EFFECT OF TUNE MODULATION ON THE DYNAMIC APERTURE OF THE SSC LATTICE

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Abstract

We model the effects of magnet power supply ripple on the long term dynamic aperture of the SSC lattice by modulating the tunes. The lattice is represented by a Taylor map of twelfth order in the phase space coordinates. The transverse tunes parametrizing the rotation matrix (obtained from the linear terms of the map) are sinusoidally modulated at different choices of amplitude and frequency. Particles are tracked through this modulated map for over a million turns. The tune modulation results in a decrease of the dynamic aperture. The extent of this decrease depends largely on the tune of the lattice and to a secondary extent on the amplitude and frequency of the ripple.

Introduction

The long term dynamic aperture of particles circulating in a storage ring depends on various factors. To a large extent it is determined by the nominal tunes of the machine and the strengths of the nonlinear fields inclusive of field errors. Other factors such as power supply ripple, noise, chromaticity setting etc. are also known to affect the dynamic aperture to varying degrees. Here we study the effect of power supply ripple on the dynamic aperture of the SSC collider lattice. We model the ripple as a modulation of the tune at the ripple frequency. We determine the dynamic aperture by tracking particles for a million turns in our computer simulations. In order to study the effect at different ripple frequencies (due to the presence of different harmonics) we need a method for the fast tracking of particles. Recently we have shown [1] that Taylor maps of 11th or 12th order in the phase space variables provide an accurate representation of the SSC collider lattice. The time for tracking these maps is about 30–40 times less than the time needed for the usual element by element tracking routines for the time scales of interest. Consequently for the study reported here we have used a Taylor map of 12th order to represent the collider lattice.

Model and Results

Let $\vec{z}_i$ denote the initial phase space coordinates of a particle and $\mathcal{M}(\nu_{x0}, \nu_{y0})$ represent the one turn map around the ring for the choice of nominal tunes ν_{x0}, ν_{y0}. For the collider these tunes are $\nu_{x0} = 107.285$, $\nu_{y0} = 106.2649$. The coordinates $\vec{z}_{i+1}$ after one turn around the ring is obtained from

$$\vec{z}_{i+1} = \mathcal{M}(\nu_{x0}, \nu_{y0}) \circ \vec{z}_i \tag{1}$$

For greater accuracy we update only the transverse variables and the time of flight i.e the variables $(x, p_x, y, p_y, c\tau)$ by a Taylor map. The change in the relative momentum deviation $\delta = (p - p_0)/p_0$ through an rf cavity is calculated exactly at the start of the turn before the map is evaluated. To study the effect of changing the tune, we multiply the map by an appropriate matrix and then track the modified map. Here we have chosen to study the effects of modulating only the horizontal tune. The modified map we study is

$$\mathcal{M}_R(\nu_{x0} + \Delta\nu_x, \nu_{y0}) = \mathcal{M} \circ R(\Delta\nu_x, 0) \tag{2}$$

where $R(\Delta\nu_x, 0)$ is a 5x5 matrix since $\mathcal{M}$ is a polynomial in the five variables (x, p_x, y, p_y, δ). The elements of this matrix are

$$R = \begin{bmatrix} C_x + \alpha_x S_x & \beta_x S_x & 0 & 0 & 0 \\ -\gamma_x S_x & C_x - \alpha_x S_x & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{3}$$

where

$$C_x = \cos 2\pi\Delta\nu_x$$
$$S_x = \sin 2\pi\Delta\nu_x \ .$$

The change in the horizontal tune can be written as

$$\Delta\nu_x = \Delta\nu_{x0} + a_r \cos(2\pi f_r t + \phi_0) \tag{4}$$

$\Delta\nu_{x0}$ changes the nominal tune of the machine in the absence of ripple to $\nu_{x0} + \Delta\nu_{x0}$. The ripple is represented by a_r, the amplitude of the tune modulation, f_r the frequency of the modulation and ϕ_0 the initial phase. It has been determined [2] that 60 Hz and 720 Hz are the main ripple frequencies. In addition to these we have also studied effects at 10Hz and 120 Hz. 10 Hz was chosen since previous studies [3] have shown that low frequencies have the largest effect on the dynamic aperture. 120 Hz was chosen as another harmonic of the main ripple frequencies. The tunes corresponding to these frequencies are 0.0029, 0.017, 0.0348 and 0.209 respectively. At each ripple frequency we chose ten different values of $\Delta\nu_{x0}$ such

