Digital Computer Simulation of Axisymmetric Electron Beams and Guns of any Energy

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Abstract

SPEED, a program for the simulation of relativistic (or non-relativistic) electron guns with either space-charge limited (Childs' Law), field emitted (Fowler-Nordheim) or specified injection emitters will be described and demonstrated with several examples. The program can be fully relativistic with induced and applied magnetic fields and crossing trajectories. Basic relaxation techniques are employed on a rectangular array of squares to keep the program small and efficient, permitting large arrays for accurate and/or elaborate simulations. Smooth electrodes are created by extending fields into electrodes, which are described by quadratic equations. It is written in FORTRAN 77 employing Calcomp-Versatec type plotting routines and can be run on virtually any computer from high end PC's (with DOS Extenders) to Cray's with only minor modifications. Excellent agreement has been obtained with both theoretical and experimental results.

I. HISTORY

The digital computer program **SPEED** (for Simulation Program for Energetic Electron Devices) was developed over a period of 30 years. The original version of the program was written at the University of Michigan, Ann Arbor starting in 1960. The next version was developed between 1967 and 1971 at Sandia National Laboratories, Albuquerque, for the simulation of high power flash X-ray diodes. There was little development of the code over the next 17 years, while the SNOW and SNOW3D codes were being developed for ion beam systems. With the development of the 80386 PC's the program was resurrected in 1988 and many improved features added.

II. THEORY

The code employs relaxation techniques while alternately computing trajectories and voltages. The voltages in the configuration are simulated on a rectangular array of squares. The space-charge densities are stored on an array identical to the Voltage array and are computed from representative trajectories from the cathode. It uses double precision (8 Byte - 64 bit) arithmetic for adequate precision.

The voltages are iteratively relaxed by solving Poisson's equation (developed in axisymmetric coordinates) at each point of the array,

$$\nabla^2 V = \frac{-\rho}{\epsilon_0} \tag{1}$$

where V is the voltage, ρ is the space charge density and ϵ_0 is the permittivity of free space, (all in SI units). The trajectories are computed using a solution of the relativistic Lagrangian

$$L = eV - e(A \cdot v) - m_0 c^2 \left(1 - \frac{v^2}{c^2}\right)^{\frac{1}{2}} \tag{2}$$

where e is the electronic charge, V the potential, $\mathbf{A}$ the magnetic vector potential, $\mathbf{v}$ the velocity, m_0 the rest mass, and c the velocity of light; or the non-relativistic Lorentz force equation

$$F = -e(E + v \times B) \tag{3}$$

where **F** is the force on the particle, **E** the electric field, and **B** the magnetic field; again in axisymmetric, cylindrical coordinates. The difference equations were developed some time ago [1] and will not be repeated here.

Current densities from the cathode can be computed using either Child's Law for space-charge limited emission,

$$J = 4\frac{\epsilon_0}{9}\sqrt{2\frac{e}{m}}\frac{V}{x^2} \tag{4}$$

or the Fowler-Nordheim equation for field emission,

$$J = 1.54\times10^{-6}\frac{E^2}{W}e^{\left(\frac{9.52}{\sqrt{W}}\right)}e^{-6.39\times10^{9}\frac{W^{1.57}}{E}} \tag{5}$$

where J is the current density, V the voltage at the distance x from the cathode, E the field at the cathode, and W the work function of the metal. A multiplying factor is available for the electric field (typically 2.0 to 3.5) to increase current to

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observed experimental values (field emitters are not all that smooth).

III. Development of the Program

The general flow chart for the digital computer program is shown in Fig. 1. The main divisions are seen to be:

1. The reading of data and setting of initial conditions (or restoring of earlier data).
2. The relaxation of the potentials.
3. The calculation of the trajectories.
4. A test for convergence or end of run with either a return to the relaxation or the output of the results.
5. The plotting, printing, and saving of results

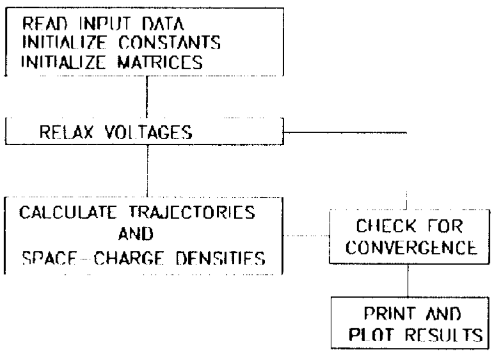

Figure 1. Basic Flow Chart for SPEED.

