The Extension of the OSCAR2D Code to Compute Azimuthally Dependent Modes of Axially Symmetric Cavities

P.Fernandes G. Sabbi
Istituto per la Matematica Applicata del CNR
via L.B. Alberti,4 - 16132 Genova - Italy
R.Parodi
Istituto Nazionale di Fisica Nucleare - Sezione di Genova
via Dodecaneso,33 - 16146 Genova - Italy

Abstract

The extension of our OSCAR2D code to the computation of azimuthally dependent modes of axially symmetric cavities is presented. The code makes use of E_r and E_z as independent field components to numerically solve the Maxwell's equations. Enforcing of $\vec{\nabla}\cdot\vec{E}=0$ both ensures that spurious modes cannot arise and reduces the number of field components to be considered in the solution process. This formulation of the problem leads to a linear eigenvalue problem for a matrix which is then solved by the Rayleigh quotient iteration. The code has been validated by comparing its results with those obtained by analytical tools. Comparisons with measured values have been performed, too.

1 Introduction.

Axially symmetric cavities are widely used to accelerate charged particles in both linear and circular accelerators. In principle, the working mode of these accelerating devices is usually an axially symmetric one, nevertheless azimuthally dependent modes are invariably present because, in practice, symmetry is always broken by couplers and tuners. Any asymmetry in the beam leads to excitation of azimuthally dependent modes, too, as well as electron discharge, if present, does (because trajectories involved lie in the r-z plane). Since excitation of azimuthally varying fields may lead to beam breakup, the importance for the cavity designer of being able to compute them is apparent.

Programs already exist, indeed, that can compute these modes, the most popular of them being certainly URMEL-T [1]. Anyway, because of the good properties shown by the approach used in the OSCAR2D code [2] to compute TM and TE monopolar (i.e. axially symmetric) modes, this code has been extended to the computation of hybrid multipolar (i.e. azimuthally dependent) ones by keeping the same discretization and solution scheme [3]-[5].

2 Problem formulation.

In monopolar modes one of electric and magnetic fields has one nonzero component only, thus it is quite natural to use this component as the unknown quantity so reducing the problem to a scalar one. On the contrary, in multipolar modes both electric and magnetic fields have three nonzero components each, so that the way Maxwell's equations are reduced to a system of two scalar equations is a crucial point. As a matter of fact different ways to perform such reduction lead to problems ranging from a quite standard linear eigenvalue problem [1] to a nonlinear eigenvalue problem with singularities (a formidable one!) [6]-[7]. Moreover it is just at this stage that you must take care the system has strictly the same solutions as the original problem. In fact, any formulation someway lacking in this respect will be, more or less, plagued by spurious modes [8].

In order to formulate the problem of computing resonant modes of a cavity we write the Fourier transformed Maxwell's equations in vacuum with suitable boundary conditions for a finite region of space Ω surrounded by a perfectly conducting surface Γ_1 and possible symmetry and skew-symmetry planes (for the E_z component) that we will indicate by Γ_2 and Γ_3, respectively. Thus, by assuming without any loss of generality that $\vec{\mathcal{H}}(\vec{r},t)=\vec{H}(\vec{r})\cos(\omega t)$ and $\vec{\mathcal{E}}(\vec{r},t)=\vec{E}(\vec{r})sin(\omega t)$ to deal only with real quantities and by posing $\omega/c=k$, the problem to solve is

$$\begin{cases} \vec{\nabla}\wedge\vec{E}=k\vec{H} & \text{in } \Omega \\ \vec{\nabla}\wedge\vec{H}=k\vec{E} & \text{in } \Omega \\ \vec{\nabla}\cdot\vec{E}=0 & \text{in } \Omega \\ \vec{\nabla}\cdot\vec{H}=0 & \text{in } \Omega \\ \vec{E}\wedge\vec{n}=0 & \text{on } \Gamma_1\cup\Gamma_2 \\ \vec{H}\wedge\vec{n}=0 & \text{on } \Gamma_3 \end{cases} \quad (1)$$

with $k\neq 0$, $\Gamma_1\cup\Gamma_2\cup\Gamma_3=\partial V$, $\vec{n}$ unit vector normal to ∂V.

