A DIFFERENT APPROACH TO BEAM-BEAM INTERACTION SIMULATION

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Abstract

Particle tracking simulation studies of the beam-beam interaction (BBI) in circular colliders require large amounts of CPU time to determine particle distributions out to large amplitudes. This is due to the limited number of superparticles simulated and the rarity of correlated events which can drive a particle to such amplitudes. An alternative approach for determining the final particle distribution out to large amplitudes is explored. The method employs a combination of particle tracking over selected regions of the amplitude space followed by the solution for the equilibrium distribution of the particle flow through this space. The technique is described and preliminary results, when applied to the two-dimensional case, are given.

I. INTRODUCTION

Poor beam lifetimes caused by the BBI limit the peak luminosity performance in many e^+e^- colliding beam machines. The highly nonlinear periodic kicks from the BBI can resonantly drive some particles to very large oscillation amplitudes, amplitudes so great that these particles strike a machine aperture limit and are lost from the beam. However, even in cases where the beam lifetime may be considered unexceptably low (≤ 10 min.) the fraction of particles actually lost during 1 damping time is very small and is of the order $\Delta N/N \approx$ (damping time)/(10 min.) $\approx 10^{-4} \rightarrow 10^{-5}$ for typical machine parameters. This implies very low beam densities at these amplitudes.

One method of studying the ails of colliding beam machines is by using particle tracking simulations. Although this method has been reasonably successful at describing the central core region of the colliding beams, because of statistics it is severely limited in its ability to simulate the low particle densities seen at very large amplitudes. Due to this limitation the method requires large amounts of computer time to be useful at predicting lifetimes > 1 minute.

An alternative to standard particle tracking is suggested by observation of a particle's oscillation amplitudes projected onto the transverse amplitude space. Most notably the works of Tennyson [1] (resonant streaming) and Gerasimov [2] (phase convection) have been very thought-provoking, as well as a novel tracking technique recently used by Irwin [3]. Their results show that the nonlinear beam-beam resonances perturb the amplitude space in such a way as to produce very correlated flows within the space.

There is no reason a simulation should be blind to the large amplitude motion and flow of particles. By selective sampling one can determine the behavior of the particle flow over the entire transverse amplitude space out to arbitrary amplitudes. With this knowledge one can then apply a suitable updating process to bring an initial distribution into equilibrium. We have designed a simulation which does this. It first tests how the BBI, damping and quantum excitation redistribute particles in the amplitude space. Using these "redistribution rules" an arbitrary initial pseudo-continuous particle distribution is updated until the distribution has relaxed to a stationary state. The final particle distribution and flow through the space is then used to determine beam lifetimes as a function of the aperture location.

It will be assumed throughout that the reader has a basic understanding of standard accelerator physics, the BBI problem and their associated terminology at the level of Sand's [4]. For clarity, the initial evaluation of the method has been restricted to a study of the weak/strong 2-dimensional BBI (transverse coordinates only) in electron storage rings. Here, only the basics of the method and some preliminary results are given. A more thorough treatment will be made available in the near future [5].

II. DESCRIPTION OF THE METHOD

Testing the Amplitude Space

The first phase of the simulation uses standard particle tracking to sample the motion of particle amplitudes at discrete points in the amplitude space. Let β' (the longitudinal variation of the machine beta function) be zero at the observation point of the particle. The following definitions of the dimensionless particle coordinates and amplitudes are used.

$$\frac{z}{\sigma_{zo}} = A_z \cos\theta_z \;\; ; \;\; \frac{z'}{\sigma'_{zo}} = A_z \sin\theta_z \;\; ; \;\; A_z^2 = \frac{z^2}{\sigma_{zo}^2} + \frac{z'^2}{\sigma'^2_{zo}}$$

z = x or y, the particle's transverse displacement coordinate,
z' = longitudinal rate of change of the transverse coordinate,
σ_{zo} and σ'_{zo} are the nominal rms sizes of the beam, and
θ_z is the particle phase in z/σ_{zo}, z'/σ'_{zo} phase space.

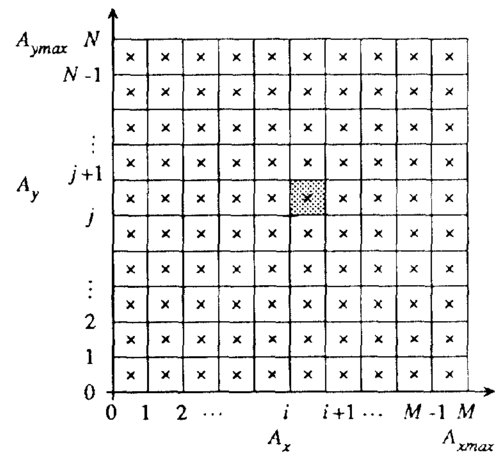

Figure 1: The Amplitude space grid.