Depending on the complexity of the problem the voltage array can vary from 100 by 50 to 600 by 600 squares and is only limited by the memory available on the computer and the time available. Jobs as small as 100 by 50 often run in 10 minutes or less on a 25 Mhz PC, but a matrix 600 by 400 may require 10 hours or more. The program will require 200 to 1000 passes through the voltage matrix (depending on matrix size) on the first cycle, with reduction of the number of passes on succeeding cycles. 5 to 20 cycles through the program (depending on the importance of the space-charge densities) are usually needed to obtain convergence.

The electrodes are defined by quadratic equations which describe their surfaces. Potentials near and just inside the electrodes are set by extending potentials into them by using a quadratic fit to potentials just outside the electrodes. This creates smooth voltage distributions which are needed to compute trajectories acurately, and eliminates the need to relax partial matrix squares. Relaxation is carried out by iteritively passing up each matrix column from left to right and then backwards through the voltage matrix solving Poisson's equation. An intense over-relaxation is carried out on the first cycle through the program to speed the calculations. Full convergence of this calculation is neither desirable nor necessary for the first few cycles.

For the space-charge limited emission the voltage employed in the calculation is at a matrix point between 1 and 2 matrix increments in front of the cathode. It is important that this calculation be made as close to the cathode surface as possible as the variations in current density along the cathode can be masked by using a larger spacing. In the actual device the current density is determined a differential distance off the cathode. Going twenty percent of the distance to the anode, as seems to be popular with finite element codes, will almost certainly mask a sharp rise (or fall) in current density frequently found at the edge of a cathode. The electric field at the cathode surface, required for the field emission calculation, is computed from the derivative of a quadratic fit to the potentials just in front of the surface.

Each trajectory carries a current fixed at the cathode and stores the space-charge and current densities at adjacent matrix points along its path. The current distribution is used to compute the induced magnetic field for the relativistic calculations. Since each trajectory is computed independently of the others, crossing is not a problem. The smoothness of the space-charge distribution is a function of the number of trajectories that are computed. One trajectory per matrix square is the minimum requirement at the cathode, while 4 to 200 (for field emitters) per matrix square will produce the best results.

Under relaxation is not needed for low space-charge (non space-charge limited) problems. But damping, of space-charge densities and voltage relaxations, for space-charge limited problems is available if it is needed.

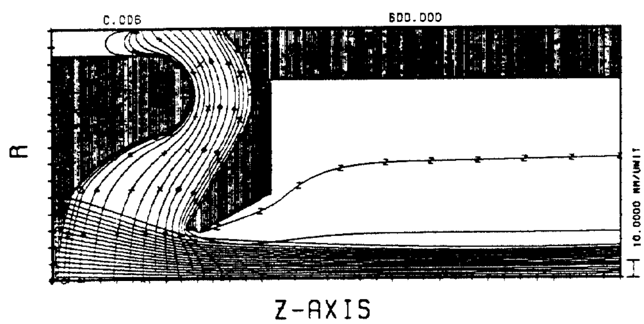

Figure 2. Beam and equipotentials for 600 kV, 500 A, relativistic beam.

IV. Examples

The first example, shown in Fig. 2, is a 600 Kv, 500 A relativistic electron gun designed for the Phermex accelerator

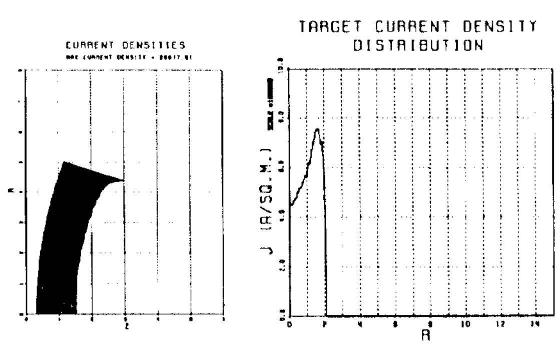

Figure 3. Current density distribution along cathode and at exit plane.

at Los Alamos National Laboratory. The matrix is 351 points long (and 151 wide) in order to extend the simulation past the beam's waist about 250 mm from the cathode. It has been the author's experience, that under most circumstances, it is the size and position (all of which is rolled into the Emittance) of the waist that is most desired by the beam designer. This data set, which consists of 38 lines, required about 360 minutes to run on the 25 MHz PC. The current densities along the cathode (with the sharp rise at the edge) and the current density distribution at the exit plane are shown in Fig 3. An emittance plot for the beam near the exit plane is shown in Fig. 4. The current and emittance are very close to the experimental values.

Figure 5 shows a typical Micro-field emitter with a radius of 100 nm. There is 100 volts on the extractor and 1 Kv on a target 250 μm from it leaving 112.5 volts at the right edge of the plot. A current of 5.5 μA is obtained in the beam with a field enhancement factor of 2.5 The current distribution from the cathode is shown in Fig. 6, where the length of the lines is proportional to current density.