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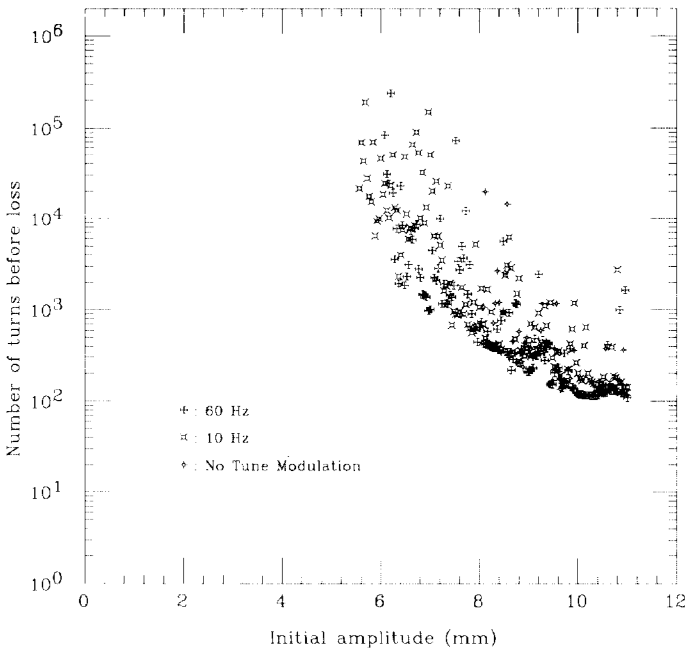

Figure 1: Survival Plots at the nominal tune ν_x=0.1012, a_r=0.001

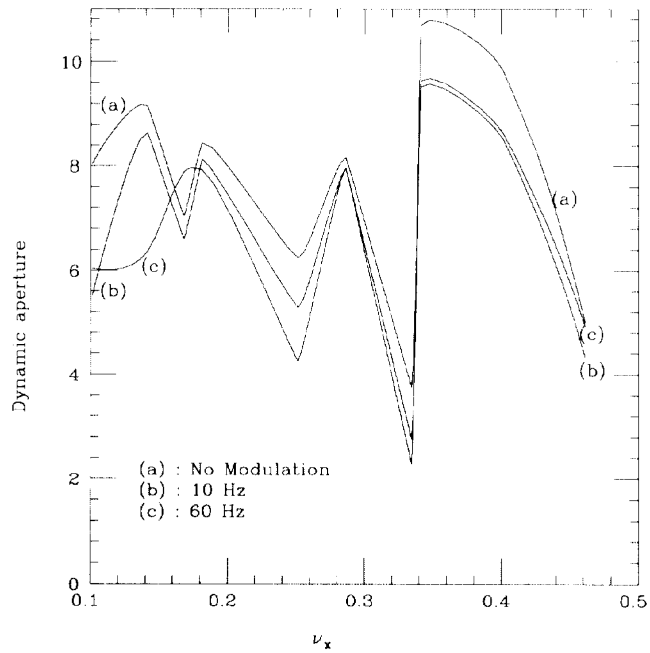

Figure 2: Variation of the Dynamic Aperture with frequency at the amplitude a_r=0.001. (a) No tune Modulation; (b) f_r=10 Hz, (c) f_r=60Hz .