A regular Cartesian grid is superimposed upon the amplitude space (figure 1). The grid extends from 0 amplitude out to amplitudes A_{xmax}, A_{ymax} in M, N steps and is used to separate the space into many identically-sized cells. The granularity of the grid is chosen empirically and is set sufficiently coarse to minimize computational time while not so coarse as to introduce undesired spurious effects in the results.

Particles are initialized with amplitudes equal to the value at the center of each grid cell (A_i, A_j). Many particles, each with different initial horizontal and vertical phase are started with amplitudes (A_i, A_j). The initial phases of the particles are set in the following manner. Let L, odd, equal the number of phases sampled in each transverse dimension. The initial phases are

$$\theta_{xm} = \frac{2\pi}{L}\left(m + \frac{1}{8}\right), \quad \theta_{yn} = \frac{2\pi}{L}\left(n + \frac{1}{8}\right)$$

where m and n are all integer values from 0 to $L - 1$. For each horizontal phase sampled there are L vertical phases associated with it, similarly with the vertical phases, making a total of L^2 particles sampled at each set of amplitudes (A_i, A_j). An example is shown in figure 2. As with the grid granularity, L is also chosen empirically. It should be small enough to minimize computing time, but large enough to properly sample any resonance features in the phase space.

After initialization each particle is transported through T full machine turns. Each of the machine turns consists of the following. One half of the BB kick from a gaussian shaped 2-dimensional opposing bunch is applied to the particle's primed coordinate. The particle's phase space coordinates are then rotated by a linear R_z matrix. The rotation angle of the matrix

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R_z^{φ} is $Q_z\pi$, where Q_z is the tune. Radiation excitation and damping, appropriate to maintaining the nominal beam sizes, are then applied to the particle's coordinates. The R_z^{φ} matrix is again used to rotate the phase space coordinates and thus transport the particle back to the interaction point where, once again, 1/2 the BB kick is applied. At this point the particle amplitudes are determined and stored so that they may be used in the amplitude averaging described below.

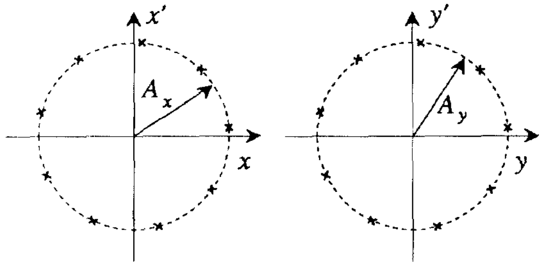


Figure 2: The distribution of particles in phase space.

If viewed on a turn-by-turn basis a particle's motion in the amplitude space, when subjected to the BBI, can jump around quite dramatically. To prevent these large excursions from appearing as a diffusion, the particle's amplitude is averaged over the T turns. (T is made large enough to allow the particle to sample any resonance effects within the space, but is significantly less than 1 damping period.) At the end of this period the particle's average position in amplitude space relative to its starting position is used to create the "density redistribution rules" for each cell.

To smooth out the effects of the discretized space, a cloud-in-cell representation of the particle (particles are given dimensions the size of a single cell) is used. As many as four cells can then be thought of as containing a fraction of a particle. In this way the finite sized particle is passed smoothly from cell to cell.

The "Density Redistribution Rules"

The L^2 particles within each cell, each of which originally had amplitudes centered within the cell, have, at the end of T turns, had their average positions spread about in the amplitude plane. This is shown graphically in figure 3. On the left is the initial distribution for the cell i, j and on the right the distribution of cell i, j's particles after the T turns. This mapping represents for cell i, j the rules for redistributing the cell "mass" (defined as the particle density at the center of the cell times the cell area $\Delta x \Delta y$) to the rest of the amplitude plane [6]. Every cell within the space has its own redistribution rule dependent upon the characteristics of the amplitude space at that point. It is this set of rules for the entire amplitude space which ultimately determines the final particle distribution.

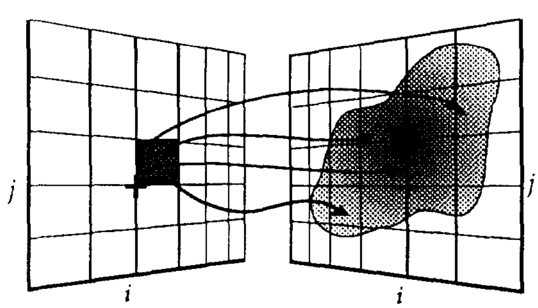


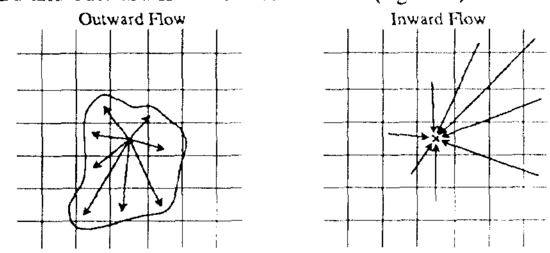
Figure 3: Graphical representation of the redistribution rules for cell i,j.

