Chromaticity Modelling in the Fermilab Main Ring

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ABSTRACT

It has been traditional to measure the transverse tunes of an accelerator beam *vs.* the momentum error dp/p and call the linear slope when the beam is near the center of the machine the chromaticity. As computer simulations of accelerators improve, it becomes important to measure the properties of existing machines better to verify that the simulations are indeed accurate. The Fermilab Main Ring magnets are known to be rich in harmonic content, but they were not individually measured. We have measured the non-linear dependence of tune *vs.* momentum error and interpret these data in terms of the octupole content of the main quadrupoles and the decapole content of the dipoles.

INTRODUCTION

A number of past investigations of tune in the Main Ring have identified several sources of tune dependence on momentum error:

(1) the "natural chromaticity" of the quadrupoles;
(2) the sextupoles introduced to compensate for (1).

the sextupole component of the dipoles, which adds three separate contributions:

(3) geometrical (poleface and assembly errors);
(4) remanent field at low beam momentum;
(5) eddy current contribution at high dp/dt.

Of these errors, the first three scale with the beam momentum, while the remanent field and eddy current effects do not.

Chromaticity is controlled in the Main Ring by using a model to predict the slope of tune *vs.* momentum error for a given sextupole current. The model is solved for the sextupole currents that will give a slope chosen by the operator. Usually the slope is chosen to be –5 to –10 chromaticity units. The model incorporates all five factors listed above, although the natural chromaticity of the quadrupoles and the geometrical contribution from the dipoles are not treated separately:

$$\xi = a + \frac{1}{B\rho}\left(b\, I_h + c\, I_v + B''\right). \qquad (1)$$

I_h and I_v are the horizontal and vertical sextupole currents. B" includes the remanent field and eddy current contributions to the sextupole component of the ring dipoles:

$$B'' = a_{rem} + a_{eddy}\ dB/dt \qquad (2)$$

The parameters for these models are derived by measuring the tune shift as a function of momentum error in the Main Ring. The orbit radius is varied by shifting the RF frequency offset (ROF). Then the tune is measured by pinging the beam and taking a fast fourier transform of the coherent oscillation. The ROF is converted to a momentum error by a calibration table. It is independently measured by fitting the known dispersion function of the Main Ring to the observed closed orbit as measured by the beam position monitors. The second method was used for the measurements of momentum error.

MEASUREMENT OF CHROMATICITY

We measured the chromaticity at injection (8.9 GeV/c) and at the ramp flattop (150 GeV/c). We took supplementary measurements at 37 GeV/c with no ramping and also with a 150 GeV/c–sec ramp in order to identify the eddy current contribution. The data at 150 GeV/c included different settings of I_h and Iv, the sextupole currents. For some settings of the sextupoles we observed strong coupling of the vertical and horizontal betatron tunes. The data presented here, however, are taken with only very weak coupling.

Thus the model used for both the vertical and horizontal tunes is:

$$\nu = a_0 + a_1\, dp + a_2\, dp^2 + a_3\, dp^3 + a_4\, dp^4 + a_5\, I_h\, dp + a_6\, I_v\, dp + a_7\, I_h + a_8\, I_v\ , \qquad (3)$$

where dp is the momentum error in percent, and I_h and I_v are the horizontal and vertical sextupole currents in Amperes. The interpretation of the coefficients is as follows: a_1 is normally called the chromaticity, a_2 is due to the sextupole component, and a_3 to octupole component and a_4 to decapole component of the guide field. The sextupole currents I_h and I_v are expected to act through the terms a_5 and a_6. We included a_7 and a_8 because we found that including all the first order effects was necessary in order to get a good fit. As expected the resulting values of a_7 and a_8 were small. The fitted coefficients is shown in Table 1. The standard errors shown understate the uncertainties in the coefficients because of correlations between parameters.

