by C. Pellegrini and D. Robin. Such a beam would find applications in synchrotron radiation sources for the production of picosecond long high brightness radiation pulses, and in lepton colliders, like the meson factories, where it can lead to larger luminosity for the same beam current, since a shorter bunch allows the use of stronger focusing and smaller transverse area at the interaction point.

This method uses a ring in which the revolution period is weakly dependent on the particle energy, Quasi Isochronous Ring (QIR), in other words a ring with a momentum compaction nearly zero. In this paper we will extend the previous analysis of the conditions for stable

single particle motion in such a ring, and give simple criteria for the estimate of the energy spread and phase acceptance of a QIR.

Single Particle Motion

We consider only the uncoupled longitudinal motion, using as variables the phase ϕ respect to the RF system, and the relative energy change respect to the reference particle, $\delta = (E - E_r)/E_r$. In our approximation we neglect the betatron oscillation amplitude, and the transverse displacement of a particle is a function of δ only, and can be written as

$$x = \eta_0 \delta + \eta_1 \delta^2 \quad (1)$$

where η_0 is the linear dispersion function and η_1 describes the first non linear correction.

The change in phase with energy respect to the reference particle is determined by the change in revolution time, i.e. the change in velocity, and the change in the length of the trajectory.

The change in trajectory length is

$$\Delta L = \int_{RT} ((1 + x/\rho)^2 + x'^2)^{1/2} ds - L \quad (2)$$

where the integration is done on the reference trajectory (RT) of length L, and a prime indicates a derivative respect to the RT arc length, s. Assuming x, x' to be small, expanding (2) to second order in δ and using (1) we have

$$\Delta \phi = \omega_{RF} \Delta T = \omega_{RF} T (\alpha_1 \delta + \alpha_2 \delta^2) \quad (3)$$

where

$$\alpha_1 = \frac{1}{L} \int_{RT} \frac{\eta_0}{\rho} ds - \frac{1}{\beta_r^2 \gamma_r^2} \quad (4)$$

$$\alpha_2 = \frac{1}{L} \int_{RT} \left(\frac{1}{2} \eta_0'^2 + \frac{\eta_1}{\rho} \right) ds + \frac{3}{2} \frac{1}{\beta_r^2 \gamma_r^2} \quad (5)$$

The term α_1 is the momentum compaction as defined in the linear theory, and α_2 is the first order non linear correction, which we call the longitudinal chromaticity. Notice that the first term in the integral in equation (5),

the square of the derivative of the linear dispersion, is always positive, while the second term, which can be controlled with sextupoles, can be either positive or negative, and allows us to control the value of the longitudinal chromaticity and make it zero. Using these notations, the equations for the longitudinal motion (synchrotron oscillations) can be written as

$$\phi' = h(\alpha_1 \delta + \alpha_2 \delta^2) \quad (6)$$

$$\delta' = -\frac{eV_0}{2\pi E_r} \{ \sin(\phi + \phi_r) - \sin(\phi_r) \} - \frac{\delta}{\tau} + \Phi \quad (7)$$

where a prime now indicates a derivative respect to $\omega_0 t$, $h = \omega_{RF} / \omega_0$ is the harmonic number, V_0 is the peak RF voltage, τ the damping time, Φ the fluctuation term, and ϕ_r the phase of the reference particle.

We study initially equations (6) and (7) neglecting damping and fluctuations, to determine the phase space area for stable single particle motion (closed, bounded phase space trajectories). In this case the system can be described by the Hamiltonian

$$H = H_0 + \frac{1}{3} h \alpha_2 \delta^3 \quad (8)$$

$$H_0 = \frac{1}{2} h \alpha_1 \delta^2 - \frac{eV_0}{2\pi E_r} \{ \cos(\phi + \phi_r) + \phi \sin \phi_r \} \quad (9)$$

When the longitudinal chromaticity is zero (6) and (7) are the usual synchrotron oscillation equations. The space phase trajectories, as shown in fig. 1, are characterized by one stable and one unstable fixed points. Assuming $\cos \phi_r > 0$, the small oscillation frequency around the stable fixed point is

$$\nu_s = \{ h \alpha_1 eV_0 \cos \phi_r / 2\pi E_r \}^{1/2} \quad (10)$$

The separatrix through the unstable fixed point defines the stable oscillation region. The maximum stable energy displacement is

$$\delta_M = \left\{ \frac{2eV_0}{\pi h \alpha_1 E_r} \left(\cos \phi_r + \left(\frac{\pi}{2} - \phi_r \right) \sin \phi_r \right) \right\}^{1/2} \quad (11)$$

When we reduce the value of α_1 , the energy acceptance given by (11) becomes larger. However the longitudinal chromaticity term becomes now important, and the phase-space trajectories are modified, as shown in fig. 2. There are now two stable fixed points and two unstable fixed points, located at $\phi = 0$, $\phi = \pi - 2\phi_r$, and $\delta = 0$, $\delta = -\alpha_1 / \alpha_2$. These points will define two separatrices, as shown in fig. 2, passing through the two unstable points and surrounding the stable points. For $\cos \phi_r > 0$ the point at $(0,0)$ is stable and the point $(0, -\alpha_1 / \alpha_2)$ is unstable. If the distance between these two

points is smaller than the energy acceptance given by (11), the separatrix through this unstable point will determine the corrected acceptance, which is now given approximately by

$$\delta_M \sim -\frac{\alpha_1}{\alpha_2} \quad (12)$$

If the opposite is true, we will have two separatrices of the usual form, as in fig. 1, separated by an increasing distance as we make the ratio of the α 's larger.