Relaxing the Particle Distribution

After creation of the redistribution rules the program enters into the second phase. Single particles are no longer tracked, but a pseudo-continuous distribution is "relaxed" on the amplitude space grid using the redistribution rules. An initial distribution is chosen and each cell within the grid assigned a particle number density accordingly. For simplicity, this number density when summed over all cells equals 1 (i.e. a total mass of 1). The redistribution rules are then applied to every cell and the results summed and stored in a separate grid array (this separate array is always initialized to zero before the rules are applied to the original array). The roles of the two grid arrays are then interchanged and the process repeated until the distribution has relaxed to a stationary state. In our case, where there is damping and random excitation included in the particle motion, it is sufficient to update the distribution for a few damping periods to reach an equilibrium distribution.

Further Exploitations

The information contained within the final stationary distribution and redistribution rules can be exploited further. For example, the flow of particles through the amplitude space is easily observed. Each cell i, j has a net mass flow out given by its own redistribution rule multiplied by the mass contained within the cell and a net mass flow inward given by the redistribution rules of the other cells which place mass into cell i, j multiplied by the mass within these cells. A flow vector for each cell is produced by taking the average of the equilibrium inward and outward flow vectors of a cell (figure 4).



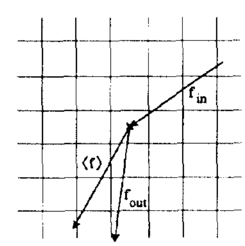


Figure 4: Calculating the flow of particles through a cell.

$$\langle r \rangle = \frac{f_{out} + f_{in}}{2}$$

For equilibrium

$$|f_{out}| = |f_{in}|$$

Lifetimes as a function of machine aperture are also readily calculated. Consider the vertical direction. A boundary is chosen at some grid amplitude, say J. Every cell below J contains some fraction of the total distribution. Let n_a be the sum of all mass within cells with grid amplitudes less than or equal to J. The distribution is now updated once using the redistribution rules. However, rather than updating the entire distribution to the maximum vertical amplitude, only cells with $j \leq J$ are used in the update. If the redistribution rules for a cell move some fraction of the cell mass to grid amplitudes larger than J then that mass is lost (figure 5). Resumming the distribution after the update then gives n_b, and the lifetime at this amplitude is

$$\frac{\Delta t}{\tau} = \frac{\Delta n}{n} \quad \text{or} \quad \tau = \frac{n_a}{n_a - n_b}$$

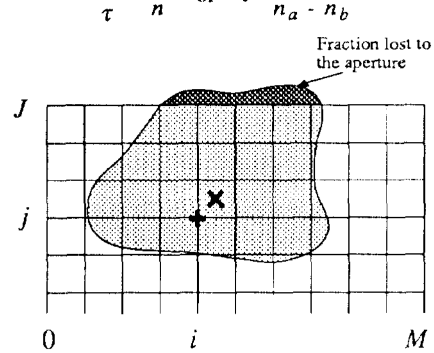


Figure 5: Loss of "mass" at an aperture limit.

III PRELIMINARY RESULTS

BBI Off

The program was tested for the case when there was no BBI. The results are shown in figures 6a-b and 7. Contours of constant (particle density)/($A_x A_y$) are shown in 6a. Each contour is spaced by x10 in density. Figure 6b shows the resulting flow and figure 7 the beam lifetime as a function of vertical machine aperture. The agreement of the simulated distribution with the expected $\exp(-x^2/2-y^2/2)$ gaussian distribution is quite good considering the coarseness of the grid used (3 cells/σ). Also shown in figure 7 is the calculated lifetime for a gaussian beam of rms size 1. Agreement, although not perfect, is very good, particularly at large amplitudes. (Note: If the revolution period of a machine is 1μsec, then a 1 hr. lifetime corresponds to 3.6×10^9 turns. This is shown as the horizontal dashed line in figure 7. The vertical scale of this figure thus extends from 1 msec to ≈ 30 years!) The flow vector of 6b shows no correlated motion as is to be expected for this case where a balance exists between the quantum excitation and damping.