that $0.1 \leq \nu_{x0} + \Delta\nu_{x0} \leq 0.5$. The tune modulation will have a significant effect on the dynamics only over several synchrotron periods. For the collider the synchrotron period corresponds to about 256 revolutions around the ring. We chose to track particles for 1.5 million turns at each value of $\Delta\nu_{x0}$ for two choices of amplitude, $a_r = 0.001$ and $a_r = 0.002$ for each of the four ripple frequencies. Figure 1 shows three survival plots at one of these tunes. From this data we determine the dynamic aperture after over 10^6 turns to be near 8.04 mm for the case with no tune modulation, 5.52 mm at 10 Hz and 6.04 mm at 60 Hz. There is clear evidence of greater instability at smaller amplitudes even after a few thousand turns for the cases with ripple. Thus the effects of the tune modulation are already felt within a few synchrotron periods – at least for this tune

The above result does show that at this tune the lowest ripple frequency brings about the largest reduction in the dynamic aperture. However, the same may not be true at other choices of the nominal tune and indeed we do find that the most damaging ripple frequency depends very much on the tune. To compare the effect of the ripple frequencies we study the variation of the dynamic aperture at a given amplitude across a range of tunes. Figure 2 shows this variation at the ripple amplitude $a_r = 0.001$ for the ripple frequencies of 10 Hz and 60 Hz . For comparison we have also shown by curve (a) the corresponding data in the absence of ripple. All curves show that the dynamic aperture drops sharply at the second, third and fourth order resonances. Near the sixth order resonance ($\nu_x = 0.166$) while the curves (a) and (b) go through a minimum, curve (c) (f_r= 60 Hz) goes through a maximum. Across the range of tunes between 0.12 to 0.25 the tune modulation at 60 Hz causes the largest reduction in the dynamic aperture. At the nominal tune (ν_x=0.285) both the 10 Hz and the 60 Hz ripple cause the same slight reduction in aperture. Beyond this tune, the 10 Hz ripple has the slightly worse effect compared to the 60 Hz ripple.

In Figure 3 we show the corresponding data for the ripple frequencies 120 Hz and 720 Hz and for comparison we also plot the 60 Hz curve. Here there are fewer surprises. Across the whole range of tunes the two higher ripple frequencies affect the dynamic aperture much less than the 60 Hz ripple with the 720 Hz ripple having the weakest effect.

We have also studied the variation of the dynamic aperture with the amplitude of ripple at a fixed ripple frequency. Figure 4 shows the data at the 60 Hz ripple frequency for the two amplitudes a_r=0.001 and a_r=0.002. The larger amplitude has the stronger effect but it is more pronounced between the tunes 0.1 to 0.25. This is a qualitatively similar result to what was seen in the previous two figures. Our data thus leads to the conclusion that the influence of the ripple on the dynamic aperture depends strongly on the tune, the reduction in the dynamic aperture being larger at the low tunes.

To identify the reasons behind why any particular frequency causes the largest reduction in the dynamic aperture at a given tune, we can examine the Fourier spectra of the phase space coordinates. The strength of the coupling between the degrees of freedom determine the resonances most responsible for particle loss. In addition it is known

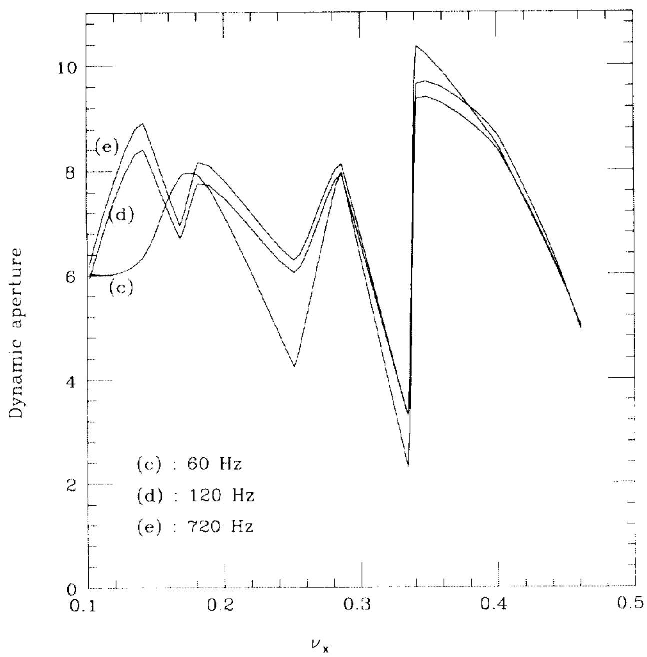

Figure 3: Variation of the Dynamic Aperture with frequency at the amplitude a_r=0.001. (c) f_r=60 Hz, (d) f_r=120 Hz, (e) f_r=720 Hz