It can be shown that problem (1) is strictly equivalent to

0-7803-0135-8/91$01.00 ©IEEE

$$
\begin{cases}
\nabla^2\vec{E}+k^2\vec{E}=0 & \text{in } \Omega \\
\vec{\nabla}\cdot\vec{E}=0 & \text{in } \Omega \\
\vec{E}\wedge\vec{n}=0 & \text{on } \Gamma_1\cup\Gamma_2 \\
(\vec{\nabla}\wedge\vec{E})\wedge\vec{n}=0 & \text{on } \Gamma_3 \\
\vec{E}\cdot\vec{n}=0 & \text{on } \Gamma_3
\end{cases} \qquad (2)
$$

in the sense that if electric field is determined by problem (2) and magnetic field is recovered from it as

$$
\vec{H}=\frac{1}{k}\vec{\nabla}\wedge\vec{E} \qquad (3)
$$

then all the solutions of problem (1) and just them are obtained so that no spurious solutions are introduced.

By rewriting the equations by components, separating variable ϕ in such a way that

$$
\begin{aligned}
E_r(r,\phi,z) &= e_r(r,z)\cos n\phi \\
E_\phi(r,\phi,z) &= e_\phi(r,z)\sin n\phi \\
E_z(r,\phi,z) &= e_z(r,z)\cos n\phi
\end{aligned} \qquad (4)
$$

dropping some dependent equations, solving $\vec{\nabla}\cdot\vec{E}=0$ for e_ϕ and substituting it in the remaining equations, we obtain the following 2D problem:

$$
\begin{aligned}
\frac{\partial^2 e_r}{\partial r^2}+\frac{3}{r}\frac{\partial e_r}{\partial r}+\frac{(1-n^2)e_r}{r^2}+\frac{\partial^2 e_r}{\partial z^2}+ & \\
+\frac{2}{r}\frac{\partial e_z}{\partial z}+k^2 e_r &= 0 \quad \text{in } \Omega' \\
\frac{\partial^2 e_z}{\partial r^2}+\frac{1}{r}\frac{\partial e_z}{\partial r}-\frac{n^2 e_z}{r^2}+\frac{\partial^2 e_z}{\partial z^2}+k^2 e_z &= 0 \quad \text{in } \Omega'
\end{aligned} \qquad (5)
$$

with boundary conditions

$$
\begin{aligned}
r\frac{\partial e_r}{\partial r}+r\frac{\partial e_z}{\partial z}+e_r=0 \quad & (e_\phi=0) && \text{on } \Gamma_1'\cup\Gamma_2' \\
e_z\sin\alpha-e_r\cos\alpha=0 \quad & (\vec{e}\cdot\vec{t}=0) && \text{on } \Gamma_1'\cup\Gamma_2' \\
\frac{\partial e_r}{\partial z}=0 \quad & (h_\phi=0) && \text{on } \Gamma_3' \\
e_z=0 \quad & (\vec{e}\cdot\vec{n}=0) && \text{on } \Gamma_3' \\
e_z=0 \quad \frac{\partial e_r}{\partial r}=0 \quad & (n=1) && \text{on z-axis} \\
e_z=e_r=0 \quad & (n>1) && \text{on z-axis}
\end{aligned} \qquad (6)
$$

where $\Omega'=\Omega\cap P$, $\Gamma_i'=\Gamma_i\cap P$ and P is any (r,z) half plane, while α is the angle between $\vec{n}$ and the z-axis and $\vec{t}$ is the unit tangent vector to the curve Γ_i'. The conditions on the z-axis are obtained by taking the limit as $r\to 0$ of Maxwell's equations written by components. See Fig. 1 for an example of an integration domain.

Once e_r and e_z have been found by solving problem (5)-(6), then e_ϕ can be computed from $\vec{\nabla}\cdot\vec{E}=0$, $\vec{E}$ from equations (4) and $\vec{H}$ from equation (3).

