APPLICATIONS OF ZMAP TO THE SSC

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Abstract

Zmap is a differential-algebraic High-order map extraction program for the systematic circular accelerator program Teapot. Its application to the SSC is outlined in this paper.

I. INTRODUCTION

At present, serious long-term stability studies of circular accelerator machines require element-by-element tracking of charged particles. For example, a dynamic aperture study of the Superconducting Super Collider (SSC) usually requires 10^5 turn computational tracking of approximately a hundred protons through more than 10^4 elements each turn. Each of these simulations requires about 20 hours Cray CPU time [1]. Since a circular accelerator machine is periodic, the static nonlinear lattice might be represented by a one-turn map. The one-turn map must have the following properties: (a) it must be simpler than the original element-by-element description of the nonlinear lattice so that enormous computer time can be saved for the study of the circular accelerator; and, (b) all (or certain) important information in the original element-by-element nonlinear static lattice must be included, that is, the one-turn map must represent the original element-by-element lattice well enough for specific studies (though it can not represent the original lattice exactly).

Zmap [2] was developed in the hope that the two properties mentioned above would be satisfied. Zmap is a differential-algebraic [3] map extraction program. It was developed for extracting one-turn Taylor maps from the systematic circular accelerator program Teapot [4]. The post-Teapot tracking program Ztrack [5] was used to take advantage of the available set-up of the two necessary input files. One is the machine file named Zfile provided by Teapot, which describes the nonlinear lattice. The other is the command file named Zcmd which in Zmap describes how the one-turn map is to be extracted, e.g. the order and the dimension of the Taylor map. Arithmetic operations for initializations, kicks, and drifts of Teapot are directly translated into calls to the corresponding multivariable polynomial operation subroutines of ZLIB [6]. If requested by the command file Zcmd, Zmap will find the dispersed closed orbit and then extract a one-turn map with respect to this closed orbit. Note that the dispersed closed orbit is expressed as a polynomial of the energy deviation to a specified order.

** Operated by Universities Research Association, Inc., for the U.S. Department of Energy under the Contract No. DE-AC02-89ER40486.

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II. THE TAYLOR MAP

For a 3-dimensional case, a closed-orbit Taylor map extracted with Zmap up to the Ω order can be expressed as follows (adapting the same notational convention as in ZLIB manual [6]).

$$m : \vec{z} = \vec{U}(\vec{z}) = \sum_{k=1}^{\Omega} \vec{u}(\vec{k})\vec{z}^{\vec{k}}$$

where $\vec{z}$ represents the conanical coordinates and their conjugate momenta, i.e. its transpose is given by

$$\vec{z}^T = [z_1, z_2, \ldots, z_6] = [x, P_x, y, P_y, \delta, P_\delta];$$

the transpose of the VTPS (vector truncated power series) is given by

$$\vec{U}^T(\vec{z}) = [U_1(\vec{z}), U_2(\vec{z}), U_3(\vec{z}), U_4(\vec{z}), U_5(\vec{z}), U_6(\vec{z})];$$

and

$$\vec{k}^T \equiv [k_1, k_2, \ldots, k_6];$$

$$\vec{z}^{\vec{k}} \equiv z_1^{k_1} z_2^{k_2} \ldots z_6^{k_6};$$

$$k = \sum_{i=1}^{6} k_i\ , \qquad \text{for } 0 \le k_i \le \Omega;$$

$$\sum_{k=1}^{\Omega} \equiv \text{summation over all } \vec{k}\text{'s for } k = 1, 2, \ldots, \Omega.$$

Note that P_x, P_y, and P_δ are the conjugate momenta of x, y, and δ respectively, where x and y are the horizontal and vertical deviations from the closed orbit and $\delta = \Delta E/E$ is the energy deviation from the on-momentum energy. Computationally, only the coefficenets $\vec{u}(\vec{k})$ of the VTPS $\vec{U}(\vec{z})$ are stored and manipulated. The coefficients $\vec{u}(\vec{k})$ is given by

$$\vec{u}^T(\vec{k}) = [u_1(\vec{k}), u_2(\vec{k}), u_3(\vec{k}), u_4(\vec{k}), u_5(\vec{k}), u_6(\vec{k})],$$

where, for limited memory available in the computers, each of the $u_i(\vec{k})$ corresponding to the TPS $U_i(\vec{z})$, for i = 1, 2, ...