Table 1
Fit to Tune vs. Momentum Error at 150 GeV/c

The tune was measured as a function of the horizontal and vertical sextupole currents, and momentum error. The fit has the form of Equation (3).

term	nx	ny
a_0	0.363 ± 0.002	0.427 ± 0.001
a_1	–0.311 ± 0.011	–0.145 ± 0.006
a_2	0.190 ± 0.026	–0.073 ± 0.013
a_3	0.426 ± 0.070	–0.176 ± 0.036
a_4	–0.611 ± 0.226	0.515 ± 0.116
a_5	0.030 ± 0.001	–0.008 ± 0.001
a_6	0.008 ± 0.001	–0.025 ± 0.001
a_7	–0.001 ± 0.001	0.000 ± 0.001
a_8	–0.001 ± 0.001	–0.000 ± 0.001

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Ignoring the insignificant terms a_7 and a_8 and keeping only the first order terms in dp we have, for ν_x:

$$\nu_x = 0.363 - 0.311\ dp + (0.030\ I_h\ dp + 0.008\ I_v\ dp) \qquad (4)$$

and for ν_y:

$$\nu_y = 0.427 - 0.145\ dp + (-0.0081\ I_h\ dp - 0.0247\ I_v\ dp) \qquad (5)$$

from which we can immediately identify the coefficients of I_h and I_v above (remember that dp is in percent. $B\rho$ is in T•m). The remanent field and eddy current effects of Equations (1) and (2) are not included here.

We corrected the slopes of ν_x and ν_y vs. dp for the effects of sextupole current by assuming that the coefficients of the sextupole current terms at 37 and 8.9 GeV/c were the same as those measured in Eqs. (4) and (5). Then C_x and C_y, shown in Table 2, include only the natural chromaticity of the quadrupoles plus a component from sextupole errors in the dipole magnetic field.

Table 2
Chromaticity at Several Beam Momenta

Linear slopes of horizontal (C_x) and vertical (C_y) betatron tune *vs.* momentum error measured at several beam momenta, corrected for differing sextupole currents as described in the text. For the large values at 8.9 GeV/c see explanation in text.

p (GeV/c)	C_x	C_y
8.9	−157.3	95.8
37 (unramped)	−44.2	−2.1
37 (ramped)	−15.3	−25.7
150	−31.1	−14.5

The large values of C_x and C_y, especially at low energy, are thought to be due to the large contribution of remanent fields and eddy currents at low energies, and not to an abnormally large (and unbelievable) natural chromaticity. The ring sextupoles are used to compensate for this remanent field.

The effects of remanent fields and eddy currents in the dipoles can be separated out by assuming that they do not scale with the beam momentum. A least squares fit to the data of Table 2 gives:

$$C_x = -17.5 + \frac{1}{B\rho}\ (-4120) \qquad (6)$$

and

$$C_y = -25.8 + \frac{1}{B\rho}\ (3580). \qquad (7)$$

The second term is the B" term of Equation (1). This term can be broken into the remanent field and eddy terms of Equation (2) by assuming that the eddy current term contributes only to the ramped case at 37 GeV. We can see the necessity of the eddy current term in the data of Fig. 1. Ramping the beam has a large effect on the horizontal tune. The coefficients of the polynomial fit of Fig. 1 are shown in Table 4. There is a substantial change in the sextupole dp^3 term and in the higher orders as well, although the uncertainty of the higher multipole coefficients is greater in the ramped case because the data are confined to a smaller range of momentum errors and hence to a smaller radius.

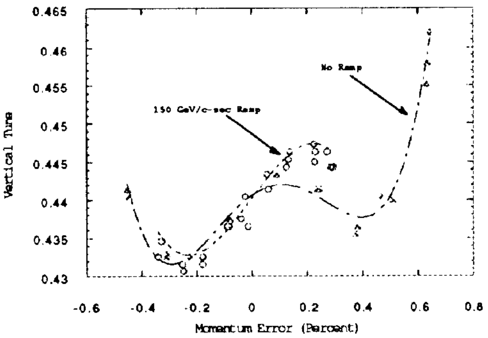

Fig. 1 Effect of Ramping at 37 GeV/c Beam Momentum. The horizontal tune is shown as a function of momentum error with (open circles) and without (solid triangles) a 150 GeV/c–sec ramp. Note the evidence for decapole moment to the ring magnetic field (quartic polynomial). The coefficients for the fit to the ramped data (dashed curve) and unramped data (dot-dash curve) are listed in Table 4.