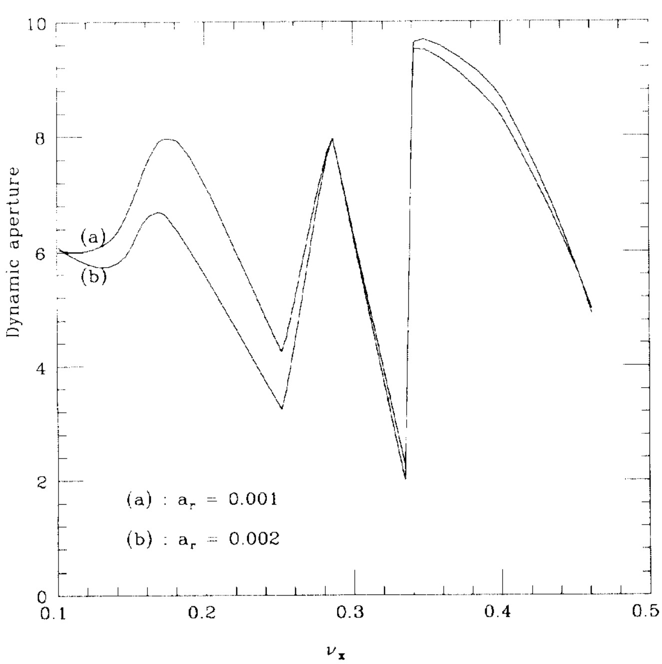

Figure 4: Variation of the Dynamic Aperture with amplitude at the ripple frequency f_r=60 Hz. (a) a_r=0.001, (b) a_r=0.002.

that the number of additional resonances excited about the main resonances due to tune modulation is inversely proportional to the modulation frequency. By this argument the possibility for chaotic motion due to resonance overlap increases at lower frequency and hence lower ripple frequencies are expected to cause a greater reduction in the beam lifetime. The FFT spectrum shows that the coupling is strongest at all tunes between the horizontal (x) and longitudinal (s) degrees of freedom so resonances likely to be most responsible for particle loss lie in the $\nu_x - \nu_s$ plane. However there is not a significant difference in the harmonic content of the spectrum with 10 Hz ripple compared to the spectrum at 60 Hz. One plausible reason could be that the effect of the power supply ripple has been somewhat diluted due to the presence of a rf cavity. Even in the absence of an external source of tune modulation, a non-zero chromaticity will cause the tune to be modulated at the synchrotron frequency due to momentum oscillations. Thus the influence of the ripple is determined by the relative strengths of the amplitudes of modulation due to the ripple and the cavity.

SUMMARY

Overall we have found that the effect of the ripple at different frequencies and amplitude has been to reduce the dynamic aperture across the range of tunes studied, the effects being stronger within the tune range 0.1-0.25. Various issues have been brought up by our observations, such as what determines the relative effect of the 10 Hz and 60 Hz ripple at any particular tune. At this point we will not venture theoretical speculations about their causes. That is left as a matter for further numerical studies and analysis, both of which are being pursued.

REFERENCES

[1] T.Sen, Y.T. Yan, A.W. Chao and E. Forest, Taylor Maps for long term tracking at the SSC, SSC Report in preparation

[2] Site Specific Conceptual Design Report SSCL-SR-1056

[3] M. Furman and F. Schmidt, Reduction of the Dynamic Aperture due to Tune Modulation, CERN SPS/89-1 (AMS), SSCL-N-718