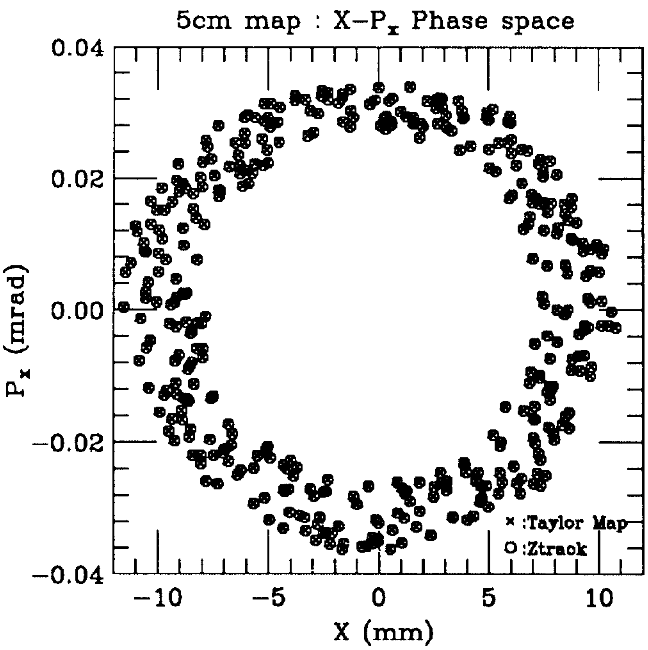

Figure 1: turn-by-turn phase space plots of a element-by element tracking and its associated 12th-order Taylor map tracking for 400 turns

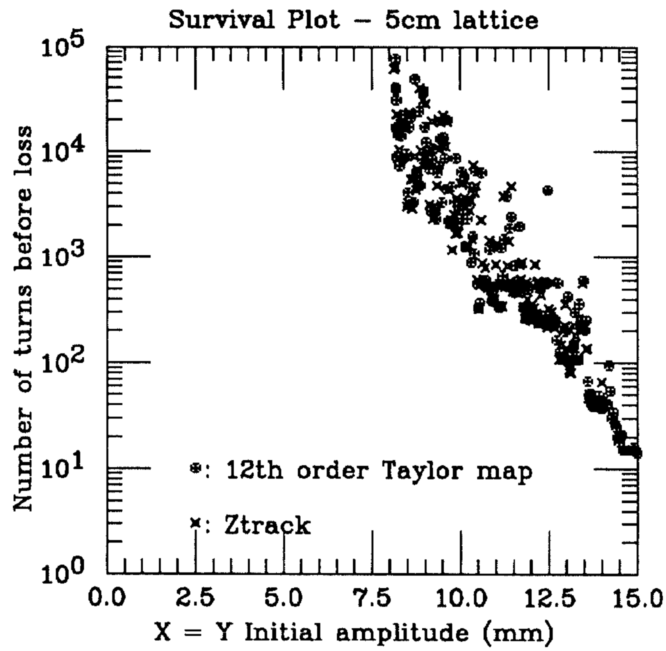

Figure 2: 10^5-turn survival plots for a 2-TeV, 5cm-diameter dipole injection lattice of the SSC, comparing the data from an 12th-order Taylor-map (extracted with Zmap) tracking with the data from its associated "Ztrack" element-by-element tracking.

6 is stored as a one-dimensional array although the index $\vec{k}$ is a vector.

III. Taylor Map Tracking

One serious question is whether the one-turn Taylor map contains enough information for the study of the long-term stability. In view of the fact that the nonlinear effects in the SSC (or other alternating gradient synchrotrons) are due to chromaticities, magnet errors and orbit errors and their associated corrections, which must be small enough compared to the leading linear effects in the domain of interest, there must exist a one-turn Taylor map, truncated at suitably high order that works. The question is therefore not whether the one-turn Taylor map works, but how high the truncated order of a Taylor map should be in order to ensure its reliability. One simple way of checking this is to use the Taylor map for particle tracking. The tracking formula is given by

$$\vec{z}_i = m : \vec{z}_{i-1} = \vec{U}(\vec{z}_{i-1}) = \sum_{k=1}^{\Omega} \vec{u}(\vec{k}) \vec{z}_{i-1}^{\vec{k}},$$

where the coefficients $\vec{u}(\vec{k})$ are kept constant and the turn-by-turn phase space coordinates $\vec{z}_i$ are obtained by substituting the previous turn phase space coordinates $\vec{z}_{i-1}$ into the truncated multi-variable polynomial of order Ω. Taylor map tracking subroutines in ZLIB can be called to perform these map trackings.