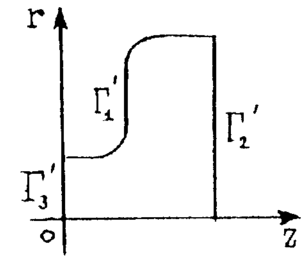

Figure 1: Example of integration domain.

3 Discretization and Solution.

Problem (5)-(6) is still a linear eigenvalue problem, even though more complicate than those for monopolar modes, so it can be discretized and solved by the same methods that have been already implemented in OSCAR2D. Since a detailed description of these methods can be found in previous papers [3]-[4], just an outline of the solution process is given here.

Problem (5)-(6) is discretized by a standard finite difference method on a square mesh with an accurate treatment of boundary conditions [3]-[4]. The resulting matrix eigenvalue problem is worked out by a Rayleigh quotient iteration where the biconjugate gradient method is used to solve the linear algebraic system involved. Initial approximations for the Rayleigh quotient iteration are provided by an overrelaxation technique where any iterate is kept orthogonal to all the previously computed eigenmodes [5].

4 Tests and Comparisons.

In order to validate our program, some dipolar modes of spherical and cylindrical resonators have been solved by using 2000 and 3300 mesh points, respectively, and fields and frequencies so obtained have been compared with corresponding analytical values. Some results of this comparison are shown in table 1 and 2, where errors in frequencies and fields are reported.

Table 1: Errors in frequencies and fields for some dipolar modes of the spherical resonator (2000 mesh points).

Mode θ,r,ϕ	Relat. error in frequency	Max.abs.normalized error in e_z	in e_r
TM111	$3.8\cdot10^{-4}$	$2.0\cdot10^{-3}$	$2.5\cdot10^{-3}$
TM211	$4.0\cdot10^{-5}$	$1.0\cdot10^{-2}$	$1.4\cdot10^{-2}$
TE111	$5.8\cdot10^{-4}$	$2.9\cdot10^{-3}$	$2.0\cdot10^{-3}$
TM311	$3.2\cdot10^{-4}$	$8.9\cdot10^{-3}$	$6.8\cdot10^{-3}$
TE211	$1.1\cdot10^{-3}$	$1.8\cdot10^{-2}$	$3.4\cdot10^{-2}$
TM411	$1.0\cdot10^{-3}$	$1.0\cdot10^{-2}$	$2.3\cdot10^{-2}$

Since field components can be zero, absolute errors normalized to the maximum value of the strongest component have been used for them, while relative errors have been used for frequencies. Table 1 and 2 show that a good agreement is achieved even with a limited number of mesh points; better results are obtained, of course, with a larger number of points.

Table 2: Errors in frequencies and fields for some dipolar modes of the cylindrical resonator (3300 mesh points). Starred errors correspond to zero components.

Mode	Relat. error	Max.abs.normalized error	
ϕ, r, z	in frequency	in e_z	in e_r
TE111	$3.5 \cdot 10^{-5}$	$5.1 \cdot 10^{-7}$ (*)	$2.6 \cdot 10^{-3}$
TE112	$3.7 \cdot 10^{-4}$	$1.4 \cdot 10^{-7}$ (*)	$1.0 \cdot 10^{-2}$
TM110	$3.6 \cdot 10^{-4}$	$2.9 \cdot 10^{-4}$	$4.5 \cdot 10^{-5}$ (*)
TM111	$3.1 \cdot 10^{-4}$	$2.9 \cdot 10^{-4}$	$4.8 \cdot 10^{-3}$
TM112	$8.5 \cdot 10^{-6}$	$2.9 \cdot 10^{-4}$	$3.2 \cdot 10^{-2}$
TE113	$1.0 \cdot 10^{-3}$	$1.7 \cdot 10^{-6}$ (*)	$2.3 \cdot 10^{-2}$

A comparison between some computed and measured dipolar mode frequencies for a cavity of the synchrotron light source ELETTRA [9] has been performed, too. Results are reported in table 3. About 3000 mesh points have been used in these computations. All computed frequencies agree with measured ones within about $3 \cdot 10^{-2}$, an acceptable value since several hard to control perturbing effects are always present in a real cavity. Fig. 2 shows the contour plot of e_z for one mode of the ELETTRA cavity.