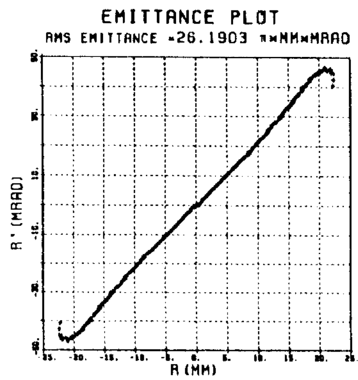

Figure 4. Emittance plot for beam at exit plane.

Nearly all the emission current is seen to come from the round tip with nano Amps to the extractor. Ten percent variations in the computed fields caused by the coarse mesh (and 5 matrix square radius) cause the variations in the current density.

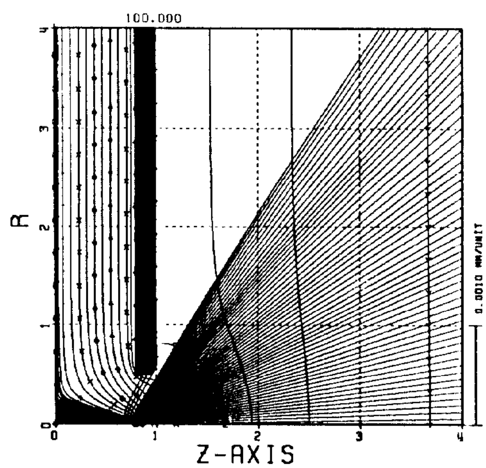

Figure 5. Micro-field emitter simulation.

The current density distribution at the exit plane of the plot is also shown in Fig. 6, where most of the current is seen to be going forward with only a fraction of a μA being lost to the sides. And finally the Emittance plot, Fig. 7, is shown at the target (.25 mm, 0.010 in. from the exit plane) after the acceleration (also calculated by the program) in a uniform field to 1 Kv.

A last example is for a focused version of the field emitter. Focusing would be desireable for some devices, such as cathode ray tubes, that need higher current densities. Here a second anode has been added after the extraction anode

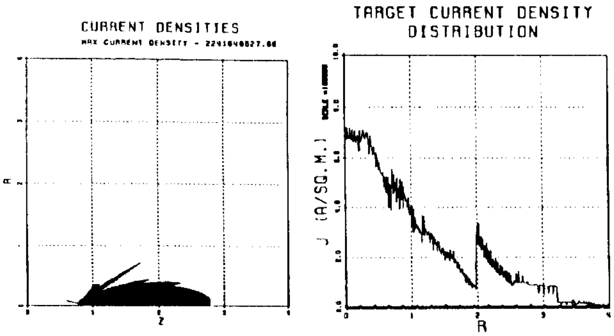

Figure 6. Emission current density, and exit plane current density, distributions.

forming an Einzel Lens. The emission has been restricted to a 20 degree (half angle) cone around the tip of the field emitter, if the same distribution of current from the cathode were allowed as in Fig. 5 about 90 percent of the electrons end up on the extraction anode. There might be some justification for this if one assumes there is some thermal heating of the tip (the current density is of the order of 10^{10} A/m^2) and most of the current comes from there. With these large assumptions the focusing of the beam can be seen in Fig. 8.

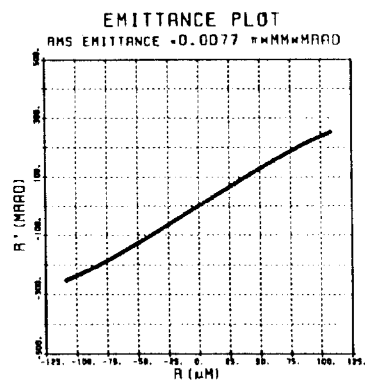

Figure 7. Emittance at target 250 μm from anode after 1 kV acceleration.

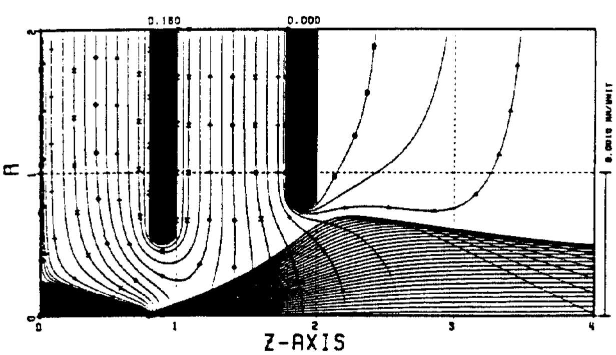

Figure 8. Trajectories and equipotential for a field emitter with restricted emission and focusing Einzel lens.

V. REFERENCES

[1] J.E. Boers, "Computer Simulations of Space-Charge Flows," University of Michigan, Ann Arbor, Mich., PhD Dissertation, 1968.