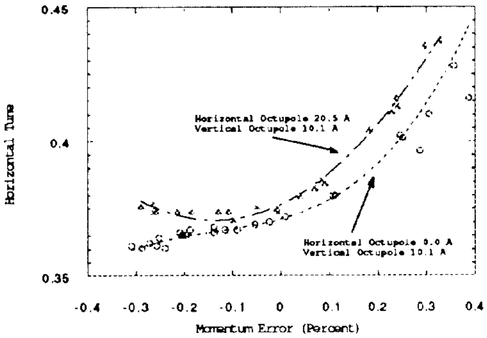

Fig. 2 Effect of Horizontal Octupoles on Horizontal Tune. The horizontal octupoles were excited to 20.1 A for the first data set (solid triangles) and then turned off for the second (open circles). The coefficients of the polynomial fit to the octupole on data (dot-dash curve) and the octupole off data (dashed curve) are shown in Table 3.

Table 3
Polynomial Fits to Horizontal Tune *vs.* Momentum Error

Fits to the horizontal tune data of the form $\nu_x = a_0 + a_1\, dp + a_2\, dp^2 + a_3\, dp^3 + a_4\, dp^4$ where dp is the momentum error in percent. r^2 is the square of the correlation coefficient. a_1, corrrected for the effects of sextupole current as described in the text, is the horizontal chromaticity

	150 GeV/c HorizontalO ctupole on	150 GeV/c HorizontalO ctupole off	37 GeV/c no ramp	37 GeV/c ramped	8 GeV/c
Horizontal Sextupole Curmet (A)	16.5	16.5	3.9	3.9	2.5
Vertical Sextupole Current (A)	−16.1	−16.1	−1.8	−1.8	1.6
a_0	0.378 ± 0.001	0.370 ± 0.001	0.360 ± 0.001	0.365 ± 0.001	0.370 ± 0.001
a_1	0.095 ± 0.003	0.056 ± 0.004	−0.015 ± 0.004	−0.016 ± 0.005	−0.071 ± 0.006
a_2	0.313 ± 0.016	0.196 ± 0.015	0.295 ± 0.014	0.223 ± 0.035	0.264 ± 0.057
a_3	—	0.435 ± 0.075	0.185 ± 0.021	0.432 ± 0.078	—
a_4	—	—	−0.776 ± 0.050	−0.576 ± 0.351	—
r^2	0.987	0.977	0.983	0.936	0.936

Table 4
Polynomial Fits to Vertical Tune *vs.* Momentum Error

Fits to the vertical tune data of the form $\nu_y = a_0 + a_1\, dp + a_2\, dp^2 + a_3\, dp^3 + a_4\, dp^4$ where dp is the momentum error in percent. r^2 is the square of the correlation coefficient. a_1, corrrected for the effects of sextupole current as described in the text, is the vertical chromaticity

	150 GeV/c HorizontalO ctupole on	150 GeV/c HorizontalO ctupole off	37 GeV/c no ramp	37 GeV/c ramped	8 GeV/c
Horizontal Sextupole Curmet (A)	16.5	16.5	3.9	3.9	2.5
Vertical Sextupole Current (A)	−16.1	−16.1	−1.8	−1.8	1.6
a_0	0.432 ± 0.001	0.432 ± 0.001	0.441 ± 0.001	0.441 ± 0.001	0.434 ± 0.001
a_1	0.089 ± 0.162	0.111 ± 0.008	0.022 ± 0.003	0.048 ± 0.003	−0.087 ± 0.003
a_2	—	−0.026 ± .017	−0.110 ± 0.011	−0.018 ± 0.007	—
a_3	0.409 ± 0.162	−0.129 ± 0.091	−0.114 ± 0.018	−0.316 ± 0.041	—
a_4	—	—	0.501 ± 0.041	—	—
r^2	0.976	0.979	0.968	0.962	0.987

FUTURE PLANS

We have measured the chromaticity of the Main Ring at a number of energies and found a substantial contribution from higher order multipoles. The data obtained so far suggest a wealth of information can be obtained about the multipole distribution around the ring by making tune measurements with the beam as a probe of the horizontal aperture. Detailed understanding of these multipoles will require more extensive measurements as well as computer simulations of the strength and distribution of multipoles required to reproduce the observed effects of varying the beam radius on the betatron tune. In addition, we plan to explore the conditions under which tune coupling occurs and the effects of varying the ring octupole magnets.