In all the tests that have been carried out, no spurious solution was produced.

Table 3: Computed and measured [9] frequencies for some dipolar modes of the ELETTRA cavity (3000 mesh points).

Frequency (Mhz)			
computed	measured	computed	measured
751	743.1	754	748.7
1128	1120.4	1230	1221.3
1282	1248.0	1324	1306.6
1608	1561.1	1690	1637.8
1733	1712.6	1751	1720.1

5 Conclusions

The OSCAR2D code has been extended to the computation of hybrid multipolar modes of cylindrically symmetric cavities. No spurious mode is produced by the program. A good agreement with analytical solutions has been found.

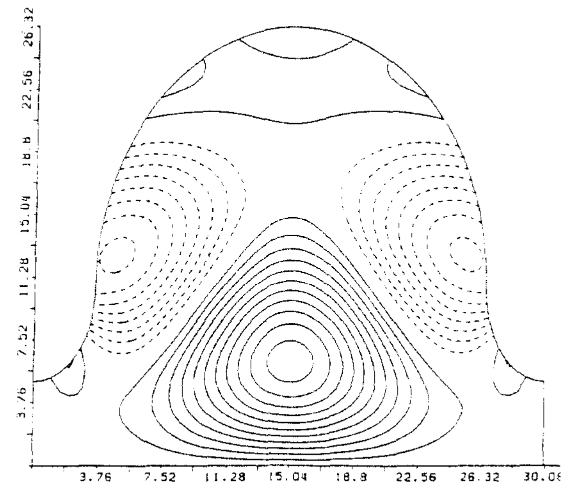

Figure 2: e_z contour plot for the 9^{th} dipolar mode of ELETTRA cavity.

References

[1] U. Van Rienen, T. Weiland, "Triangular discretization method for the evaluation of RF-fields in cylindrically symmetric cavities", IEEE Transactions on Magnetics, vol.MAG-21, pp.2317-2320, November 1985.

[2] P. Fernandes, R. Parodi, "Features and applications of the RF cavity code OSCAR2D." To be published in IEEE Transactions on Magnetics, September 1991.

[3] P. Fernandes, R. Parodi, "Computation of electromagnetic fields in multicell resonant structures for particle acceleration", IEEE Transactions on Magnetics, Vol.MAG-19, pp.2421-2424, November 1983.

[4] P. Fernandes, R. Parodi, "Computation of electromagnetic fields in TE and TM resonator and in waveguides", IEEE Transaction on Magnetics, Vol.MAG-21, pp.2246-2249, November 1985.

[5] P. Fernandes, R. Parodi, "Higher order modes computation in RF cavities", IEEE Transaction on Magnetics, Vol.24, pp.154-157, January 1988.

[6] R.L. Gluckstern, R.F. Holsinger, K. Halbach, G.N. Minerbo, "Ultrafish - generalization of Superfish to $m > 1$", in Proceeding of the 1981 Linear Accelerator Conference, Report LA-9234 C, Los Alamos, USA, 1981.

[7] A.G. Daikowsky, Yu.I. Portugalov, A.D. Ryabov, "PRUD-code for calculation of the nonsymmetric modes in axial symmetric cavities", Particle Accelerators, Vol.12, pp.59-64, 1982.

[8] A. Bossavit, "Solving Maxwell equations in a closed cavity, and the question of spurious modes", IEEE Transactions on Magnetics, Vol.26, pp.702-705, March 1990.

[9] E. Karantzoulis, A. Wrulich, "Multibunch instability investigation for the ELETTRA cavities", in Proceedings of EPAC90 Conference, Nice, France, June 1990, pp.1618-20.