Shown in Figure 1 are two phase space (x, p_x) plots for 400 turns, in the same frame, one from element-by-element tracking with the post-Teapot tracking program Ztrack for an SSC injection lattice with 5cm-diameter dipole multipole errors and the other from its associated 12th-order Taylor-map tracking. A particle is launched at a betatron amplitude x = y = 8.1 mm in each of the two trackings. Note that dynamic aperture of this injection lattice is about 8.1 mm. The two phase space plots are hardly distinguishable within the resolution of this figure. Indeed, the accuracy of one turn Taylor map tracking for this case is about 8 digits compared to its associated element-by-element tracking.

Figure 2 shows two survival plots, in the same frame, comparing tracking data for a hundred thousand turns from "Ztrack" element-by-element tracking for the same SSC injection lattice and from its associated 12th-order one-turn-Taylor-map tracking. Although the map tracking results and the element-by-element tracking results are not identical, they agree in the global behaviour up to one million turns, as well as in many other details. Including the time for extracting the one-turn Taylor map, obtaining a survival plot with reasonable detail up to 10 million turns (the required proton-coasting time of the SSC injection lattice), a 12th-order Taylor-map tracking is more than 10 times faster than the corresponding "Ztrack" element-by-element tracking for the SSC injection lattice. However, such a comparison of computational speed is not suitable for short-term trackings.

Based on our experience [7], an 11th-order Taylor map is sufficient for the SSC dynamic aperture study up to 10^6 turns. However a 10th or a 9th order maps would not be suitable. Futher studies [8] show that the problem of the 10th and 9th order Taylor maps is that they are not symplectic enough [9]. The effects from artificial diffusion due to the missing of higher-order effects become significant after 10^5 turns for the 10th and 9th order map trackings. However, a 9th-order Taylor map for the SSC lattice contains enough important information for long-term study. If one symplectifies the 9th-order Taylor map by adding small artificial terms for orders larger than the 9th order using Dragt-Finn factorization [10] and reexpands the map into an 11th-order or a 12th-order Taylor map, one obtains roughly the same dynamic aperture at 10^6 turn as before [8]. Therefore, for long-term stability study, one could conclude that for a Taylor map to be symplectic enough requires a higher truncation order than the truncation order required for the same Taylor map to contain sufficient important information. Thus, one can always extract a Taylor map with a truncation order that is high enough to contain sufficient important information (but not necessarily symplectic enough) and then symplectify it for long-term tracking [11] [12].

IV. Analysis Of A Taylor Map

The most difficult issue for using Taylor map as a long-term tracking tool is the choice of the truncation order. If one must rely on the comparison of the Taylor map tracking results with its associated element-by-element tracking results to determine the truncation order, the Taylor map tracking would not be necessary at all. While this issue is still under investigation, there is little doubt that a modest-order Taylor map can always be used for short-term studies in calculating the smear, tune shifts, resonances, and other quantities of interest [13].

V. Summary And Conclusion

A differential algebraic high-order map extraction program, Zmap, has been developed for extracting one-turn Taylor maps from the systematic circular accelerator program Teapot. Extraction of the 12th-order Taylor maps is practical for the SSC. These maps can be used for the analysis of the short-term behavior of the charged particles in the SSC as well as long-term stability studies. Most of the higher order terms are kept to satisfy the symplectic condition. This is consistent with the fact that in the Fourier representation of the mixed-variable generating function of the map, very few Fourier modes are required [14].

VI. Acknowledgments

The authors thank G. Bourianoff, A.J. Dragt, and J. Irwin for many discussions.

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[9] Here "symplectic enough" may sound strange to the readers. It has no nothing to with bit truncation error. Given a computer with infinite accuracy, there exists a critical order Ω_c such that the Taylor map truncated at order Ω_c can be considered symplectic if the higher order ($> \Omega_c$) contributions are all small enough that they do not modify the outcome of the calculation.

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