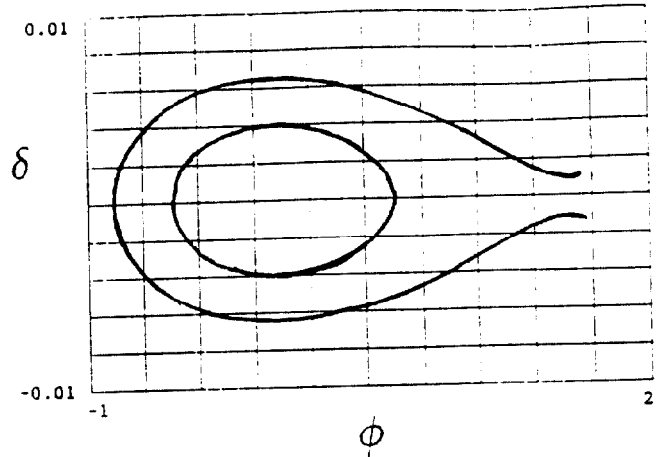


Figure 1: Longitudinal phase space trajectories. In this case $\alpha_2 = 0$.

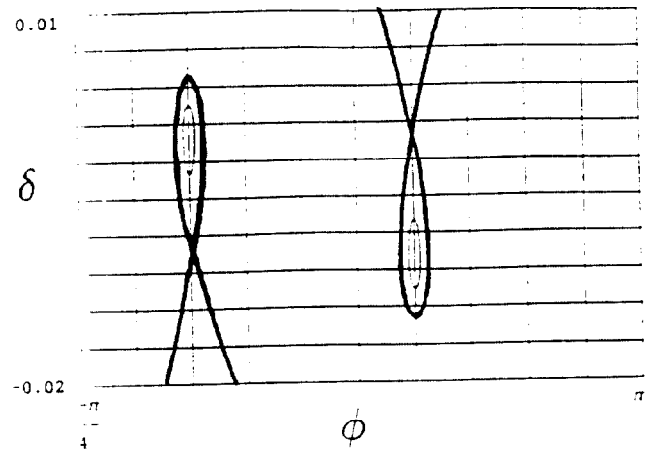


Figure 2: Longitudinal phase space trajectories. In this case $\alpha_2 / \alpha_1 = 100$.

Notice that in all cases we have now two stable regions in phase space. In the usual case they look like in Fig. 1, RF buckets, and their separation is so large that particles in the second bucket would have a very different energy and could not stay in the ring. In the case of small momentum compaction, and an acceptance determined by (12), the two regions have a different shape, Fig. 2. We will call them Alpha-buckets. The difference in the central energy of the two Alpha-buckets is the same as the expression (12) of the energy acceptance. These two regions could both contain particles which can survive in the ring. However one can still fill only one of them.

The phase acceptance can be also easily determined by finding the intersection of the separatrices with the curves $\delta = 0$, $\delta = -\alpha_1 / \alpha_2$.

Conclusions

We have established the phase space structure for a QIR, and given a simple expression for the ring energy acceptance. We must also notice that these results are based on a simplified model of the QIR. We are neglecting the effect of betatron oscillations and expanding the equation only to second order in δ . A more complete description requires a full non linear tracking of the particle motion in the ring. These results can however be used as a guide to the design of the ring lattice. It is for instance apparent the importance in the ring design of controlling simultaneously both the momentum compaction and the longitudinal chromaticity, so that their ratio will remain large enough to provide the needed energy and phase acceptance. This requires that we now control and make zero or nearly zero the two transverse chromaticities and the longitudinal chromaticity. This can be done using at least three families of sextupoles. As usual these sextupoles may reduce the ring dynamic aperture, and their placement and number will need to be properly selected to keep this reduction to a minimum. We are now studying the lattice of rings of the QIR type that can produce bunch length in the millimeter or submillimeter region.

The peak current in such short bunches can be made large by using the longitudinal coupling impedance reduction with bunch length, and the synchrotron radiation damping, as discussed in [2].

References

1. See for instance Proc. Of the ICFA Workshop on Low Emittance Beams, J.B. Murphy and C. Pellegrini eds., Brookhaven National Laboratory Report (1987).
2. C. Pellegrini and D. Robin, Nucl. Instr. and Meth. A301, 27-36 (1991).