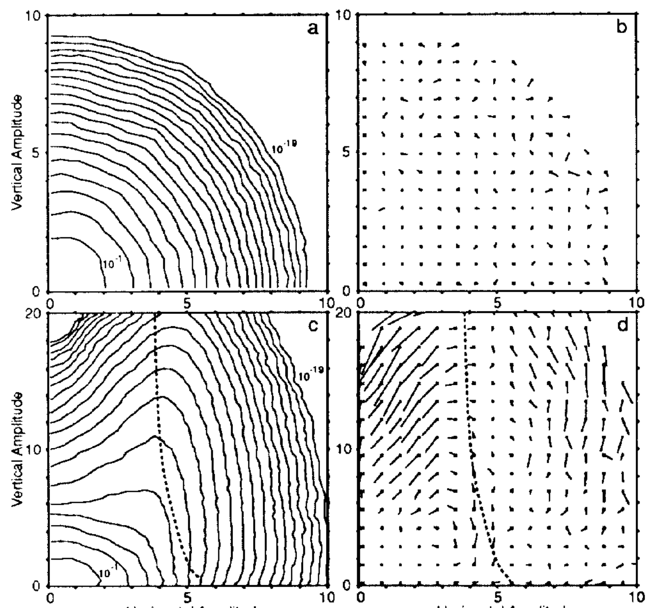


Figure 6: a) Resultant (particle density)/($A_x A_y$) and b) mass flow velocities, No BBI. c) & d) Same as a) & b) except $\xi_x = \xi_y = 0.04$ (Note the change of scale). The dashed line is the $4Q_x + 2Q_y = 4$ resonance. Other parameters: $Q_{xo} = 0.690$, $Q_{yo} = 0.609$, damping time = 1000 turns, $\sigma_{xo}/\sigma_{yo} = 33$.

BBI On

Figures 6c-d and 7 show the results when the BBI is turned on. Except for the BB tune shift parameters, which were $\xi_x = \xi_y = 0.04$, all other parameters were identical to those used to generate figures 6a&b. The BBI has clearly perturbed the resulting distribution and there is an obvious correlated flow of particles in the amplitude plane. Lifetimes are also much worse than the BBI off case at any given machine aperture.

At this set of tunes, the $4Q_x + 2Q_y = 4$ beam-beam driven coupling resonance was determined responsible for driving the particles out to such large vertical amplitudes. The location of this resonance in the amplitude space is shown as the dashed line in figures 6c&d.

Other locations in the tune plane were also explored. Other resonances and their effects on the resulting distribution tend to show the same characteristic features seen in figure 6c: a large increase in the vertical beam density at large vertical amplitudes and moderate horizontal amplitude. This suggest a simple measurement to verify this finding. A very small Be probe, "finger", as used in a previous experiment [7] could be inserted vertically into the beam. Its horizontal position could then be varied and the bremsstrahlung photons created by the interaction of the large amplitude particles with the probe could be counted. One should easily be able to measure the greater than nine order of magnitude difference between the density at amplitude $A_x = 0$, $A_y = 18$ and $A_x = 4.5$, $A_y = 18$ shown in figure 6c.

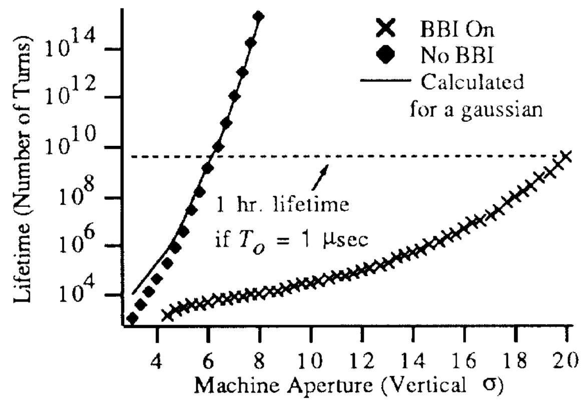


Figure 7: Lifetimes vs. vertical machine aperture for the distributions and flows of figures 6a-d

IV CONCLUDING REMARKS

This new method of sampling the amplitude space of the particles and using the resulting redistribution rules to determine the particle density in amplitude space has been shown to be capable of predicting beam lifetimes out to a large number of turns (> 1 hr.). It also has the added feature of being able to provide a clear picture of the particle dynamics within the amplitude plane through visualization of the resulting beam densities and particle flows. The method is still, however, in its infancy and many checks of its accuracy and limitations must still be performed. The technique must also be further developed to include the longitudinal motion of the particles since this added motion is known to have a significant influence of the particle dynamics.

Further exploitations of the technique should also be explored; it is by no means limited to the study of the BBI. It can, for instance, be used to determine the final particle distribution of an electron beam in the presence of machine nonlinearities or, with suitable modification, the particle distribution evolution of a hadron beam for similar conditions.

ACKNOWLEDGEMENTS

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